Non-Instantaneous Deterioration Inventory Model with Inflation and Stock-Dependent Demand

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ABSTRACT

In this paper, an inventory model for an item is presented with inflation and stock dependent demand under non-instantaneous deterioration without allowing shortages. In real life conditions, freshness and quality ofsome products can be maintaining Here in this inventory model we assume that some products maintain originality for some time. The necessary and sufficient conditions are used to find the optimal solutions and the corresponding maximum profits for the different value sets of the given numerical data with sensitivity analyses and presented graphically.

Keywords

Non-instantaneous deterioration, Inflation, Inventory, purchasing cost, Sales revenue cost, Stock-dependent demand.

1. INTRODUCTION

Inflation plays an important role in the optimal order policy and influences the demand of certain products. Now a days it is observed that in the supermarket, display of the consumer goods in large quantities attracts more customers and generates higher demand. As a result, the effect of inflation and stock dependent demand cannot be ignored for determining the optimal inventory policy. Effect of inflation and stock dependent demand is also well established in inventory problems. First**Buzacott (1975)**discussed the inflation subject to different types of pricing policies.**Gurnani (1983)** developed economic analysis of inventory systems.First**Gupta** and **Vrat (1986)**discussed the inventory models for stock-dependent consumption rate.

Hariga and M. Ben-daya (1996) presented an optimal time-varying lot sizing inventory models under inflationary conditions.**Abad (2003)**investigated the inventory models of this type of item. **Chang (2004)**discussed an inventory model by taking into account the inflation and finite time horizon with large quantity of purchase orders. **Yang (2004)**presented an inventory model with different pricing policies.**Jaggi et al. (2006)**developed an inventory model in which units are deteriorating at constant rate and demand rate is increasing exponentially due to inflation over a finite planning horizon using discount cash flow approach. **Soni and Shah (2008)** discussed the optimal ordering policy for an inventory model with stock-dependent demand under progressive payment scheme.

Most recently, **Chang et al. (2010)** developed an optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand.**Sana, S.S., (2012)** developed an EOQ model for perishable items with stock-dependent demand.

This paper deals with an inventory model for non-instantaneous deteriorating items with inflation and stock-dependent demand. Considering the realistic conditions, the problem of finding the optimal replenishment policy for non-instantaneous deteriorating items with inflation and stock-dependent demand is considered in this study. The necessary and sufficient conditions of the existence and uniqueness of the optimal solution are given. A numerical example, graphical illustration and sensitivity analysis are used to illustrate the model.

2. NOTATION AND ASSUMPTIONS

The following assumptions and notations are used in this paper:

$D(t) = a + bQ(t)$ Demand rate at time t, Where a, b are positive constants and Q(t) is the inventory level at time t.

t_1 Length of fresh product time

α Deterioration rate

A Ordering cost per order

C_h Inventory holding cost per unit time

C_p Purchasing cost per unit

C_d Deteriorating cost per unit

C_s Sales revenue cost per unit

r Discount rate, representing the time value of money

i Inflation rate

R Net discount rate of inflation; R = r-i

Q_1 Inventory level at time [0, t_1] in which the product has no deterioration.

Q_2 Inventory level at time [t_1, t_2] in which the product has deterioration.

TP Total present value of profit per unit time of inventory system.

3. MATHEMATICAL MODEL

The inventory levels are governed by the following differential equations:

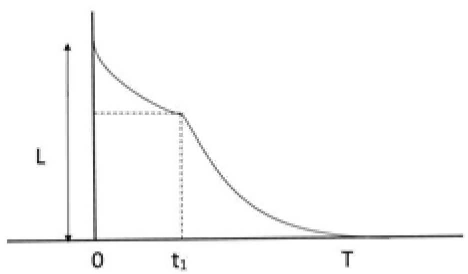

Fig 1: Inventory Model

$$\frac{dQ_1(t)}{dt} = -[a + b\, I_1(t)] \qquad 0 \le t \le t_1 \ldots (1)$$

$$\frac{dQ_2(t)}{dt} + \alpha\, I_2(t) = -[a + b\, I_2(t)] \quad t_1 \le t \le T \ldots (2)$$

withtheboundaryconditions $Q_1(0) = L, Q_2(T) = 0$ respectively. Solving these differential equations, we get the inventory level as follows:

$$Q_1(t) = \frac{a}{b}\left(e^{-bt} - 1\right) + Le^{-bt}, \qquad 0 \le t \le t_1 \quad \ldots (3)$$

$$Q_2(t) = \frac{a}{b+\alpha}\left(e^{(b+\alpha)(T-t)} - 1\right), \qquad t_1 \le t \le T \ldots (4)$$

Considering continuity of Q(t) at $t=t_1$, it follows from Equations (3) and (4) that $Q_1(t_1) = Q_2(t_1)$

$$\Rightarrow L = \frac{a}{b+\alpha}\left(e^{(b+\alpha)t_2} - 1\right)e^{bt_1} - \frac{a}{b}\left(1 - e^{bt_1}\right) \ldots (5)$$

Now the total present value of profit per cycle consists of the following elements:

