Superior Multibrots for Multicorns for Positive Values

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ABSTRACT

The Multibrots for Multicorns is a modification of the classic Mandelbrot and Julia sets and it is given by the complex function $A_c = \bar{z}^d + c; d \geq 2$ where d and c is a constant. The Multibrot fractal type is particularly interesting, with beautiful shapes and lots of spirals. In this paper we have presented different characteristics of Multibrot function for Multicorns using superior iterates. Further, different properties like trajectories, fixed point, its complex dynamics and its behaviour towards Julia set are also discussed in the paper.

Keywords

Superior Multibrot, Tricon and Multicorns

1. INTRODUCTION

The Mandelbrot set ***M*** is defined by a family of quadratic polynomial named after Benoit Mandelbrot whose coefficients are complex numbers [8]. In terms of mathematical function the Mandelbrot function could be defined as:

$$z_{n+1} = z_n{}^2 + c; c \in C$$

A given complex number ***c*** either belongs to the set ***M*** or it does not. The Mandelbrot set is simply a collection of complex numbers [1]. For any complex number ***c***, we determine whether or not *c* is belongs to that collection or set ***M*** by repeatedly applying a particular procedure in order to generate a sequence. If that sequence diverges from the origin or does not belong to the set ***M***, then ***c*** is not a member of the Mandelbrot set.

The anti-polynomial $\bar{z}^n + c$ of the complex polynomial $z^n + c$ has attracted several researchers working on the related Mandelbrot sets. While discussing the similarities of the Mandelbrot set for the antipolynomials, especially, when ***n*** = 2 or ***n*** = 3, the dynamics of the antipolynomial leads to interesting study of Tricorns and Multicorns. The purpose of this paper is to generate new Superior Multibrot for Multicorns by using Mann iterates, also called as superior iterates, by Rani and Kumar [6] and [11]. In this paper we have introduced a new set of Multibrot named as Superior Multibrot for Multicorns for positive powers and explored their corresponding Julia sets.

2. PRELIMINARIES

Definition 2.1: Julia sets

French mathematician Gaston Julia [5] investigated the iteration process of a complex function. Now, Julia set has been applied widely in computer graphics, biology, engineering and other branches of mathematical sciences. Consider the complex-valued quadratic function

$$z_{n+1} = z_n^2 + c; c \in C$$

where C be the set of complex numbers and n is the iteration number. The Julia set for parameter C is defined as the boundary between those of z_0 that remain bounded after repeated iterations and those escape to infinity. The Julia set on the real axis are reflection symmetric, while those with complex parameter show rotation symmetry with an exception to $c = (0,0)$ see Rani and Kumar [6] and [11].

Definition 2.2: Multibrot set

The Multibrot function is given by complex function $z_{n+1} = z_n^d + c; d \geq 2$ where ***n*** and ***c*** is a constant. So, the collection of point that satisfies the relation $z_{n+1} = z_n^d + c; d \geq 2$ is called Multibrot set. The Mandelbrot function is defined in terms of the function $z_{n+1} = z_n^d + c; d = 2.$ It is always possible for ***d*** to have any value other than 2 *i.e.* an integer, negative, rational, irrational, imaginary or complex value. As the exponent varies interesting collection of multibrot sets are obtained. The complex dynamics of Multibrot function, generally known as Multibrot fractal, is a modification of the classic Mandelbrot and Julia sets.

Definition 2.3: Superior Orbit

Let A be a subset of real or complex numbers and $f : A \to A$. For $x_0 \in A$, construct a sequence $\{x_n\}$ in A in the following manner

$$x_1 = s_1 f(x_0) + (1 - s_1) x_0$$
$$x_2 = s_2 f(x_1) + (1 - s_2) x_1$$
$$\vdots$$
$$x_n = s_n f(x_{n-1}) + (1 - s_n) x_{n-1}$$

where $0 < s_n \le 1$ and $\{s_n\}$ is convergent to a non-zero number. The sequence $\{x_n\}$ constructed above is called Mann sequence of iterates or superior sequence of iterates. Let z_0 be an arbitrarily element of C. Construct a sequence $\{z_n\}$ of points of C in the following manner:

$$z_n = sf(z_{n-1}) + (1-s) z_{n-1,} n = 1, 2, 3...$$

where f is a function on a subset of C and the parameter s lies in the closed interval $[0,1]$. The sequence $\{z_n\}$ constructed above, denoted by $SO(f, z_0, s)$, is the superior orbit for the complex valued function f with an initial choice z_0 and parameter ***s***. We may denote it by $SO(f, x_0, s_n)$. Notice that $SO(f, x_0, s_n)$ with $s_n = 1$ is $O(f, x_0)$. We remark that the superior orbit reduces to the usual Picard orbit when $s_n = 1$.

