Superior Multibrots for Multicorns for Negative Values

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ABSTRACT

The Multibrots for Multicorns is given by the function $A_c = \bar{z}^d + c; d \geq 2.$ where d and c are constant. The given function is modification of the classic Mandelbrot and Julia sets. The Multibrots for multicorn function has beautiful shapes with symmetry, along real axis. In this paper we have presented different characteristics of Multibrot function for Multicorns using superior iterates.

Keywords

Superior Multibrot, Tricon and Multicorns

1. INTRODUCTION

The Mandelbrot set ***M*** is defined by quadratic polynomial named after Benoit Mandelbrot whose coefficients are complex numbers [8]. Mandelbrot function can be defined as the function $z_{n+1} = z_n^{\ 2} + c; c \in C$ where ***c*** is a complex number. For any complex number ***c***, we determine whether or not *c* is belongs to that collection or set ***M*** by repeatedly applying a particular procedure in order to generate a sequence. If that sequence diverges from the origin or does not belong to the set ***M***, then ***c*** is not a member of the Mandelbrot set. If, on the other hand, the sequence stays near the origin or in other words belongs to the set ***M***, then ***c*** is a member of the Mandelbrot set [2]. Each complex number can be represented as a point on the complex plane [4]. The Mandelbrot set is one of the best-known examples of mathematical visualization. A picture of the Mandelbrot set can be made by coloring all the points '***c***' which belongs to ***M*** black, and all other points white [2]. The more colorful pictures usually seen are generated by coloring points not in the set according to how quickly or slowly the sequence $|Z_c^n(0)|$ diverges to infinity. The anti-polynomial $\bar{z}^n + c$ of the complex polynomial $z^n + c$ has lots of similarities of the Mandelbrot set for the antipolynomials, especially, when ***n*** = 2 or ***n*** = 3, the dynamics of the antipolynomial leads to interesting study of Tricorns and Multicorns. The purpose of this paper is to generate new Superior Multibrot for Multicorns by using Mann iterates, also called as superior iterates, by Rani and Kumar [6] and [11]. Further, different properties like trajectories [7], fixed point, its complex dynamics and its behaviour towards Julia set [5], [9] and [10] are also discussed in the paper. In this paper we have introduced a new set of Multibrot named as Superior Multibrot for Multicorns for negative powers and explored their corresponding Julia sets.

2. PRELIMINARIES

Definition 2.1: Julia sets

French mathematician Gaston Julia [5] investigated the iteration process of a complex function. Now, Julia set has been applied widely in computer graphics, biology, engineering and other branches of mathematical sciences. Consider the complex-valued quadratic function

$$z_{n+1} = z_n^2 + c; c \in C$$

where C be the set of complex numbers and n is the iteration number. The Julia set for parameter c is defined as the boundary between those of z_0 that remain bounded after repeated iterations and those escape to infinity. The Julia set on the real axis are reflection symmetric, while those with complex parameter show rotation symmetry with an exception to $c = (0,0)$ see Rani and Kumar [6] and [11].

Definition 2.2: Multibrot set

The Multibrot function is given by complex function $z_{n+1} = z_n^d + c; d \geq 2$ where ***n*** and ***c*** is a constant. So, the collection of point that satisfies the relation $z_{n+1} = z_n^d + c; d \geq 2$ is called Multibrot set. The Mandelbrot function is defined in terms of the function $z_{n+1} = z_n^d + c; d = 2.$ It is always possible for ***d*** to have any value other than 2 *i.e*. an integer, negative, rational, irrational, imaginary or complex value. As the exponent varies interesting collection of multibrot sets are obtained. The complex dynamics of Multibrot function, generally known as Multibrot fractal, is a modification of the classic Mandelbrot and Julia sets.

Definition 2.3: Superior Orbit

Let A be a subset of real or complex numbers and $f : A \to A$. For $x_0 \in A$, construct a sequence $\{x_n\}$ in A in the following manner

$$x_1 = s_1 f(x_0) + (1 - s_1) x_0$$
$$x_2 = s_2 f(x_1) + (1 - s_2) x_1$$
$$\vdots$$
$$x_n = s_n f(x_{n-1}) + (1 - s_n) x_{n-1}$$

where $0 < s_n \le 1$ and $\{s_n\}$ is convergent to a non-zero number. The sequence $\{x_n\}$ constructed above is called Mann sequence of iterates or superior sequence of iterates. Let z_0 be an arbitrarily element of C. Construct a sequence $\{z_n\}$ of points of C in the following manner:

$$z_n = sf(z_{n-1}) + (1 - s) z_{n-1,} n = 1, 2, 3...$$

where f is a function on a subset of C and the parameter s lies in the closed interval $[0,1]$. The sequence $\{z_n\}$ constructed above, denoted by $SO(f, z_0, s)$, is the superior orbit for the complex valued function f with an initial choice z_0 and parameter ***s***. We may denote it by $SO(f, x_0, s_n)$. Notice that $SO(f, x_0, s_n)$ with $s_n = 1$ is $O(f, x_0)$. We remark that the superior orbit reduces to the usual Picard orbit when $s_n = 1$.