1) Ordering cost per cycle is OC= A. …(6)

2)Holding cost per cycle is given by

$$HC = C_h \left(\int_0^{t_1} e^{-Rt}\, Q_1(t)dt + \int_{t_1}^{T} e^{-Rt}\, Q_2(t)dt \right)$$

$$= C_h\left[\frac{ae^{-Rt_1}(R + b - \mathrm{Re}^{-bt_1}) - ab}{bR(R+b)} + \frac{L}{R+b}\left(1 - e^{-(R+b)t_1}\right) + \frac{a}{b+\alpha}\left\{\frac{(b+\alpha)e^{-RT} + e^{-Rt_1}(\mathrm{Re}^{(b+\alpha)t_2} - R - b - \alpha)}{R(b+\alpha+R)}\right\}\right] \ldots (7)$$

3) Deterioration cost per cycle is given by

$$DC = C_d \int_{t_1}^{T} \alpha\, e^{-Rt}\, Q_2(t)dt$$

$$= \alpha\, C_d\left[\frac{a}{b+\alpha}\left\{\frac{(b+\alpha)e^{-RT} + e^{-Rt_1}(\mathrm{Re}^{(b+\alpha)t_2} - R - b - \alpha)}{R(b+\alpha+R)}\right\}\right] \ldots (8)$$

4) Purchasing cost per cycle is given by

$$PC = C_p \times L = C_p\left[\frac{a}{b+\alpha}\left(e^{(b+\alpha)t_2} - 1\right)e^{bt_1} - \frac{a}{b}\left(1 - e^{bt_1}\right)\right] \ldots (9)$$

5) Sales Revenue cost per cycle is given by

$$SRC = C_s \int_0^{T} e^{-Rt}\, D(t)dt$$

$$= C_s\left[\frac{a}{R}\left(1 - e^{-RT}\right) + \frac{ae^{-Rt_1}(R + b - \mathrm{Re}^{-bt_1}) - ab}{R(R+b)} + \frac{Lb}{R+b}\left(1 - e^{-(R+b)t_1}\right) + \left(\frac{ab}{b+\alpha}\right)\left(\frac{(b+\alpha)e^{-RT} + e^{-Rt_1}(\mathrm{Re}^{(b+\alpha)t_2} - R - b - \alpha)}{R(b+\alpha+R)}\right)\right] \ldots (10)$$

Thus the total present value of profit per cycle per unit time is given by

$$TP = \frac{1}{T}\left[SRC - OC - HC - DC - PC\right] \ldots (11)$$

Substituting equations (6–10) in the above equation (11), we get

$$TP = \frac{1}{T}\Bigg[C_s\left\{\frac{a}{R}\left(1 - e^{-RT}\right) + \frac{ae^{-Rt_1}(R + b - \mathrm{Re}^{-bt_1}) - ab}{R(R+b)} + \frac{Lb}{R+b}\left(1 - e^{-(R+b)t_1}\right) + \left(\frac{ab}{b+\alpha}\right)\left(\frac{(b+\alpha)e^{-RT} + e^{-Rt_1}(\mathrm{Re}^{(b+\alpha)t_2} - R - b - \alpha)}{R(b+\alpha+R)}\right)\right\}$$

$$- A - C_h\left\{\frac{ae^{-Rt_1}(R + b - \mathrm{Re}^{-bt_1}) - ab}{bR(R+b)} + \frac{L}{R+b}\left(1 - e^{-(R+b)t_1}\right) + \frac{a}{b+\alpha}\left(\frac{(b+\alpha)e^{-RT} + e^{-Rt_1}(\mathrm{Re}^{(b+\alpha)t_2} - R - b - \alpha)}{R(b+\alpha+R)}\right)\right\}$$

$$- \alpha\, C_d\left\{\frac{a}{b+\alpha}\left(\frac{(b+\alpha)e^{-RT} + e^{-Rt_1}(\mathrm{Re}^{(b+\alpha)t_2} - R - b - \alpha)}{R(b+\alpha+R)}\right)\right\}$$

$$- C_p\left\{\frac{a}{b+\alpha}\left(e^{(b+\alpha)t_2} - 1\right)e^{bt_1} - \frac{a}{b}\left(1 - e^{bt_1}\right)\right\}\Bigg] \ldots (12)$$

The total present value of profit per unit time is maximumif $\frac{dTP}{dt_2} = 0$ …. (13)

and $\frac{d^2TP}{dt_2^2} < 0$ …. (14)

4. SOLUTION ALGORITHM FOR PROPOSED MODEL

Step.1. Input A, C_h, C_p, C_s, C_d, α, R, a, b, t_1;

Step.2. From equation(13) compute t_2 and from Relation (12) compute TP;

Step.3. Put the value of t_2 in equation (14) to check the optimal solution. If satisfied then go to stop otherwise go to step 1 for changing the parameters values.

5. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

To illustrate the above results, we consider the following example: A=500, R=0.01, C_s= 30 per unit, Cp = 15 per unit, C_h= 0.50 per unit, C_d= 0.2 per unit, α=0.60,*a*= 100 and *b*=0.5 units. From Table 1, we observe that the system cost (TP) is Maximum when t_1=1/5 and t_2=1.76 (month).

