Quotient Spaces Vs Quotient Mappings

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ABSTRACT

This paper is to study the impact of projection mappings on various types of quotient mappings between quotient spaces.

KEYWORDS

Quotient spaces, quotient maps, α-quotient maps, strongly α -quotient maps, α^* -quotient maps

1. INTRODUCTION

There are many situations in Topology where we build a topological space by starting with some spaces and doing some kind of `gluing' or `identification'. Quotient spaces that are extremely important allows us to construct interesting spaces and maps between them. The notion of quotient spaces allows us to glue simple spaces to create complicated spaces and define `spaces of things'. M.Lellis Thivagar [3] introduced the concepts of α-quotient mappings, strongly α -quotient mappings and α^*-quotient mappings between topological spaces. The purpose of this paper is to study the impact of projection mappings on various types of quotient mappings between quotient spaces.

2. PRELIMINARIES

Definition 2.1 [9]: Let(X,τ) be a topological space and let r be an equivalence relation on X so that X is divided into disjoint equivalence classes denoted by r(x) or [x] for any $x \in X$. Let X/r denote the family of equivalence classes. Then X/r is called the quotient set of X by r. Consider $p: X \to X/r$ given by $p(x) = r(x)$ for all $x \in X$. Since each $x \in X$ belongs to exactly one equivalence class, p is well defined and surjective. p is called the projection of X onto X/r. Let $\tau_p =$

$\{ U \subseteq X/r : p^{-1} (U) \in\tau \}$.Then τ_p is a topology on X/r, called the quotient topology and ($X/r,\tau_p$) is called the quotient space of X by the equivalence relation r.

Remark 2.2: If $U\subseteq X/r$, U is a collection of equivalence classes. If union of these equivalence classes belong to τ , then $U \in\tau_p$.

Remark 2.3: Let (X,τ) be a topological space and r be an equivalence relation on X. Consider the quotient space X/r of X by the equivalence relation r with the quotient topology $\tau_p = \{ U\subseteq X/r : p^{-1} (U) \in \tau \}$ where $p: (X,\tau) \to (X/r, \sigma)$ is a surjective map where σ is an arbitrary topology on X/r. Then $\tau_p \subseteq \sigma$, if p is an open map and $\sigma \subseteq \tau_p$, if p is continuous and hence is the strongest topology in X/r.

Theorem 2.4 [9]: [Quotient Theorem]

Let (x,τ) and (Y,σ) be topological spaces and r be an equivalence relation on X and r' be an equivalence relation on Y. Consider the quotient spaces X/r and Y/r' of X and Y respectively by r and r'. If f: $(X, \tau)\to (Y,\sigma)$ is continuous and relation preserving, then f : $(X/r,\tau_p)\to (Y/r', \sigma_q)$ given by

f ([x]) = [f(x)] is continuous.

Definition 2.5 [3]: A subset S of (X,τ) is called and α-set if $S \subseteq\tau (\mathrm{int}\tau(\mathrm{Cl}\tau(\mathrm{int}\, S)))$ and the family of all α-sets in (X, τ) is denoted by τ^{α} or $\alpha O(X)$.

Definition 2.6 [7]: A function f: $(X,\tau) \to (Y, \sigma)$ is called

i) α-continuous if the inverse image of each open set in Y is an α-set in X.

ii) α-open if the image of each open set in X is an α -set in Y.

iii) α-irresolute if the inverse image of every α -set in Y is an α -set in X.

Definition 2.7 [3]: A function f: $(X,\tau) \to (Y,\sigma)$ is strongly α-open if the image of every α-set in X in an α-set in Y.

Definition 2.8 [4]: Let f: $(X,\tau) \to (Y,\sigma)$ be a surjective map. Then f is said to be a quotient map if a subset U of (Y,σ) is open if and only if $f^{-1}(U)$ is open in (X, τ).

