Stochastic Analysis of a System with Preventive Maintenance and Common Cause Failure

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ABSTRACT
This paper Study the reliability and availability characteristics of the system with PM and CCF. The failure times , replacement times, PM times and CCF times of a components are assumed to be exponentially distributed. We derive the mean time to failure(MTTF) and the steady state availability( ) in this system. Some Special cases have been studies theoretically and graphically to observe the effect of the preventive maintenance (PM) and Common Cause Failure (CCF) on system performance. Certain important results have been derived as special cases.

Keywords
Mean Time to System Failure (MTSF), Steady-state availability, Preventive Maintenance (PM) and the Kolmogorov’s forward equations method, Common Cause Failure (CCF).

1. INTRODUCTION
Several authors [1, 3, 4, 5, 7, 8] studied the reliability analysis of different behaviors' systems. Goel & Shrivastava [6] studied comparison reliability characteristics of two systems with bivariate exponential lifetimes. Sing and Rawal [9] studied the availability analysis of series system with human failure under different repair policies. Researchers in reliability have shown a keen interest in the analysis of two (or more) component parallel systems owing to their practical utility in modern industrial and technological set-ups. In these systems, more commonly used are those in which the failure in one component affects the failure rates of other components. For example we can consider engine failure in two engine planes, wear of two pens on an executive's desk or the performance of individual's eyes, ears, kidneys and other paired physical organs. A CCF is defined as the failure of single unit or multiple units due to a single common-cause. Some of the CCF may occur due to the following reasons.

1- Wrong designing of equipment during design phase.

2- Improper maintenance of machines by workers.

3- Natural catastrophe likes flood, earthquake, fire… etc.

4- High temperature of computer chips.

The purpose of this study is to study the system with two dissimilar components arranged in parallel with Preventive Maintenance (PM) and Common Cause Failure (CCF). We analyzed the system by using Kolmogorov,s forward equation method.

We derived measures of system effectiveness like MTSF and the steady-state availability . Graphical Studies of effect the Preventive Maintenance (PM) and Common Cause Failure (CCF) on the measures mentioned above are also given.

2. Material and Method
In this study, the system is analyzed by using Kolmogorov , s equations method. Various measures of system effectiveness such mean time to system failure MTSF and Steady State Availability in this system has been obtained.

α_1	The failure rate of component A
β_1	The failure rate of component B
α_2	The failure rate of component A when B has already failed
β_2	The failure rate of component B when A has already failed
γ	The replacement rate of component A
δ	The replacement rate of component B
θ	The replacement rate of component A , B
λ	The rate of time for taking a unit into preventive maintenance
μ	The preventive maintenance rate
η	The CCF of unit A.
τ	The CCF of unit B.
$P_i(t)$	Probability the components are working at time $t, \ (t \geq 0)$ at state S_i
A_N	Component A in normal mode and operative
B_N	Component B in normal mode and operative
A_F	Component A in failure mode and needs

	replacement
B_F	Component B in failure mode and needs replacement
A_{NP}	Component A in normal mode and under preventive maintenance
B_{Np}	Component B in normal mode and under preventive maintenance
A_{CCF}	Component A under CCF
B_{CCF}	Component B under CCF

Assumptions

1-The system consists of a single unit having two dissimilar parallel

Components, say A and B.

2- The system remains operative even if a single component operates.

3- The failure of a component changes the life time parameter of the other.

4- Each component can be replaced with a similar component with both the Components (when failed) can also be replaced simultaneously

5- After replacement of each component, the system is as good as new.

6- CCF and other failures are statistically independent.

7- CCF, other failure, replacement and preventive maintenance rates are constant.

8- Preventive maintenance (e.g. overhaul, inspection, minor repairs, etc.)

