Vanishing Moments of a Wavelet System and Feature Set in Face Detection Problem for Color Images

S. Patilkulkarni, PhD.
Associate Professor
Dept. of Electronics & Communication,
S J College of Engineering, Mysore, INDIA

H C Vijay Lakshmi
Dept. of Computer Science & Engineering,
S J College of Engineering, Mysore, INDIA

ABSTRACT
A systematic study of correlation between wavelet features and vanishing moments of wavelet system is considered for the problem of face detection in color images. It is concluded that the wavelet system with lower vanishing moments is better suited in problems such as face detection as higher vanishing moments in wavelet used actually implies redundant feature set.

General Terms
Image Processing, Face detection, Wavelet

Keywords
Feature extraction, threshold, wavelets, vanishing moments

1. INTRODUCTION
Face recognition and face detection in pattern recognition are very important and challenging problems. Correct classification of the pattern calls for considering the important dominant striking features of the face. In order to extract prominent features of the objects under consideration for classification, a good feature extraction procedure is essential. For detailed survey of face detection and recognition using different features proposed by different researchers refer [1], [2]. Wavelets coefficients based on only grayscale images do not account for variation in illumination and also fail to address the problem of representing different colors. When converted into grayscale, two different colors may get represented with the same pixel intensity in grayscale. Vanishing moments is an important property of wavelets which helps in extracting smoother features of the image. The role of vanishing moments of wavelet transforms in feature extraction with respect to face detection is explored in this paper. Very few researchers have explored the role of vanishing moments in their application, notable exceptions being [3-5].

2. WAVELETS AND VANISHING MOMENTS
The Discrete Wavelet Transform (DWT) has become a very versatile signal processing tool over the last decade. In fact, it has been effectively used in signal and image processing applications ever since Mallat [6] proposed the multiresolution representation of signals based on wavelet decomposition. Wavelets is widely used in many image processing applications such as image compression, image denoising, image enhancement, image recognition, feature detection, texture classification. The advantage of DWT over other traditional transformations is that it performs multiresolution analysis of signals with localization both in time and frequency. Orthogonal or biorthogonal wavelet transform has often been used in many image processing applications, because it makes possible multiresolution analysis and does not yield redundant information [6]. Wavelet transform is a representation of a signal in terms of a set of basis functions, which is obtained by dilation and translation of a basis wavelet. The advantage of DWT over other traditional transformations is that it performs multiresolution analysis of signals with localization both in time and frequency. The wavelet consists of two components, the scaling function which describes the low-pass filter for the wavelet transform, and the wavelet function which describes the band-pass filter for the transform. For orthogonal wavelets, the scaling function Φ and mother wavelet Ψ are given by the recursion relations.

$$\Phi(x) = \sqrt{2}\sum_k h_k \Phi(2x-k) \tag{1}$$

$$\Psi(x) = \sqrt{2}\sum_k g_k \Phi(2x-k) \tag{2}$$

Their scaled translates which form the set for n^{th} level resolution are denoted by

$$\Phi_k^n\,(x) = 2^{\frac{n}{2}}\Phi(2^n\,x-k) \tag{3}$$

$$\Psi_k^n\,(x) = 2^{\frac{n}{2}}\Psi(2^n\,x-k) \tag{4}$$

Wavelet transforms can be applied in a number of scientific research areas such as feature extraction, edge and corner detection, partial differential equation solving, transient detection, filter design, electrocardiogram (ECG) analysis, texture analysis, business information analysis and gait analysis. Transforms in image processing are two-dimensional. Discrete wavelet transform is calculated by applying the corresponding one-dimensional transform to the columns first, and then to the rows. When filtering, we have four possibilities

- low-pass filter to rows, followed by low-pass filter to columns (LLcoefficients)

- low-pass filter to rows, followed by high-pass filter to columns (HLcoefficients)
- high-pass filter to rows, followed by low-pass filter to columns (LHcoefficients)
- high-pass filter to rows, followed by high-pass filter to columns (HHcoefficients)

In wavelet decomposition, the image is split into an approximation and details images. Approximation image is obtained by low pass filtering and detail images are obtained by high pass filtering. Further decomposing the approximation image (LL1 subband), we will get second level LL2, HL2, LH2 and HH2 coefficients as shown in the Fig. 1a. The high pass or detail component characterizes the images' high frequency information and the low pass or approximation component contains its low frequency information.