Definition 2.9 [3]: Let f: $(X,\tau) \to (Y,\sigma)$ be a surjective map. Then f is said to be

i) an α- quotient map if f is α-continuous and $f^{-1}(U)$ is open is open in X implies U is an α -set in Y.

ii) a strongly α -quotient map provided a subset U of Y is open in Y if and only if $f^{-1}(U)$ is an α -set in X.

iii) an α^* - quotient map if f is α-irresolute and f^{-1} (U) is an α-set in X implies U is open in Y.

3. QUOTIENT MAPPINGS VS QUOTIENT SPACES

Theorem 3.1: Let f: $(X,\tau) \to (Y,\sigma)$ be a relation preserving, quotient map. Then the mapping $f^* : (X/r, \tau_p) \to (Y/r', \sigma_q)$ given by $f^*(A) = [f(A)]$ for every $A \in X/r$ is also a quotient map.

Proof: The quotient topology τ_p on X/r is given by $\tau_p = \{U\subseteq X/r : p^{-1}(U) \in\tau\}$ where $p: X \to X/r$ is the projection mapping. Similarly, the quotient topology σ_q on Y/r' is given by $\sigma_q = \{V \subseteq Y/r' : q^{-1}(V) \in\sigma \}$ where $q: Y \to Y/r'$ is the projection mapping. Since f is relation preserving and continuous, f * is continuous. That is, whenever a subset $U \subseteq Y/r'$ is open,

$(f^*)^{-1}$ (U) is open in X/r. If $U \subseteq Y/r'$ such that $(f^*)^{-1}(U)$ is open in X/r, then $p^{-1}((f^*)^{-1}(U))$ is open in X. That is, $(f^*\circ p)^{-1}(U)$ is open in X.

$$\begin{tikzcd} X \arrow[r, "f"] \arrow[d, "p"'] & Y \arrow[d, "q"] \\ X/r \arrow[r, "f^*"'] & Y/r' \end{tikzcd} \qquad f^*\circ p = q\circ f$$

$(q o f)^{-1}(U)$ is open in X. That is, $f^{-1}(q^{-1}(U))$ is open in X and $q^{-1}(U) \subseteq Y$. Therefore $q^{-1}(U)$ is open in (Y, σ), since f is a quotient mapping. That is, $q^{-1}(U) \in \sigma$ and hence $U \in \sigma_q$. That is, U is open in Y/r'. Thus, a subset U is open in Y/r' if and only if $(f^*)^{-1}(U)$ is open in X/r. Therefore f * is a quotient map.

Example 3.2 : Consider the topological spaces (X,τ) and (Y,σ) given by X = {a,b,c,d}, τ ={ X, ϕ, {c}, {c,d} } and Y ={x,y,z} ; σ = { Y, ϕ, {x}, {y},{x,y}}. Let f: X $\to$ Y be defined as f(a) = z = f(b), f(c) = x, f(d) = y. f is surjective, but is not a quotient mapping, since {y} is open in Y but $f^{-1}(\{y\})$ = {d} is not open in X. Let X/r = { {a,b}, {c}, {d} } and Y/r' = { {x,y}, {z}}. Consider the projection mappings p: X $\to$ X/r given by p(a) = {a,b} = p(b) , p(c) ={c}, p(d) = {d} and q: Y $\to$ Y/r' given by q(x) = {x,y} = q(y), q(z) = {z}. Then, the quotient topology τ_p on X/r is given by τ_p = { X/r, ϕ, {{c}}, {{c}, {d}} } and the quotient topology σ_q on Y/r' is given by σ_q = { Y/r',ϕ , { {x,y}} }. Define f * : $(X/r, \tau_p) \to (Y/r', \sigma_q)$ as f * (A) = [f(A)]. That is, f * ({a,b}) = {z}, f *({c}) = {x ,y},

f *({d}) = {x, y}. f *is surjective and a subset U of Y/r' is open if and only if $(f^*)^{-1}(U)$ is open in X/r. Therefore, f * is a quotient map, even though f is not a quotient map but relation preserving. That is, the converse of the previous theorem is not true.