A unit can be in one of the following states at time t is

Up States :

$S_0 = (A_N, B_N)$, $S_1 = (A_F, B_N)$, $S_2 = (A_N, B_F)$,
$S_4 = (A_{Np}, B_{Np})$

Down States : $S_4 = (A_F, B_F)$, $S_5 = (A_{CCF}, B_{CCF})$

Table (1):- Transition rates

	S_0	S_1	S_2	S_3	S_4	S_5
S_0		α_1	β_1	λ		η
S_1	γ				β_2	
S_2	δ				α_2	
S_3	μ					
S_4	θ					
S_5	τ					

2. Mean time to system failure

From table (1) above, Let $P_i(t)$ to be the Probability that the system at time t $(t \geq 0)$ is in state S_i. Let $P(t)$ be the probability row vector at time t, we have the following initial condition.

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4, P_5(t)] = [1, 0, 0, 0, 0, 0]$$

We obtain the following differential equations

$$P_0'(t) = -(\alpha_1 + \beta_1 + \lambda + \eta) P_0(t) + \gamma P_1(t) + \delta P_2(t) + \mu P_3(t) + \theta P_4(t) + \tau P_5(t),$$

$$P_1'(t) = \alpha_1 P_0(t) - (\gamma + \beta_2) P_1(t),$$

$$P_2' = \beta_1 P_0(t) - (\delta + \alpha_2) P_2(t),$$

$$P_3'(t) = \lambda P_0(t) - \mu P_3(t),$$

$$P_4'(t) = \beta_2 P_1(t) + \alpha_2 P_2(t) - \theta P_4(t),$$

$$P_5'(t) = \eta P_0(t) - \tau P_5(t).$$

(1)

The above equations can be written in the matrix form as

$$\dot{P} = QP,$$

(2)

Where,

$$Q = \begin{pmatrix} -(\alpha_1+\beta_1+\lambda+\eta) & \gamma & \delta & \mu & \theta & \tau \\ \alpha_1 & -(\gamma+\beta_2) & 0 & 0 & 0 & 0 \\ \beta_1 & 0 & -(\delta+\alpha_2) & 0 & 0 & 0 \\ \lambda & 0 & 0 & -\mu & 0 & 0 \\ 0 & \beta_2 & \alpha_2 & 0 & -\theta & 0 \\ \eta & 0 & 0 & 0 & 0 & -\tau \end{pmatrix}$$

It is difficult to evaluate the transient solutions hence we delete the rows and columns of absorbing state of matrix Q and take the transpose to produce a new matrix, say A**.**

The expected time to reach an absorbing state is obtained from

$$MTSF = P(0)(-A^{-1})\begin{pmatrix}1\\1\\1\\1\end{pmatrix} \qquad (3)$$

Where,

$$A = \begin{pmatrix} -(\alpha_1+\beta_1+\lambda+\eta) & \alpha_1 & \beta_1 & \lambda \\ \gamma & -(\gamma+\beta_2) & 0 & 0 \\ \delta & 0 & -(\delta+\alpha_2) & 0 \\ \mu & 0 & 0 & -\mu \end{pmatrix}$$

We obtain the MTSF of the form

$$MTSF = \frac{L+\mu[K+Z]}{\mu[\beta_2 K + G]} \qquad (4)$$

Where,

$$K = \alpha_1(\alpha_2+\delta),$$
$$Z = (\beta_2+\gamma)(\alpha_2+\beta_1+\delta),$$
$$L = \lambda(\beta_2+\gamma)(\alpha_2+\delta),$$
$$G = (\beta_2+\gamma)\{\delta\eta + \alpha_2(\beta_1+\eta)\}$$

