Matrix Inversion Method for Solving Fully Fuzzy Linear Systems with Triangular Fuzzy Numbers

S. Muruganandam
Professor and Head,
Department of Mathematics,
TRP Engineering College (SRM Group),
Tiruchirappalli, India.

K. Abdul Razak
Assistant Professor,
Department of Mathematics,
TRP Engineering College (SRM Group),
Tiruchirappalli, India.

ABSTRACT

Linear systems have important applications to many branches of Science and Engineering. In many applications, atleast some of the parameters of the system are represented by fuzzy rather than crisp numbers. This paper, discusses fully fuzzy linear systems with triangular fuzzy numbers. A matrix inversion method is proposed for solving Fully Fuzzy Linear System (FFLS) of equations. Finally, the method is illustrated by solving a numerical example.

Keywords: Triangular fuzzy numbers, fuzzy arithmetic, fully fuzzy linear systems, matrix inversion method.

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1. INTRODUCTION

Systems of linear equations are important for studying and solving a large proportion of the problems in many topics in applied mathematics. Systems of simultaneous linear equations play a major role in various areas such as operational research, physics, statistics, engineering and social sciences. Usually, in many applications some of the parameters in the problems are represented by fuzzy number rather than crisp, and hence it is important to develop mathematical models and numerical procedures that would appropriately treat general fuzzy linear systems and solve them. The concept of fuzzy numbers and fuzzy arithmetic operations were first introduced by Zadeh [16] and Dubois and Prade [9].

Friedman et al. [10] proposed a general model for solving a $n \times n$ fuzzy linear system, whose coefficient matrix is crisp and right hand side column is an arbitrary fuzzy number vector. To find the solution, the original $n \times n$ fuzzy linear system is replaced with $2n \times 2n$ crisp linear system. Some of the most interesting numerical methods can be seen in [5,6,8,9]. They are proposed to find the solution of the $2n \times 2n$ crisp linear systems. Buckley and Qu [8] extended several methods to obtain linear system of interval equations. The exact solution of FFLS can be found by solving these interval systems [2,3,16] proposed fully fuzzy linear systems. Whose all parameters are triangular fuzzy numbers. In [16] authors proposed a new method for solving fully fuzzy linear systems based on QR decomposition. Abbasbandy and Jafarian [11] applied steepest descent method for approximation of the unique solution of fuzzy system of linear equation. Abbasbandy et al. [7] proposed LV decomposition method for solving fuzzy system of linear equation. Nasseri et al. [14] used a certain decomposition of the coefficient matrix of the fully fuzzy linear system of equations to construct a new algorithm for solving these system of equations. Nasseri and Zahmatkesh [15] proposed a new method for computing a non negative solution of the fully fuzzy linear system of equations.

In this paper, our aim is to solve $\tilde{A} \otimes \tilde{x} = \tilde{b}$, where $\tilde{A}$ is a fuzzy matrix and $\tilde{x}$ and $\tilde{b}$ are fuzzy vectors with appropriate sizes. This paper is organized as follows: In section 2, we give some basic definitions on fuzzy numbers and fuzzy arithmetic operations.

In section 3, we define a fully fuzzy linear system and propose a method to solve these system of equations.

In section 4, numerical example is given for computing the solution of FFLS is proposed. The conclusion is in section 5.

2. BASIC DEFINITIONS

In this section, some basic definitions are reviewed [1].

Definition 2.1

The characteristic function $\mu_{\tilde{A}}$ of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set fall within specified range (i.e.) $\mu_{\tilde{A}} : X \to [0, 1]$. The assigned value indicate the membership grade of the element in the set.

The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$ defined by $\mu_{\tilde{A}}(x)$ for $x \in X$ is called a fuzzy set.

Definition 2.2

A fuzzy set $\tilde{A}$ defined on the universal set of real number R is said to be a fuzzy number of its membership function has the following characteristics.

(i) $\tilde{A}$ is convex if and only if for any $x_1, x_2 \in R$, the membership function of A satisfies the inequality.

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)), \quad 0 \leq \lambda \leq 1$$

(ii) $\tilde{A}$ is normal implying that there exist atleast one $x \in R$ such that $\mu_{\tilde{A}}(x) = 1$.

Definition 2.3

A fuzzy number $\tilde{A}$ is said to be non-negative fuzzy number if and only if $\mu_{\tilde{A}}(x) = 0$ for all x.

Definition 2.4 (Triangular fuzzy number)

A fuzzy number $\tilde{A}$ = (m, α, β) is said to be a triangular fuzzy number if its membership function is given by [14].

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \dfrac{m - x}{\alpha}, & m - \alpha \le x \le m, \alpha > 0 \\ 1 - \dfrac{x - m}{\beta}, & m \le x \le m + \beta, \beta > 0 \\ 0, & \text{otherwise} \end{cases}$$

Definition 2.5

A triangular fuzzy number $\tilde{A}$ = (m, α, β) is called positive. (i.e.) $\tilde{A} \ge 0$ if and only if $m - \alpha \ge 0$.

Definition 2.6

A triangular fuzzy number $\tilde{A}$ = (m, α, β) is zero triangular fuzzy number if and only if m = 0, α = 0, β = 0.