4. α - QUOTIENT MAPS VS QUOTIENT SPACES

Example 4.1: Let X = {a,b,c,d} with the topology τ = {X,ϕ, {a}, {a,b}, {a,c,d} }and Y = { x,y,z } with the topology σ =

{ Y,ϕ , {x} }. Then τ^α = {ϕ , X, {a}, {a,b}, {a,c}, {a,d}, {a,b,c}, {a,b,d}, {a,c,d} } and σ^α = { ϕ , Y, {x}, {x,y}, {x,z}}. Define f: X $\to$ Y as f(a) = x, f(b) = y, f(c) = f(d) = z. Then f is an α-quotient map. Let X/r = {a,c,d}, {b} } and Y/r' = { {x,z}, {y} }. The projection mappings p: X $\to$ X/r and q: Y $\to$ Y/r' are given by p(a) = p(c) = p(d) = {a,c,d} ; p(b) = {b} and q(x) = q(z) = {x,z}; q(y) = {y}. The quotient topology τ_p on X/r = {X/r,ϕ , {{a,c,d}} } and the quotient topology σ_q on Y/r' is {Y/r', ϕ}. Define f *: $(X/r, \tau p) \to (Y/r', \sigma q)$ as f *({a,c,d}) = [f({a,c,d})] = {x,z} and f * ({b}) = {y}. f * is α -continuous but $(f^*)^{-1}(V)$ is open in X/r for V = Y/r',ϕ , {x,z} of which {x,z} is not an α-set in Y/r'. Thus, f * is not an α-quotient map, even though f is α-quotient.

Remark 4.2: To overcome this hurdle, we impose the condition that the projection maps must be strongly α-open.

Theorem 4.3: If f: $(X,\tau) \to (Y,\sigma)$ is an α-quotient map that is relation preserving and if X/r and Y/r' are the quotient spaces by the equivalence relations r and r' respectively on X and Y, then the mapping f * : $(X/r, \tau_p)) \to (Y/r', \sigma_q)$ is also an α-quotient map, provided the projection mappings p: X$\to$ X/r and q: Y $\to$ Y/r' are strongly α-open.

Proof: Since f is α-quotient, f is α-continuous and $f^{-1}(U)$ is open in X implies U is an α-set in Y. Let V$\subseteq$ Y/r' be such that V is open in Y/r'. That is, V$\in\sigma_q$ and hence $q^{-1}(V) \in \sigma$. Since f is α-continuous, $f^{-1}(q^{-1}(V))$ is an α-se t in X. Therefore,

$(q o f)^{-1}(V)$ is an α-set in X. $(f^* o p)^{-1}(V)$ is an α-set in X. That is, $p^{-1}((f^*)^{-1}(V))$ is an α-set in X and $(f^*)^{-1}(V) \subseteq X/r$. Since p is strongly α-open, $p(p^{-1}(f^*)^{-1}(V))) = (f^*)^{-1}(V)$ is an α-set in X/r. Thus, $(f^*)^{-1}(V)$ is an α-set in X/r for every open set V in Y/r' and hence f* is α-continuous. Next, to prove that $(f^*)^{-1}(V)$ is open in X/r implies V is an α -set in Y/r'. Let V $\subseteq$ Y/r' be such that $(f^*)^{-1}(V)$ is open in X/r. $p^{-1}((f^*)^{-1}(V)) \in\tau$. That is, $(f^* o p)^{-1}(V) = (q o f)^{-1}(V) = f^{-1}(q^{-1}(V)) \in\tau$. That is,

$f^{-1}(q^{-1}(V))$ is open in X and $q^{-1}(V) \subseteq Y$. Therefore $q^{-1}(V)$ is an α-set in Y, since f is α-quotient. Since q is strongly α-open, $q(q^{-1}(V))$ is an α-set in Y/r'. That is, V is an α-set in Y/r'. Thus, f * is also an α-quotient map.