4. Availability analysis

For the availability case of table (1), using the same initial conditions

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0)] = [1,0,0,0,0,0]$$

The differential equations above can be expressed as

$$\begin{pmatrix} P_0'(t)\\ P_1'(t)\\ P_2'(t)\\ P_3'(t)\\ P_4'(t)\\ P_5'(t)\end{pmatrix} = \begin{pmatrix} -(\alpha_1+\beta_1+\lambda+\eta) & \gamma & \delta & \mu & \theta & \tau \\ \alpha_1 & -(\gamma+\beta_2) & 0 & 0 & 0 & 0 \\ \beta_1 & 0 & -(\delta+\alpha_2) & 0 & 0 & 0 \\ \lambda & 0 & 0 & -\mu & 0 & 0 \\ 0 & \beta_2 & \alpha_2 & 0 & -\theta & 0 \\ \eta & 0 & 0 & 0 & 0 & -\tau \end{pmatrix} \begin{pmatrix} P_0(t)\\ P_1(t)\\ P_2(t)\\ P_3(t)\\ P_4(t)\\ P_5(t)\end{pmatrix}$$

The system availability can be obtained from the solutions for $P_i(t)$, i =0,1,2,…5.The states 0,1,2,3 in table(1) are the only up sates of the system. The steady-state availability is given by

$$A(\infty) = P_0 + P_1 + P_2 + P_3 = 1 - [P_4(\infty) + P_5(\infty)] \qquad (5)$$

In the steady state, the derivatives of the state probabilities become zero so that

$$Q\,P(\infty) = 0, \qquad (6)$$

Which in matrix form

$$\begin{pmatrix} -(\alpha_1+\beta_1+\lambda+\eta) & \gamma & \delta & \mu & \theta & \tau \\ \alpha_1 & -(\gamma+\beta_2) & 0 & 0 & 0 & 0 \\ \beta_1 & 0 & -(\delta+\alpha_2) & 0 & 0 & 0 \\ \lambda & 0 & 0 & -\mu & 0 & 0 \\ 0 & \beta_2 & \alpha_2 & 0 & -\theta & 0 \\ \eta & 0 & 0 & 0 & 0 & -\tau \end{pmatrix} \begin{pmatrix} P_0(\infty)\\ P_1(\infty)\\ P_2(\infty)\\ P_3(\infty)\\ P_4(\infty)\\ P_5(\infty)\end{pmatrix} = \begin{pmatrix}0\\0\\0\\0\\0\\0\end{pmatrix}$$

The following normalizing condition

$$P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) + P_5(\infty) = 1 \quad (7)$$

We substitute (7) in any of the redundant rows in (6) to give

$$\begin{pmatrix} -(\alpha_1+\beta_1+\lambda+\eta) & \gamma & \delta & \mu & \theta & \tau \\ \alpha_1 & -(\gamma+\beta_2) & 0 & 0 & 0 & 0 \\ \beta_1 & 0 & -(\delta+\alpha_2) & 0 & 0 & 0 \\ \lambda & 0 & 0 & -\mu & 0 & 0 \\ 0 & \beta_2 & \alpha_2 & 0 & -\theta & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

The system of equations in matrix above to obtain the steady-state probabilities

$P_4(\infty), P_5(\infty)$

The steady state availability is given by

$$A(\infty) = \frac{\theta\tau(L+\mu(K+Z)}{\frac{\eta\theta\mu L}{\lambda} + [\theta L + \tau\{K(\beta_2+\theta) + \mu\alpha_2\beta_1(\beta_2+\gamma) + \mu\theta Z\}]} \quad (8)$$

5. Special Cases

5.1 Study the system without preventive maintenance

The mean time to system failure is given by

$$\mathrm{MTSF}_1 = \frac{K+Z}{\beta_2 K + G}$$

The steady state availability is given by

$$A_1(\infty) = \frac{\tau\theta(K+Z)}{\theta\eta\frac{L}{\lambda} + (\beta_2+\theta)K + \tau\alpha_2\beta_1(\beta_2+\gamma) + \tau\theta Z}$$

5.2 Study the system without Common Cause Failure

The results of Ref. [2] are derived.

5.3 Study the system without preventive maintenance and Common Cause Failure

The results of Ref. [2] are derived.