Wavelet details contain high frequency components. The more high-frequency components are involved in an edge, the more abrupt the edge transition will be, which corresponds to a clearer edge. Joining multiresolution details in the edge reconstruction procedure will create an effect of enhancing edges. The given input image is wavelet decomposed, the detail coefficients capturing the edge information are considered, the approximation coefficient of the wavelet decomposed image is replaced with the sum of horizontal, vertical and diagonal (HL, LH, HH) detail coefficients of the image. These details contain the pixel intensity change in horizontal, vertical and diagonal directions capturing the important features of the image in all the three directions (Fig. 1b). Images containing different hues but same intensity values have the same pixel intensity value for different in the grayscale image. Color images when converted into grayscale image, may represent different color information with the same pixel intensity, resulting in missing actual image feature. In order to capture the features without losing any vital information, the given color image is converted in to HSV space, so that variation in hue, saturation and value which represent the image features are captured. The image in HSV color space is wavelet decomposed in all the three channels and using the horizontal, vertical and diagonal detail coefficients, the image features in all the three channels which account for variation in hue, saturation as well as intensity are extracted as feature set. An important property of a wavelet function is the number of vanishing moments, which describes the effect of the wavelet on various signals. The ability of a wavelet to suppress a polynomial depends on a crucial mathematical characteristic of the wavelet called its number of *vanishing moments*. A higher vanishing moment implies that more moments (quadratic, cubic, etc.) will be captured from the signal. The "moments" part comes from the fact that this is all equivalent to saying that the first *p* derivatives of the Fourier transform of the wavelet filter all are zero when evaluated at dc frequency. This is perfectly analogous to the probabilistic idea of a "moment generating function" of a random variable, which is basically the Fourier transform, and the *n*-th derivative evaluated at zero gives the *n*-th moment of the variable (i.e. the expected value, the expected value of the square, of the cube, etc.) So these Fourier transform derivative-zeros correspond to integrals back in the time/space domain that must be zero for the wavelet. For orthogonal and conventional biorthogonal wavelets vanishing moments associated with either the $\Phi(t)$ scaling function or the wavelet function $\Psi(t)$ alone determines this smoothness condition(reconstruction filter moments) [7][8]; whereas for the class of biorthogonal wavelet systems known as Coiflets, vanishing moments are equally distributed for both scaling $\Phi(t)$ and wavelet function $\Psi(t)$ [9]. In terms of filters, the m[th] moments of $\{h_k\}$, $\{g_k\}$ are

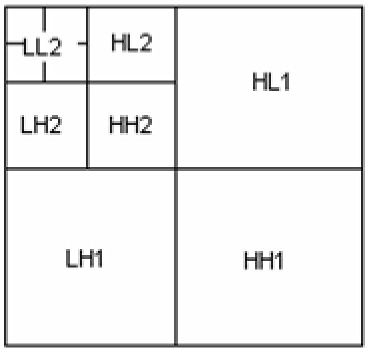

Fig 1a : Multiresolution structure of wavelet decomposition.

Fig 1b. Wavelet decomposition of a face image.

$$G_m = \sum_k k^m g_k \quad (5)$$

$$H_m = \sum_k k^m h_k \quad (6)$$

The "vanishing" part means that the wavelet coefficients are nonzero for polynomials of degree at most *p*−1. The analysis and synthesis wavelets can have different numbers of vanishing moments and regularity properties. One can use the wavelet with the greater number of vanishing moments for analysis resulting in a sparse representation, while we use the smoother wavelet for reconstruction. In theory, more vanishing moments means that scaling function can represent more complex signals accurately. In a way, p is a measure of the accuracy of the wavelet [7]. Biorthogonal wavelets feature a pair of scaling functions and associated scaling filters — one for analysis and one for synthesis. There is also a pair of wavelets and associated wavelet filters — one for analysis and one for synthesis. The analysis and synthesis wavelets can have different numbers of vanishing moments and regularity properties. The vanishing moments associated with the Bior family wavelets are represented as Nr and Nd. here Nr is the the number of vanishing moments associated with the reconstruction and Nd is the number of vanishing moments associated with the decomposition filters. In summary, more vanishing moments implies complex functions can be represented with a sparser set of wavelet coefficients.