Example 4.4: Let X = {a, b, c, d} with τ = {X,ϕ , {a}, {b}, {a,b}, {a, c, d} } and Y = {x, y, z} with σ = { Y, ϕ , {x}, {x, y}, {x, z} }.Then τ^α = { X, ϕ, {a}, {b}, {a, b}, {a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d} } and $\sigma^\alpha = \sigma$. Let f : X $\to$ Y be defined as f(a) = y, f(b) = x, f(c) = f(d) = z. Then f is not α-continuous, since {x, z} is open in Y but $f^{-1}(\{x, z\})$ = {b, c, d} is not an α-set in X and hence f is not α-quotient. Consider the quotient sets on X and Y respectively given by X/r = { {a, b}, {c, d} } and Y/r' = { {x, y}, {z}} and the projection mappings given by p(a) = p(b) = { a,b}; p(c) = p(d) = {c,d} and q(x) = q(y) ={x,y}; q(z) ={z}. Then τ_p = { X/τ_p; $\sigma_q{}^\alpha = \sigma_q$. Define f* : $(X/r, \tau_p) \to (Y/r', \sigma_q)$ as f *({a,b}) = {x,y}; f * ({c,d}) = {z}. Then f * is α-continuous and

$(f^*)^{-1}(V)$ is open for V = Y/r', ϕ , {{x,y}} and all these V are α-sets in Y/r'. Therefore f * is α-quotient even though f is not α-quotient. Also we note that p and q are strongly α-open. That is, the converse of the theorem 4.1 is not true.

5. STRONGLY α -QUOTIENT MAPPINGS VS QUOTIENT SPACES

Example 5.1: Let X = {a,b,c,d} with τ = {X,ϕ , {a}, {a,d}} and Y = {x,y,z} with σ= {Y, ϕ, {x}, {x,y}, {x,z}}. Then τ^α = {X, ϕ, {a}, {a,b}, {a,c}, {a,d}, {a,b,c}, {a,b,d}, {a,c,d}} and $\sigma^\alpha = \sigma$. Define f: X $\to$ Y as f(a) = x, f(b) = z, f(c) = y, f(d) = z. Then U$\subseteq$ Y is open in Y if and only if $f^{-1}(U)$ is an α-set in X and hence f is strongly α-quotient. Let X/r = {{a,c},{b,d}} and Y/r' = {{x,y},{z}} with the projection mappings p: X $\to$ X/r given by p(a) = p(c) = {a,c}; p(b) = p(d) = {b,d} and q: Y$\to$ Y/r' given by q(x) = q(y) = {x,y}; q(z) = {z}. Then τ_p =

{X/r, ϕ} = $\tau_p{}^\alpha$ and σ_q = {Y/r', ϕ,{x,y}} = $\sigma_q{}^\alpha$. Define

f *: $(X/r, \tau_p) \to (Y/r', \sigma_q)$ as f * ({a,c}) = {x,y} and

f* ({b,d}) = {z}. Since $(f^*)^{-1}(\{x,y\})$ = {a,c} where {x,y} is an α-set in Y/r' but {a,c} is not open in X/r, f * is not strongly α-quotient. Thus, even though f: $(X,\tau) \to (Y,\sigma)$ is strongly α-quotient, f * is not strongly α-quotient.

Remark 5.2: To overcome the above hurdle, we include the conditions that the projection mapping p is α-irresolute and strongly α-open so that f * becomes strongly α-quotient.

Theorem 5.3: If f: $(X,\tau) \to (Y,\sigma)$ is relation preserving and strongly α- quotient and if X/r and Y/r' are the quotient spaces by the equivalence relations r and r' respectively on X and Y, then the mapping f * : $(X/r, \tau_p) \to (Y/r', \sigma_q)$ given by

f *(A) = [f(A)] for all A $\in$ X/r is also strongly α-quotient, provided the projection mapping p: X $\to$ X/r is α-irresolute and strongly α -open.