6. Graphical Study

The purpose of this section studies the effect of (PM) on the system. The following numerical results are obtained by considering the following system parameters:- We fix $\beta_1 = 0.2, \beta_2 = 0.3, \eta = 0.05,\ \delta = 0.3, \gamma = 0.4, \theta = 0.4, \tau = 0.05$

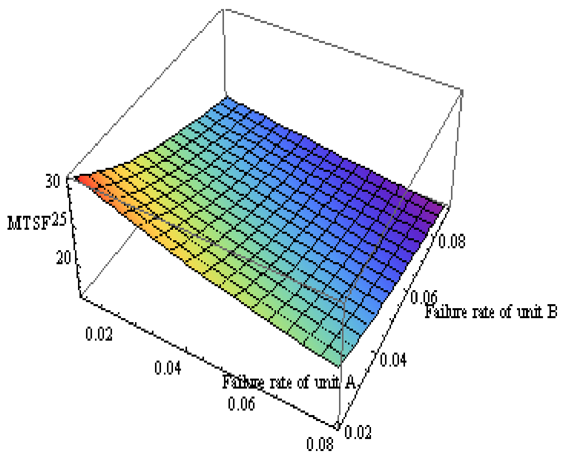
Table (2) :- The mean time to system failure MTSF by using three cases of PM.

α_1	α_2	Case 1 MTSF when $\lambda = 0.3 < \mu = 0.7$	Case 2 MTSF when $\lambda = \mu = 0.7$	Case 3 MTSF when $\lambda = 0.7 > \mu = 0.3$
0.01	0.02	30.96	39.52	59.84
0.02	0.03	26.88	34.33	51.70
0.03	0.04	23.84	33.46	45.89
0.04	0.05	21.49	27.46	41.39
0.05	0.06	19.62	25.08	37.80
0.06	0.07	18.10	23.13	34.87
0.07	0.08	16.83	21.51	32.43
0.08	0.09	15.76	20.14	30.36

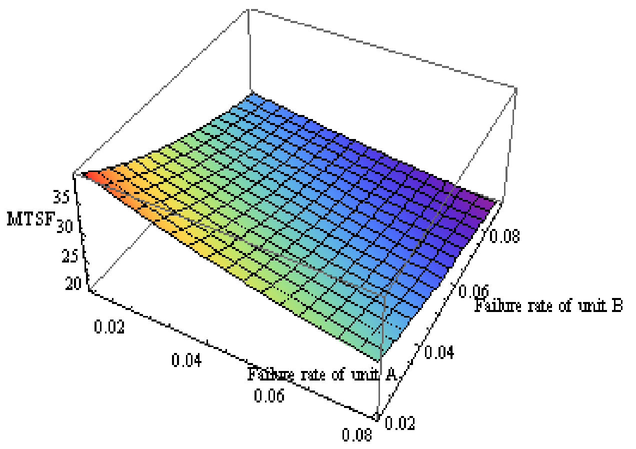
Table (3):- The steady-state availability A (∞) by using three cases of PM.

α_1	α_2	Case 1 A(∞) when $\lambda = 0.3 < \mu = 0.7$	Case 2 A(∞) when $\lambda = \mu = 0.7$	Case 3 A(∞) when $\lambda = 0.7 > \mu = 0.3$
0.01	0.02	0.664944	0.716954	0.792213
0.02	0.03	0.659152	0.711771	0.7881
0.03	0.04	0.653731	0.706892	0.784197
0.04	0.05	0.648649	0.70229	0.780488
0.05	0.06	0.643878	0.697944	0.776956
0.06	0.07	0.639394	0.693834	0.773588
0.07	0.08	0.635175	0.689941	0.770373
0.08	0.09	0.6312	0.68625	0.767298

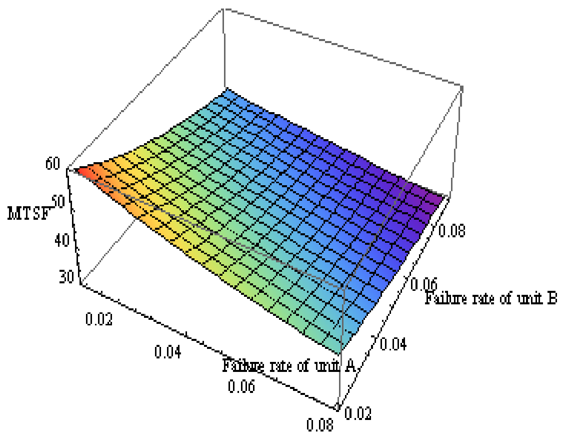
Case1 (Fig. 1)