Definition 2.4. Superior Multibrot for Multicorns

Let $A_c = \bar{z}^n + c$ denote the antipolynomial of the complex polynomial $z^n + c$, where $c \in C$, the set of complex numbers. The sequence $\{x_n\}$ constructed above is called Mann sequence of iterates or superior sequence of iterates. Let z_0 be an arbitrarily element of C. Construct a sequence $\{z_n\}$ of points of C in the following manner:

$$z_n = sf(z_{n-1}) + (1 - s) z_{n-1,} n = 1, 2, 3...$$

where f is a function on a subset of C and the parameter ***s*** lies in the closed interval $[0,1]$. Now we define the superior Multibrot set for Multicorns.

3. ANALYSIS OF SUPERIOR MULTIBROT SET FOR MULTICORNS

CASE 2. (For $d < 0$)

For $d = 2$, the Multibrot set converts to Mandelbrot set. For ***d*** = 2, we notice that the superior Multibrot fractal for the multicorns fractal converts to usual superior Mandelbrot fractal. The case is also true for the corresponding superior Julia sets. When 0< ***s*** ≤1 and ***d*** > 2, we obtain superior Multibrot for the Multicorn. Further, we investigate the dynamics of the superior multibrot sets for the transformation function $A_c(z) = \bar{z}^d + c$ and analyze the ***z*** plane fractal images generated from the iterations of this functions using Mann iteration procedure and changes that occur in the visual characteristics of the images for negative, positive and fractional powers.

Further investigate the various characteristics of the superior multibrot for multicorns. We divide our analysis in three categories on the basis of power ***d***.

Case 1. For the Positive power or $d \ge 2$

Case 2. For the Negative power or $d < 0$

Case 3. For the Non integer Power or d, i.e. fractional

In this paper we are going to present the second case i.e $d < 0$:

Here we have presented the **Superior Multibrot for Multicorn** at ***d***= -3, -4, -5 and -10 are represented in Fig. [1] - [4]. In the continuity of power function we found the new set to the superior Mandelbrot set. We found that for odd powers of ***d***, the superior Multibrot set for multicorn is symmetric across both the **x** and **y** axis, whereas for the even power of ***d*** we find the symmetry along x axis only. As we increase the power of superior multibrot function for multicorn, we get ***d***+1 ovoid in each image.