3. FEATURE EXTRACTION

Feature extraction is a vital step in pattern recognition. Finding discriminative features and accurate classification is very important. Face detection is a classification problem and it calls for classifying the selected segmented region as face or non-face. For efficient classification a robust feature set is very much essential. This calls for an efficient feature extraction method. Feature extraction is the method of capturing information about the object in compact way, using these features one should be able to retrieve and hence recognize the object under consideration. Features should contain vital information about the object we are trying to locate, the feature extraction should be easy to compute and should be robust and compact enough to represent the object under consideration uniquely. The features extracted should well depict the human perception about the object. Image features can refer to global properties of an image i.e. average gray level, shape of intensity histogram etc. and local properties of an image such as edges, textures, important features uniquely representing the object and also shape of contours etc. From object recognition purpose these image features should be local, meaningful, detectable parts of an image. Meaningful features are associated to unique information in the image (e.g., like eyes, eyebrows, mouth and nose in a human face in the context of face detection problem). They should be invariant to some variations in the image formation process (i.e. invariance to viewpoint and illumination for images captured with digital cameras). These features extracted should be detectable, they can be located or detected from images via algorithms and features should robustly capture the salient features of the object. The features extracted should be represented using a feature vector with less complexity. Robust and compact feature set [11-14],[16] yield good detection rates. In this paper we have extracted important facial features of a face using different wavelets belonging to different wavelet family with increasing vanishing moments. The features are extracted as data points and edge data points after first level wavelet decomposition of image in HSV color space. For detailed survey of color spaces refer [15].

In the first feature extraction method used for the experimentation for this work, from here after to be referred as threshold based feature extraction method, the facial features are extracted by first converting the windows containing face image with only prominent facial features from RGB color space to HSV color space. The horizontal, vertical and diagonal detail coefficients for each of the three channels (H, S, V) are obtained using discrete wavelet transform. In the proposed experiment 'Haar', 'Bior1.3', 'Bior1.5' wavelet filters are used. The eye and mouth socket regions extracted in H, S, V channel are dilated in all the three channels using morphological operation to enhance the features and make it clearer. Only the horizontal, vertical and diagonal details in all the three channels are considered. The LL subband image i.e. approximation coefficients are replaced with the sum of horizontal, vertical and diagonal details as these detail coefficients capture all the important features of the face. The image is reconstructed with enhanced detail coefficients in each channel. The reconstructed image in HSV channel which predominantly contains eyebrows, eyes, nostrils and mouth regions as shown in 'Fig. 2b'is converted back to RGB color space and then to grayscale image and an image histogram is generated. In the end, using suitable hard intensity threshold, only the prominent facial features are extracted.

Following lowpass and highpass analysis filters of 'Haar', 'Bior1.3','Bior1.5', 'Coif1' and 'Coif2' given in equations (7) to (16) were used respectively.

$$\mathrm{HaarLp} = +\,0.7071 + 0.7071\mathrm{Z}^{-1} \quad (7)$$

$$\mathrm{Bior1.3Lp} = -0.08839\mathrm{Z}^{+2} + 0.08839\mathrm{Z}^{+1} + 0.7071 + 0.7071\mathrm{Z}^{-1} + 0.08839\mathrm{Z}^{-2} - 0.08839\mathrm{Z}^{-3} \quad (8)$$