Proof: Since f is strongly α-quotient, a subset U of Y is open in Y if and only if $f^{-1}(U)$ is an α-set in X. If V$\subseteq$ Y/r' such that V is open in Y/r', then $q^{-1}(V)$ is open in Y. Since f is strongly

α-quotient, $f^{-1}(q^{-1}(V))$ is an α-set in X. That is, $(q \text{ of })^{-1}(V) = (f^* o\ p)^{-1}(V) = p^{-1}((f^*)^{-1}(V))$ is an α-set in X. Since p is strongly α-open, $p(p^{-1}((f^*)^{-1}(V)))$ is an α-set in X/r. Since p is surjective, $(f^*)^{-1}(V)$ is an α - set in X/r. Thus, V is open in Y/r' implies $(f^*)^{-1}(V)$ is an α-set in X/r. If $V \subseteq Y/r'$ such that $(f^*)^{-1}(V)$ is an α - set in X/r, since p is α-irresolute, $p^{-1}((f^*)^{-1}(V)) = (f^* o\ p)^{-1}(V)$ is an α-set in X. That is, $(q o\ f)^{-1}(V) = f^{-1}(q^{-1}(V))$ is an α-set in X. Since f is strongly α-quotient, $q^{-1}(V)$ is open in Y and hence V is open in Y/r' by the definition of quotient topology. Thus, $(f^*)^{-1}(V)$ is an α-set in X/r implies V is open in Y/r' and hence f* is strongly α-quotient.

Example 5.4: Let X = {a,b,c,d} with τ = {ϕ , X, {a}, {c}, {a,c},{c,d}, {a,c,d}, {b,c,d}} and Y = {x,y,z} with σ = {ϕ , Y, {x}, {z}, {x,y}, {x,z}}. Then τ^α= {ϕ , X, {a}, {c}, {a,c}, {b,c}, {c,d}, {a,b,c}, {a,c,d}, {b,c,d}} and $\sigma^\alpha = \sigma$. Consider f: X $\rightarrow$ Y given by f(a) = x, f(b) = f(d) = y, f(c) = z. f is not strongly α -quotient, since {x,y} is open in Y but f -1({x,y}) = {a,b,d} is not an α-set in X. If X/r = {{a,c}, {b,d}} and Y/r' = {{x,z}, {y}}, then the projection mappings p: X $\rightarrow$ X/r and q: Y $\rightarrow$ Y/r' are given by p(a) = p(c) = {a,c}; p(b) = p(d) = {b,d} and q(x) = q(z) = {x,z}; q(y) = {y}. The quotient topologies τp and σq respectively on X/r and Y/r' are given by τ_p = {ϕ, X/r, {{a,c}}} and σq = {ϕ ,Y/r', {{x,z}}} and $\tau_p = \tau_p{}^\alpha$; $\sigma_q = \sigma_q{}^\alpha$. Define f* : (X/r, τ_p) $\rightarrow$ (Y/r', σ_q) as f* ({a,c}) = [f({a,c})] = [{x,z}] = {x,z} , f *({b,d}) = {y}. Then f* is strongly α-quotient. Thus f* is strongly α-quotient, p is α-irresolute and strongly α-open but f is not strongly α-quotient. That is, the converse of the above theorem is not true.

Remark 5.5: It is known that any strongly α-quotient mapping is α - quotient but an α-quotient mapping is not necessarily strongly α-quotient. The following theorem gives a sufficient condition for an α-quotient mapping between quotient spaces to be strongly α-quotient.

Theorem 5.6: If f *: (X/r, τ_p) $\rightarrow$ (Y/r',σ_q) ia α-quotient and quasi α-open, then it is strongly α-quotient where X/r and Y/r' are quotient spaces of the topological spaces X and Y respectively by the equivalence relations r and r'.

Proof: Let V be open in Y/r'. Since f * is α- continuous,

$(f^*)^{-1}(V)$ is an α -set in X/r. Thus, for all open sets V in Y/r', $(f^*)^{-1}(V)$ is an α -set in X/r. Now, let $V \subseteq$ Y/r' be such that

$(f^*)^{-1}(V)$ is an α-set in X/r. Since f is quasi α-open,

(f *)((f *)-(V)) is open in Y/r'. That is, V is open in Y/r', as

f * is surjective. Thus, V is open in Y/r' whenever $(f^*)^{-1}(V)$ is an α -set in X/r. Hence, f * is strongly α-quotient.