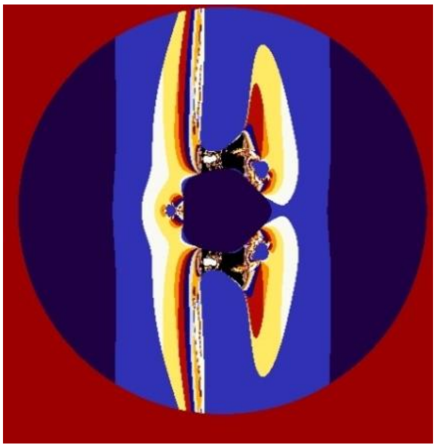

Fig. 1. Superior Multibrot set for Multicorn for *d* = -3 and *s*= 0.1

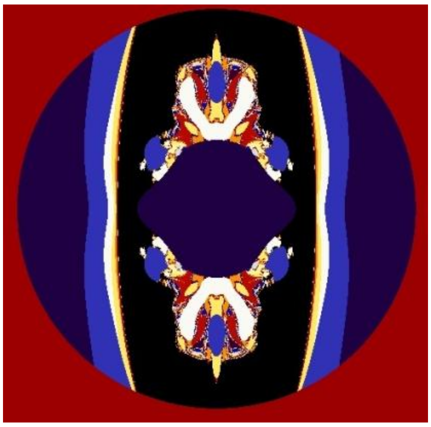

Fig. 2. Superior Multibrot set for Multicorn for *d* = -4 and *s*= 0.1

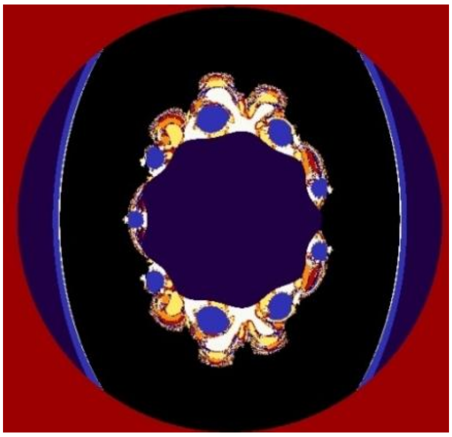

Fig. 3. Superior Multibrot set for Multicorn for *d* = 5 and *s*= 0.5

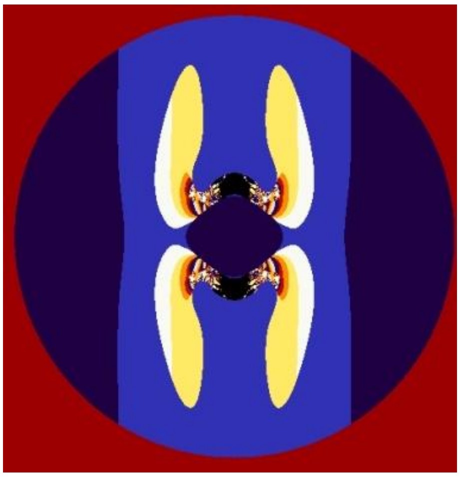

Fig. 4. Superior Multibrot set for Multicorn for *d* = -10 and *s*= 0.8

SUPERIOR JULIA SETS OF FOR MULTICORN AT $d < 2$ AND THEIR FIXED POINTS

On iterating the Multicorn function for d < 2 we obtain the symmetric image at ***z*** = (-0.0056, -1.8641), ***d***=-3 and ***s*** = 0.3 see Fig. 5. Here we also presented a figure at ***z*** = (-0.0946, -0.0809), ***d***=-10 and ***s*** = 0.1 which resembles with the wheel see Fig. 8.

Third degree superior Julia sets for Multicorns family

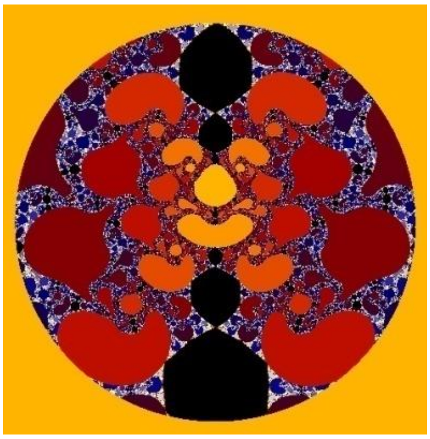

Fig. 5. Superior Julia set for (*z* = -0.0056, -1.8641) at *d*=-3 and *s* = 0.3

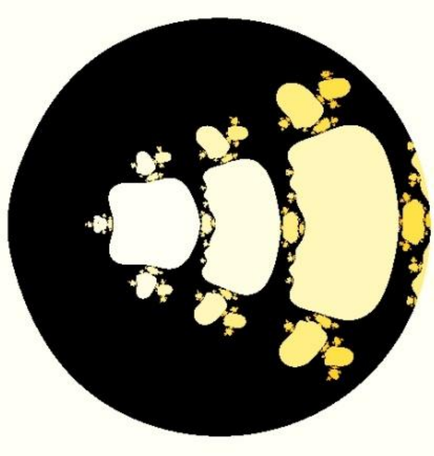

Fig. 6. Superior Julia set for (*z* = -0.0126, 3.6838) at *d*=-4 and *s* = 0.3

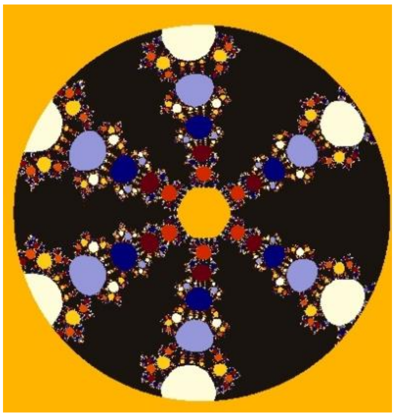

Fig. 7. Superior Julia set for (z = 0.1108, -0.0118) at d=-5 and s = 0.3

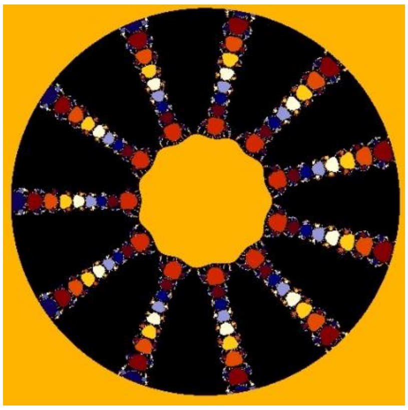

Fig. 8. Superior Julia set for (z = -0.0946, -0.0809) at d=-10 and s = 0.1

4. CONCLUSION

In this paper we have presented the analysis of superior Multibrot for the Multicorn at different power of ***d*** see Fig. [1] – [8]. Further we have presented the geometric properties of superior Julia sets along different axis. We also find an interesting image which resemble to the wheel see Fig. [8].

5. REFERENCES

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