$$\mathrm{Bior1.5Lp} = +0.01657\mathrm{Z}^{+4} - 0.01657\mathrm{Z}^{+3} - 0.1215\mathrm{Z}^{+2} + 0.1215\mathrm{Z}^{+1} + 0.7071\mathrm{Z}^{0} + 0.7071\mathrm{Z}^{-1} + 0.1215\mathrm{Z}^{-2} - 0.1215\mathrm{Z}^{-3} - 0.01657\mathrm{Z}^{-4} + 0.01657\mathrm{Z}^{-5} \quad (9)$$

$$\mathrm{Coif1Lp} = -0.07273\mathrm{Z}^{0} + 0.3379\mathrm{Z}^{-1} + 0.8526\mathrm{Z}^{-2} + 0.3849\mathrm{Z}^{-3} - 0.07273\mathrm{Z}^{-4} - 0.01566\mathrm{Z}^{-5} \quad (10)$$

$$\mathrm{Coif2Lp} = +0.01639\mathrm{Z}^{0} - 0.04146\mathrm{Z}^{-1} - 0.06737\mathrm{Z}^{-2} + 0.3861\mathrm{Z}^{-3} + 0.8127\mathrm{Z}^{-4} + 0417\mathrm{Z}^{-5} - 0.07649\mathrm{Z}^{-6} - 0.05943\mathrm{Z}^{-7} + 0.02368\mathrm{Z}^{-8} + 0.005611\mathrm{Z}^{-9} - 0.001823\mathrm{Z}^{-10} - 0.0007205\mathrm{Z}^{-11} \quad (11)$$

$$\mathrm{HaarHp} = -0.7071\mathrm{Z}^{0} + 0.7071\mathrm{Z}^{-1} \quad (12)$$

$$\mathrm{Bior1.3Hp} = -0.7071\mathrm{Z}^{0} + 0.7071\mathrm{Z}^{-1} \quad (13)$$

$$\mathrm{Bior1.5Hp} = -0.7071\mathrm{Z}^{0} + 0.7071\mathrm{Z}^{-1} \quad (14)$$

$$\mathrm{Coif1Hp} = +0.01566\mathrm{Z}^{+4} - 0.07273\mathrm{Z}^{+3} - 0.3849\mathrm{Z}^{+2} + 0.8528\mathrm{Z}^{+1} - 0.3379\mathrm{Z}^{0} - 0.07273\mathrm{Z}^{-1} \quad (15)$$

$$\mathrm{Coif2Hp} = +0.0007275Z^{+10} - 0.001823Z^{+9} - 0.005611Z^{+8} + 0.02368Z^{+7} + 0.05943Z^{+6} - 0.07649Z^{+5} - 0.417Z^{\mp 4} + 0.8127Z^{\mp 3} - 0.3861Z^{+2} - 0.06737Z^{+1} + 0.04146Z^{+0} + 0.01639Z^{-1} \quad (16)$$

In the second method of feature extraction, to be referred to as edge data point based feature extraction method, after wavelet decomposition of the HSV image and image reconstruction using the enhanced detail coefficients of each channel, the reconstructed image is converted to grayscale image and the edge data points are extracted using 'Sobel' edge detection algorithm [10],[17] to capture the points with sharp variation in pixel intensity values.

4. EXPERIMENTS AND RESULT ANALYSIS

The experiments are first conducted using 'Haar', 'Bior1.3', 'Bior1.5' and 'Bior2.2', 'Bior2.4' filters. Using threshold feature extraction method, it is found that using wavelets with higher order vanishing moment of the same family resulted in steady change in the number of feature points. When the experiment was repeated with 'Coiflet1', 'Coiflet2', 'db2' and 'db4' filters the number of feature points also tends to change steadily with higher order vanishing moments. Certain applications like face detection require only prominent features rather than minute details, where as applications like face recognition, object recognition requires every minute feature to distinguish one from the other. Lesser number of data points results in applications like object recognition may result in misclassification and more number of data points sometimes add more constraints in applications like object detection. After carefully analyzing and testing the features extracted using threshold and edge extraction method using wavelets belonging to different family with increasing vanishing moments, depending on the application, one should choose the wavelet filters from a given wavelet family with suitable vanishing moments. For face detection algorithms, features extracted using threshold with 'Bior1.3' is found to be suitable after thorough experimentation. However, for extracting edge as features of an object 'Haar' is suitable, as filters consider two immediately adjacent terms of the signal. The detail image is computed as their difference accounts for sharp change in pixel intensity value. In case of other wavelet filters, the edge may be smudged which is evident in the following figures. The features extracted using threshold and edge extraction with different wavelet family is captured in the Figure 3(a) to Figure 8(b).