Definition 2.4. Superior Multibrot for Multicorns

Let $A_c = \bar{z}^n + c$ denote the antipolynomial of the complex polynomial $z^n + c$, where $c \in C$, the set of complex numbers. The sequence $\{x_n\}$ constructed above is called Mann sequence of iterates or superior sequence of iterates. Let z_0 be an arbitrarily element of C. Construct a sequence $\{z_n\}$ of points of C in the following manner:

$$z_n = sf(z_{n-1}) + (1-s) z_{n-1,} n = 1, 2, 3...$$

where f is a function on a subset of C and the parameter ***s*** lies in the closed interval $[0,1]$. Now we define the superior Multibrot set for Multicorns.

3. ANALYSIS OF SUPERIOR MULTIBROT SET FOR MULTICORNS

CASE 1. (For $d \ge 2$)

For $d = 2$, the Multibrot set converts to Mandelbrot set. For ***d*** = 2, we notice that the superior Multibrot fractal for the multicorns fractal converts to usual superior Mandelbrot fractal. The case is also true for the corresponding superior Julia sets. When 0< ***s*** ≤1 and ***d*** > 2, we obtain superior Multibrot for the Multicorn. Further, we investigate the dynamics of the superior multibrot sets for the transformation function $A_c(z) = \bar{z}^d + c$ and analyze the ***z*** plane fractal images generated from the iterations of this functions using Mann iteration procedure and changes that occur in the visual characteristics of the images for negative, positive and fractional powers.

Further investigate the various characteristics of the superior multibrot for multicorns. We divide our analysis in three categories on the basis of power ***d***.

Case 1. For the Positive power or $d \ge 2$

Case 2. For the Negative power or $d < 0$

Case 3. For the Non integer Power or d, i.e. fractional

In this paper we are going to present the first case i.e $d \ge 2$:

Here we have presented the **Superior Multibrot for Multicorn** at ***d*** = 3, 4, 5 and 10 see Fig. [1] - [4]. The new set has some striking similarities to the superior Mandelbrot set. We observe that for odd powers, the superior Multibrot set for multicorn is symmetric across both the ***x*** and ***y*** axis. Whereas for the even powers of ***d*** we find symmetry along the ***x*** axis, as we increase the power of superior multibrot function for multicorn, we get *d+1* ovoid in each image.