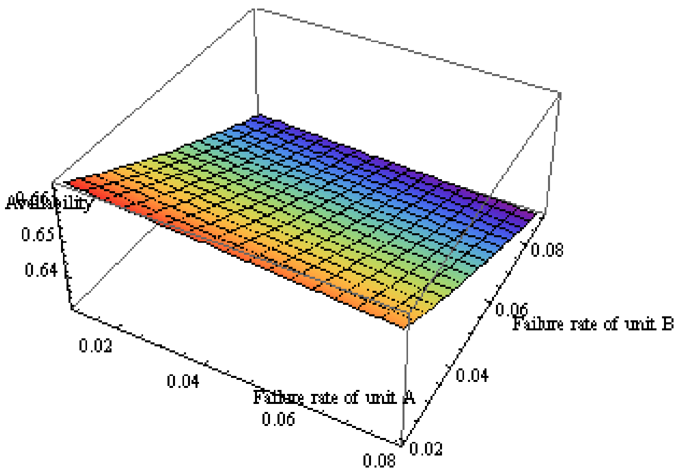
Case2 (Fig. 2)



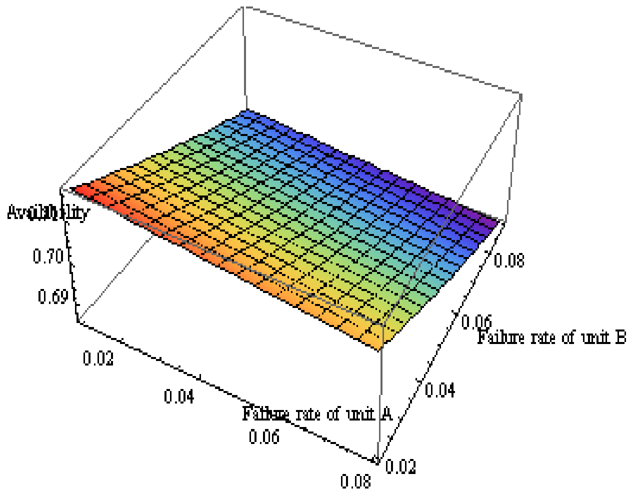
Case3 (Fig. 3)



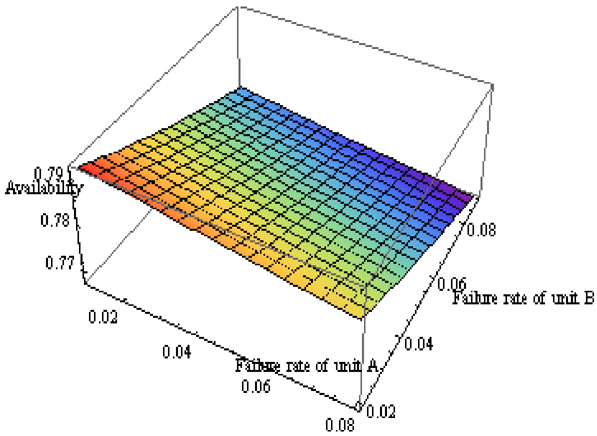
Case 1 (Fig. 4)



Case2 (Fig.5)



Case3 (Fig.6)



3. CONCLUSION

We used computer software, to compare MTSF under effect of (PM). This result show that

MTSF in (Case1) < MTSF in (Case2) < MTSF in (Case3) and

A (∞) in (Case1) < A (∞) in (Case2) < A (∞) in (Case3)

We conclude that the system after adding of preventive maintenance (PM). The system has the high reliability, in (case3).

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6. REFERENCES

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