Threshold based feature extraction method accounts for steady change in the number of data points with increase in vanishing moments when compared with edge extraction data points and is listed in Table1 and Table2. This is attributed to the filters of the scaling function. The number data points extracted using different wavelets of the same wavelet family using the threshold and edge as feature extraction procedure used in our algorithm is tabulated in Table-1.and Table-2.

Fig : 2(a) 2(b) 2(c) 2 (d) 2(e) 2(f)

Fig 2: (a)and (e). Face image; 2(b). Wavelet detail image (horizontal & vertical & diagonal) in HSV color space; 2(c) Converted wavelet detail image (horizontal & vertical &diagonal) in RGB color space; 2(d) and 2(f) Intensity threshold facial features.

Table 1. Threshold based features with different wavelet families

Haar	Bior1.3	Bior1.5		Bior2.2	Bior2.4		Coiflet1	Coiflet2		Db2	Db4
1066	1071	1080		476	546		724	592		691	610
808	845	876		617	617		736	719		790	832
743	791	800		308	331		526	351		412	294
749	769	787		374	390		573	392		368	329
884	901	917		490	509		709	604		698	509
479	493	512		366	378		473	406		387	431
937	994	1025		467	473		684	556		709	540
622	652	663		368	387		510	447		496	456
857	911	913		359	385		565	429		535	479
714	747	782		433	467		573	505		600	483
797	831	831		593	607		751	579		763	612
810	862	883		533	546		705	612		586	539

855	905	914		422	467		576	458		533	559
1105	1164	1195		513	560		728	654		771	594
947	987	997		578	600		840	673		758	598
952	1013	1023		664	701		848	623		742	579
939	995	1009		559	567		726	619		571	626
849	887	890		524	523		694	544		679	576
879	969	983		541	568		748	587		665	620

Table-2. Edges based features extraction comparison.

Edge Data points					
Haar	**Bior1.3**	**Bior1.5**		**Bior2.2**	**Bior2.4**
168	177	172		120	118
162	160	169		181	188
195	193	194		161	169
157	164	152		151	168
199	192	194		171	170
147	137	134		196	193
162	168	169		163	168
165	171	171		146	127
192	207	207		152	147
149	153	159		176	174
194	200	200		205	214
191	192	191		157	159
197	187	187		140	147
205	199	192		172	158
206	197	197		142	146
193	185	187		217	213
210	213	203		158	162
172	180	178		149	145
208	209	209		211	217