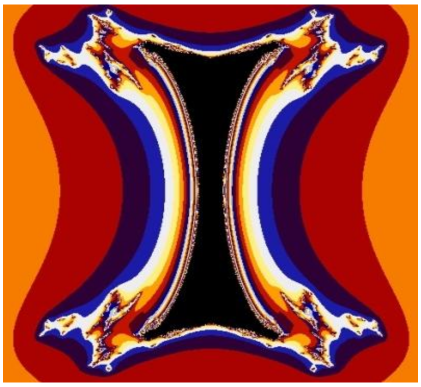

Fig. 1. Superior Multibrot set for Multicorn for *d* = 3 and *s*= 0.5

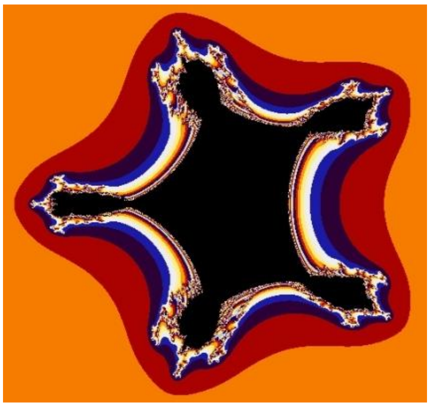

Fig. 2. Superior Multibrot set for Multicorn for *d* = 4 and *s*= 0.8

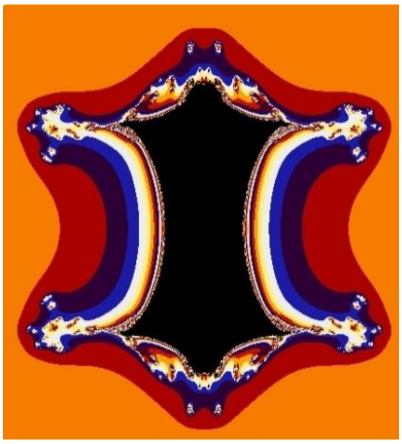

Fig. 3. Superior Multibrot set for Multicorn for *d* = 5 and *s*= 0.5

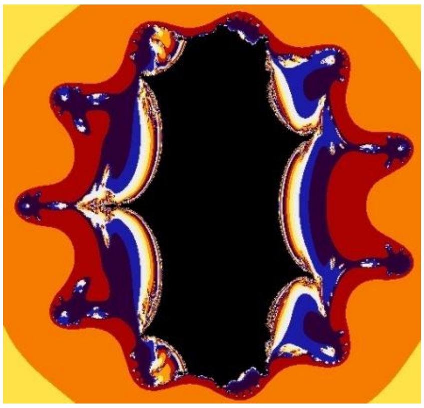

Fig. 4. Superior Multibrot set for Multicorn for *d* = 10 and *s*= 0.5

SUPERIOR JULIA SETS MULTICORN AT $d \geq 2$ AND THEIR FIXED POINTS

For ***d*** = 2, we obtain beautiful fractals at ***z*** = (0.001, -2.021), *d*=3 and ***s*** = 0.5 which has symmetry through ***y*** axis and after a few iteration orbit converges to infinity, see Fig. [5] and [6]. We also find the interesting image at ***z*** = (-1.354, -9.499), ***d***=4 and ***s*** = 0.1 which resembles with dragon and the orbit is converges to a fixed point see Fig. [7] and [8]. We also find an interesting image at ***z*** = (-0.8818, -1.1266), ***d***=5 and ***s*** = 0.5 which resembles with Hen and orbit converges to a fixed point see Fig. [9] and [10]. We also get an image at ***z*** = (-2.5120, -0.0435), ***d***=10 and ***s*** = 0.5 which is symmetric along **x** axis and we get no fixed point see Fig. [11] and [12].

Third degree superior Julia sets for Multicorn family

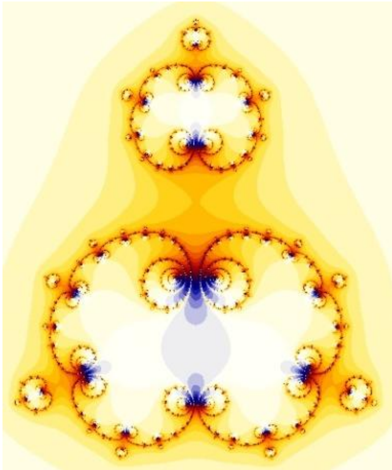

Fig. 5. Superior Julia set for (*z* = 0.001, -2.021) at *d*=3 and *s* = 0.5

Fig. 6. Orbit of Julia set for (*z* = 0.001, -2.021) at *d*=3 and *s* = 0.5

Number of iteration i	| F(z) |
1	0
2	1.0105
7	2.90E+08
8	1.22E+25
9	9.07E+74
10	3.73E+224
11	NaN
12	NaN
13	NaN
14	NaN
15	NaN

Table 1. For (z_0 = -0.0189, 19.277) at *d*=3 and *s* = 0.1

(Some Intermediate iteration has been skipped intentionally)