6. α* -QUOTIENT MAPPINGS VS QUOTIENT SPACES

Example 6.1: Let (X,τ) , (Y, σ), f : X $\rightarrow$ Y and the quotient spaces (X/r, τ_p) and (Y/r', σ_q) be an in example 5.1. f is α*-quotient but the mapping f *: X/r $\rightarrow$Y/r' as in that example is not α*-quotient.

Remark 6.2: To overcome the above situation, we include the conditions that the projection mappings p and q are α-irresolute and p is strongly α- open.

Theorem 6.3: If f : (X, τ) $\rightarrow$ (Y,σ) is relation preserving and α*-quotient and if X/r and Y/r' are the quotient spaces by the equivalence relations r and r' respectively on X and Y, then the mapping f * : (X/r, τ_p) $\rightarrow$ (Y/r', σ_q) given by f *(A) = [f(A)] for all A$\in$ X/r is α*-quotient, provided the projection mappings p and q are α-irresolute and p is strongly α-open.

Proof: Since f is α*-quotient, f is α-irresolute and $f^{-1}(V)$ is an α-set in X implies V is open in Y. Let V $\subseteq$Y/r' be an α-set in Y/r'. Since q is α- irresolute, $q^{-1}(V)$ is an α-set in Y. Since f: X $\rightarrow$ Y is α-irresolute, $f^{-1}(q^{-1}(V))$ is an α-set in X. That is,$(qof)^{-1}(V) = (f^*op)^{-1}(V) = p^{-1}((f^*)^{-1}(V))$ is an α-set in X. Since p: X $\rightarrow$ X/r is strongly α-open, $p(p^{-1}((f^*)^{-1}(V)))$ is an α-set in X/r. Since p is surjective, $(f^*)^{-1}(V)$ is an α-set in X/r. Therefore, f * is α-irresolute. Let V$\subseteq$ Y/r' be such that $(f^*)^{-1}(V)$ is an α-set in X/r. Since p is α-irresolute, $p^{-1}((f^*)^{-1}(V))$ is an α-set in X. That is, $(f^*op)^{-1}(V) = (qo\ f)^{-1}(V)$ is an α-set in X. That is ,

$f^{-1}(q^{-1}(V))$ is an α-set in X. Since f is α*-quotient, $q^{-1}(V)$ is open in Y. Therefore, V is open in Y/r'. Thus, f * : (X/r, τ_p) $\rightarrow$ (Y/r', σ_q) is α*-quotient.

Example 6.4: Let X = {a,b,c,d} with the topology τ = {ϕ, {a}, {a,b}, {a,c,d} } and Y = {x,y,z} with the topology σ = { ϕ, Y, {x}}. Then τ^α = { ϕ , X, {a}, {a,b}, {a,c}, {a,d}, {a,b,c}, {a,b,d}, {a,c,d}} and σ^α = { ϕ , Y, {x}, {x,y},

{x,z}}. Consider the mapping f: X $\rightarrow$ Y given by f(a) = x,

f(b) = z, f(c) = f(d) = y. f is not α*-quotient. Consider the quotient spaces X/r ={{a}, {b}, {c,d}} and Y/r' = {{x}, {y}, {z}} with the projection mappings p: X $\rightarrow$ X/r given by p(a) = {a}, p(b) = {b}, p(c) = p(d) = {c,d} and q: Y $\rightarrow$ Y/r' given by q(x) = {x}, q(y) = {y}, q(z) = {z}. The quotient topologies on X/r and Y/r' are given by τ_p = {X/r, ϕ , {{a}}, {{a}, {b}},{{a}, {c,d}}} and σ_q = {Y/r',ϕ,{{x}}}. Also $\tau_p{}^\alpha = \tau_p$ and $\sigma_q{}^\alpha$ ={Y/r',ϕ ,{{x}}, {{x},{y}},{{x},{z}}}.Define the mapping f *: (X/r, τ_p) $\rightarrow$ (Y/r', σ_q) as f *({a}) = {x}; f * ({b}) = {z}; f *({c,d}) = {y}. f * is α*- quotient. Also p is strongly α-open and p, q are α-irresolute. Thus, we have shown with this example that the converse of the above theorem is not true.

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