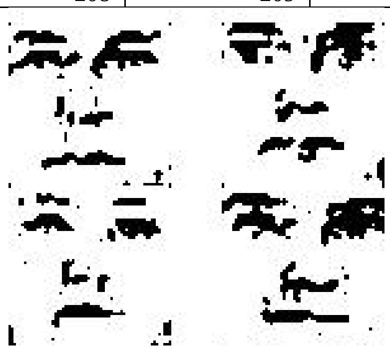

Fig 3(a): Haar Threshold Features

Fig 3(b): Haar Edge Features

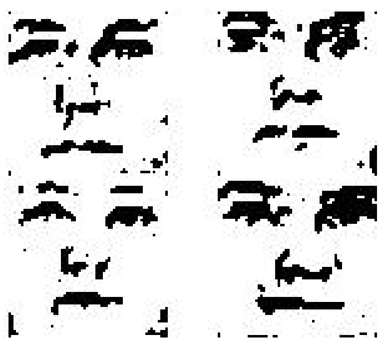

Fig 4(a): Bior1.3 Threshold Features

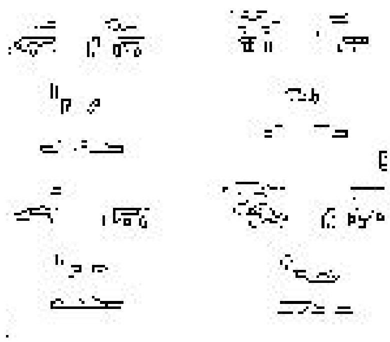

Fig 4(b): Bior1.3 Edge Features

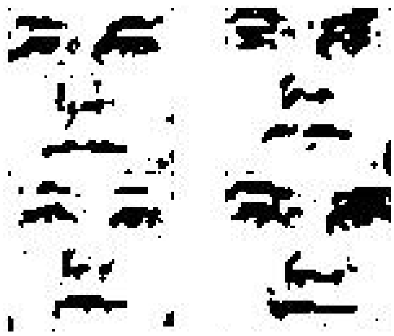

Fig 5(a): Bior1.5 Threshold Features

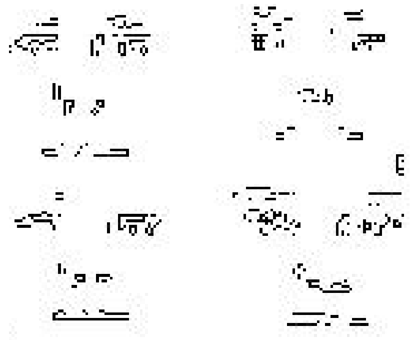

Fig 5(b): Bior1.5 Edge Features

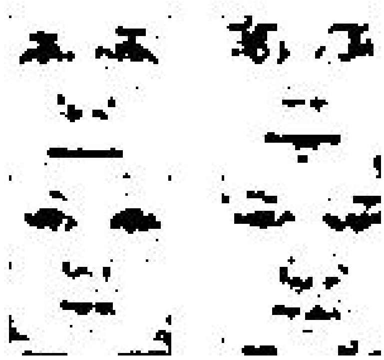

Fig 6(a): Coiflet1 Threshold Features

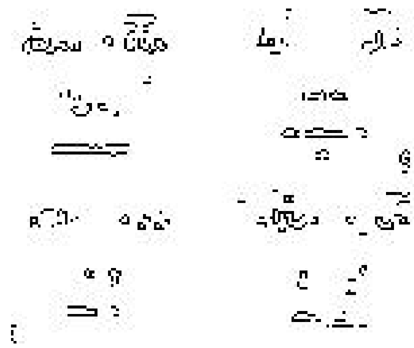

Fig 6(b): Coiflet1 Edge Features

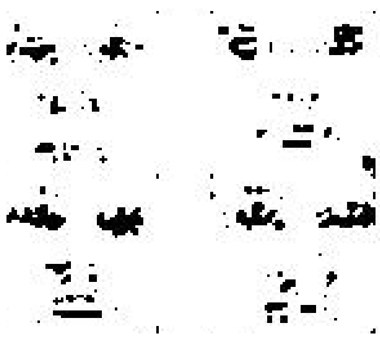

Fig 7(a): Coiflet2 Threshold Features

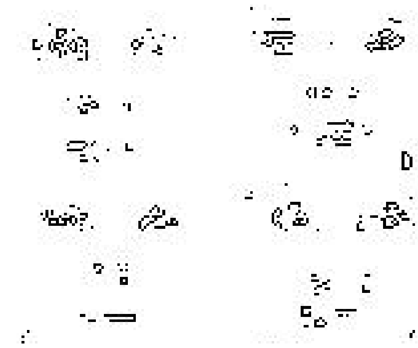

Fig 7(b): Coiflet2 Edge Features

Fig 8(a): Db2 Threshold Features

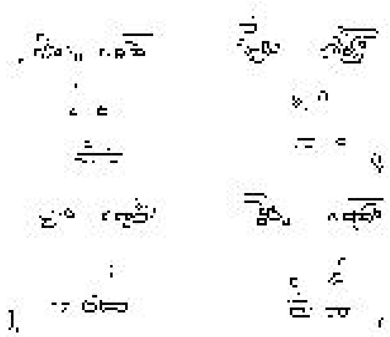

Fig 8(b): Db2 Edge Features

5. CONCLUSIONS

Notion of features depends on the problem context. For problems like face detection finer details of face are unimportant and hence one doesn’t buy much by choosing wavelet with higher vanishing moments. The experiments conducted here suggested that `Haar’ or `Bior 1.3’ wavelets yield suitable features. In fact, features obtained using wavelets with higher vanishing moments will act like `noise’ and in effect contribute to reduced performance in classification accuracy.

6. REFERENCES

[1] Y. Ming-Hsuan, D J Kreigman, and N Ahuja, Detecting Faces in images: a survey. IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 24, 2002, pp. 34-58.

[2] W. Zhao, R. Chellappa, A. Rosenfeld, and P. Phillips, Face recognition: A literature survey. ACM Computing Surveys, pp. 399-458, 2003.

[3] Al Jumah, Abdullah, Mohammed Gulam Ahamad, and Syed Amjad Ali. Denoising of Medical Images Using Multiwavelet Transforms and Various Thresholding Techniques. Journal of Signal and Information Processing 4 (2013), pp. 24-32.

[4] Li, Ren Chong, Yi Long You, and Feng Xiang You, Research of Image Processing Based on Lifting Wavelet Transform. Applied Mechanics and Materials 263 (2013), pp. 2502-2509.