Fourth degree superior Julia sets for Multicorns Family

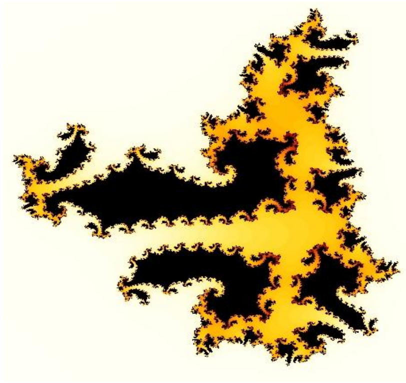

Fig. 7. Superior Julia set for (*z* = -1.354, -9.499) at *d*=4 and *s* = 0.1

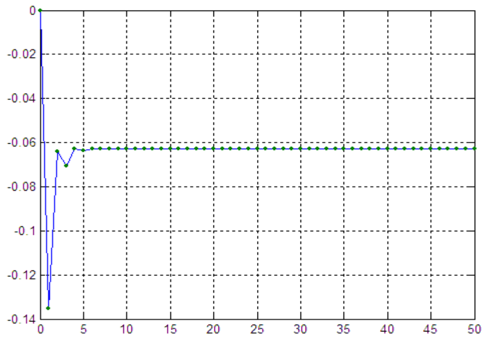

Fig. 8. Orbit of Julia set for (*z* = -1.354, -9.499) at *d*=4 and *s* = 0.1

Number of iteration i	| F(z) |
1	0
2	0.9595
3	0.90668
4	0.93376
5	0.9294
6	0.93212
7	0.93168
8	0.93196
9	0.93192
10	0.93195
11	0.93194
12	0.93194

Table 2. For (z_0 = -1.354, -9.499) at *d*=4 and *s* = 0.1

(Some Intermediate iteration has been skipped intentionally)

Higher degree superior Julia sets for Multicorns family

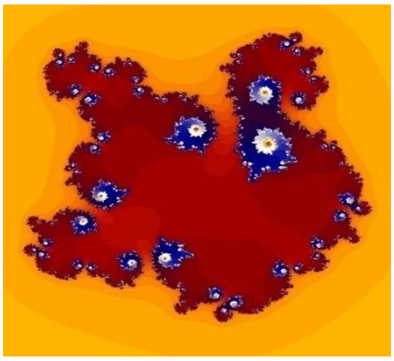

Fig. 9. Superior Julia set for (*z* = -0.8818, -1.1266) at *d*=5 and *s* = 0.5

Fig. 10. Orbit of Julia set for (*z* = -0.8818, -1.1266) at *d*=5 and *s* = 0.5

Number of iteration i	| F(z) |
1	0
2	0.71533
3	0.78076
7	1.2647
8	2.3223
6	33.782
7	2.20E+07
8	2.58E+36
9	5.68E+181
10	NaN
11	NaN
12	NaN

Table 3. For (z_0 = -0.8818, -1.1266) at *d*=5 and *s* = 0.5

(Some Intermediate iteration has been skipped intentionally)

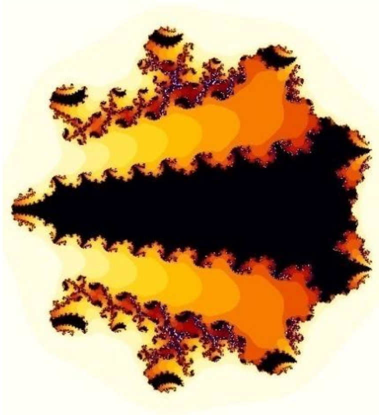

Fig. 11. Superior Julia set for (*z* = -2.5120, -0.0435) at *d*=10 and *s* = 0.5

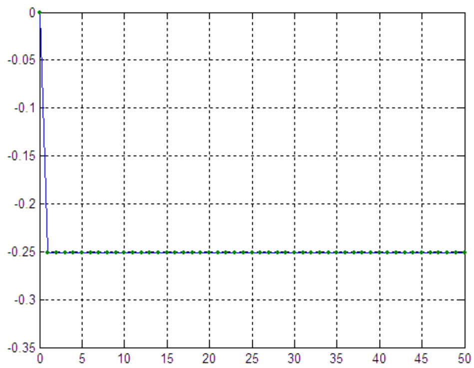

Fig. 12. Orbit of Julia set for (*z* = -2.5120, -0.0435) at *d*=10 and *s* = 0.5

Number of iteration i	| F(z) |
1	0
2	0.71533
3	0.78076
4	0.85108
8	2.32E+00
9	3.38E+01
10	2.20E+07
8	2.58E+36
9	5.68E+181
10	NaN
11	NaN
12	NaN

Table 4. For (z_0 = -2.5120, -0.0435) at *d*=10 and *s* = 0.5

(Some Intermediate iteration has been skipped intentionally)

4. CONCLUSION

In this paper we have presented the dynamics and fixed point analysis of superior Multibrot for the Multicorn at different power of ***d*** see Fig. [1] – [12]. Further we have presented the geometric properties of superior Julia sets along different axis. We also found two interesting image which resemble to dragon and hen see Fig. [7] and [9].

5. REFERENCES

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