As can be observed in the above study the sensitivity analysis of the parameters present in this inventory model, the total profit changes significantly with changes in the different demand values.

Table 1. Variation of demand 'a' according to t_2, L and TP

	Demand part*a*		
	100	110	120
t_2	1.76	1.71	1.67
L	618.2	638.4	657.8
TP	1253.7	1404.9	1556.7

If the demand rates (*a*) increase, then the lower time t_2, the longer order quantity (L) and total profit (TP) increase.

Table 2. Variation of demand 'b' according to t_2, L and TP

	Demand part *b*		
	0.50	0.49	0.48
t_2	1.76	1.63	1.54
L	618.2	520.4	454.6
TP	1253.7	1220.4	1191.2

If the demand rate (*b*) increases, then the longer time t_2, the longer order quantity (L) and total profit (TP) increases.

Table 3. Variation of Deterioration rate α according to t_2, L and TP

	Deterioration rateα		
	0.60	0.61	0.62
t_2	1.76	1.66	1.58
L	618.2	549.9	498.7
TP	1253.7	1231	1210.3

If deterioration rate (α)increases, then the time t_2, order quantity (L) and total profit (TP) decreases.

Table 4. Variation of Sales revenue cost C_s according to t_2, L and TP

	Sales revenue cost C_s		
	28	29	30
t_2	1.26	1.44	1.76
L	321.4	409.1	618.2
TP	889.1	1060.5	1253.7

If the sales revenue cost (C_s) increases, then the time t_2, order quantity (L) and total profit (TP) increases.

Table 5. Variation of Purchasing cost *Cp* according to t_2, L and TP

	Purchasing cost *Cp*		
	15	16	17
t_2	1.76	1.26	1.05
L	618.2	321.8	238.2
TP	1253.7	998.3	794.3

If purchasing cost (C_p) increase, then the time t_2, order quantity (L) and total profit (TP) decreases.

Table 6. Variation of Holding cost *Ch*according to t_2, L and TP

	Holding cost *Ch*		
	.40	.50	.60
t_2	1.87	1.76	1.67
L	705.3	618.2	554.6
TP	1277.6	1253.7	1234.1

If the holding cost (C_h)increase, then the time t_2, order quantity (L) and total profit (TP) decreases.

Table 7. Variation of Ordering cost *A* according to t_2, L and TP

	Ordering cost *A*		
	500	600	700
t_2	1.76	1.86	1.95
L	618.2	698.4	775.4
TP	1253.7	1204.0	1156.5

If the ordering cost (A) increases then it is quite natural that the total profit (TP) for this purpose decreases.

Table 8. Variation of Inflation rate *R* according to t_2, L and TP

	Inflation rate*R*		
	.010	.015	.020
t_2	1.76	1.69	1.62
L	618.2	563.5	519.3
TP	1253.7	1231.3	1210.4

If theinflation rate (R) increases then it is quite natural that the total profit (TP) for this purpose decreases.

The following graphs show the relation between total profit(TP) and time period t_1 and t_2.

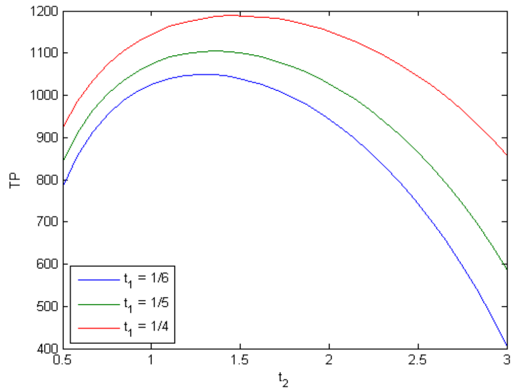

Fig 2: Total Profit TP v/s t_2 for different t_1 values

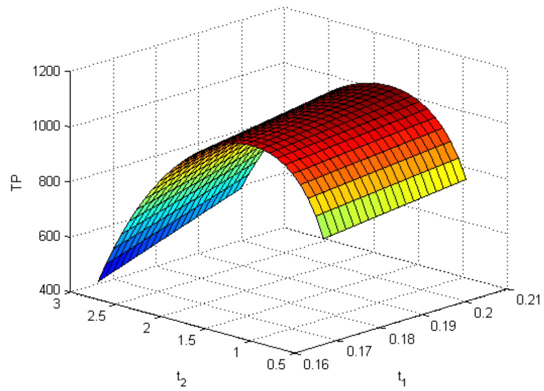

Fig 3: 3D view of Total Profit TP v/s t_2 for different t_1 values

6. CONCLUSION

This study introduces the concept of an inventory control system against the non-instantaneous deteriorating items withinflationwithout allowing shortages. Here in this paper we have used a numerical example to demonstrate sensitivity analysis and to get the optimal solution. A possible future research direction is the study of an inventory model for production rate, inflation, shortages, partial backlogging, two warehouse,permissible delay in payments, advance payment and supply-chain models etc.

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