[5] Kumar, Neeraj, and Amit Sethi. On image-driven choice of wavelet basis for super resolution. In Proc. Of Signal Processing and Communications (SPCOM), 2012 International Conference on. IEEE, 2012,Bangalore, India, pp. 1-4.

[6] S. Mallat, A Theory for Multiresolution Signal Decomposition: The Wavelet Representation. IEEE Trans. Pattern Analysis and Machine Intelligence, vol. 11, no. 7, 1989, pp. 674–693.

[7] Nguyen T., Strang G., Wavelets and Filter Banks. Wellesley-Cambridge Press, 1997.

[8] I. Daubechies, Ten Lectures on Wavelets. CBMS-NSF Lecture Notes nr. 61, SIAM, 1992.

[9] Tian J., Wells R. O. Jr, Vanishing Moments and Biorthogonal Systems.

[10] R. C. Gonzalez, R. E. Woods Digital Image processing. Second edition Prentice Hall India.

[11] P. Viola and M. Jones, Robust real time object detection. International Journal of Computer Vision, vol. 57, no. 2, pp. 137-154, 2001.

[12] Rein-Lein Hsu, Mohamed-Abdel Mottaleb, Anil K jain Face Detection in Color Images. IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 42, Issue 5, pp.696-706.

[13] C. Garcia, G. Zikos, G. Tziritas Face Detection in Color Images using Wavelet Packet Analysis**.** In Proceedings of IEEE International Conference on Multimedia Computing and Systems, Vol.1, 1999, pp. 703 – 708.

[14] Mohamed A. Berbar Hamdy M. Kelash, and Amany A. Kandeel, Faces and facial features detection in color images. In. Proc. Geometric Modeling and Imaging― New Trends (GMAI'06), 2006, pp 209 – 214.

[15] V. Veznevets, V Sazonov, and A Andreeva, A survey on pixel-based skin color detection techniques. In Proceedings Graphicon-2003 pp. 85-92.

[16] H.C. Vijaylakshmi, S. PatilKulkarni, Segmentation Algorithm for Multiple Face Detection in Color Images With Skin Tone Regions Using Color Spaces and Edge Detection Techniques. International Journal of Computer Theory and Engineering (IJCTE), Vol. 2, No. 4, pp. 552-558. Aug-2010.

[17] Raman Maini & Dr. Himanshu Aggarwal Study and comparison of various image edge Detection techniques. International Journal of Image Processing (IJIP) , Volume 3 Issue (1), pp.1-11, Jan-Feb 2009.