Definition 2.7

Two triangular fuzzy numbers $\tilde{A}$ = (m, α, β) and $\tilde{B}$ = (n, γ, δ) are said to be equal if and only if m = n, α = γ and β = δ.

Definition 2.8

A matrix $\tilde{A} = (\tilde{a}_{ij})$ is called a fuzzy matrix, if each element of $\tilde{A}$ is a fuzzy number. We may represent n × n fuzzy matrix $\tilde{A} = (\tilde{a}_{ij})_{n \times n}$ such that $\tilde{a}_{ij} = (a_{ij}, b_{ij}, c_{ij})$.

Definition 2.9

Arithmetic operations on Triangular fuzzy numbers

In this section, arithmetic operations addition, subtraction, multiplication are reviewed [1].

Let $\tilde{A}$ = (m, α, β) and $\tilde{B}$ = (n, γ, δ) be two triangular fuzzy numbers.

(i) $\tilde{A} \oplus \tilde{B}$ = (m, α, β) ⊕ (n, γ, δ)

= (m + n, α + γ, β + δ)

(ii) $-\tilde{B}$ = -(n, γ, δ) = (-n, δ, γ)

(iii) $\tilde{A} - \tilde{B}$ = (m, α, β) (n, γ, δ)

= (m – n, α + δ, β + γ)

(iv) If A ≥ 0 and B ≥ 0 then

$\tilde{A} \otimes \tilde{B}$ = (m, α, β) ⊗ (n, γ, δ)

≅ (mn, mγ + nα, mδ + nβ)

(v) For scalar multiplication,

$$\lambda \otimes \tilde{A} = \lambda \otimes (m, \alpha, \beta) = \begin{cases} (\lambda m, \lambda\alpha, \lambda\beta), & \lambda \ge 0 \\ (\lambda m, -\lambda\beta, -\lambda\alpha), & \lambda < 0 \end{cases}$$

3. PROPOSED METHOD

In this section, a definition for FFLS and direct method to solve the FFLS are given.

Consider the n × n fuzzy linear system of equations.

$$(\tilde{a}_{11} \otimes \tilde{x}_1) \oplus (\tilde{a}_{12} \otimes \tilde{x}_2) \oplus ... \oplus (\tilde{a}_{1n} \otimes \tilde{x}_n) = \tilde{b}_1$$

$$(\tilde{a}_{21} \otimes \tilde{x}_1) \oplus (\tilde{a}_{22} \otimes \tilde{x}_2) \oplus ... \oplus (\tilde{a}_{2n} \otimes \tilde{x}_n) = \tilde{b}_2$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$(\tilde{a}_{n1} \otimes \tilde{x}_1) \oplus (\tilde{a}_{n2} \otimes \tilde{x}_2) \oplus ... \oplus (\tilde{a}_{nn} \otimes \tilde{x}_n) = \tilde{b}_n$$

where $\tilde{a}_{ij}, \tilde{x}_i$ and $\tilde{b}_i$ are all fuzzy numbers.

The matrix of this linear system is represented as

$$\tilde{A} \otimes \tilde{x} = \tilde{b}$$

where the coefficient of matrix $\tilde{A} = (\tilde{a}_{ij})$, 1 ≤ i ≤ n, 1 ≤ j ≤ n is a n × n fuzzy matrix and $\tilde{a}_{ij} \in F(R)$ and $(\tilde{x}_j, \tilde{b}_i) \in F(R)$, for all i = 1 ... n & j = 1 ... n. This system is called fully fuzzy linear system (FFLS).

In this paper, the positive solution of the FFLS $\tilde{A} \otimes \tilde{x} = \tilde{b}$ by matrix inversion method (Direct Method) is proposed.

Consider $\tilde{A}$ = (A, M, N) ≥ 0, $\tilde{x}$ = (x, y, z) ≥ 0

and $\tilde{b}$ = (b, h, g) ≥ 0

Now,

(A, M, N) ⊗ (x, y, z) = (b, h, g)

Then, by using Definition 2.9,

(Ax, Ay + Mx, Az + Nx) = (b, h, g)

Using Definition 2.7,

$$\left.\begin{aligned} Ax &= b \\ Ay + Mx &= h \\ Az + Nx &= g \end{aligned}\right\} \qquad ... (3.1)$$

Therefore,

$$\left.\begin{aligned} x &= A^{-1}b \\ y &= A^{-1}[h - Mx] \\ z &= A^{-1}[g - Nx] \end{aligned}\right\} \qquad ... (3.2)$$

The solution of FFLS will be represented by

$\tilde{x}_i = (x_i, y_i, z_i)$ for i = 1, 2, ..., n

4. NUMERICAL EXAMPLE

Consider the following fully fuzzy linear system and solve it by matrix inversion method.

$(15, 1, 4) \otimes (x_1, y_1, z_1) \oplus (5, 2, 9) \otimes (x_2, y_2, z_2) = (10, 15, 25)$

$(10, 5, 6) \otimes (x_1, y_1, z_1) \oplus (25, 3, 4) \otimes (x_2, y_2, z_2) = (20, 30, 40)$

Solution

The given FFLS may be written as

$$\begin{pmatrix} (15,1,4) & (5,2,9) \\ (10,5,6) & (25,3,4) \end{pmatrix} \begin{pmatrix} (x_1, y_1, z_1) \\ (x_2, y_2, z_2) \end{pmatrix} = \begin{pmatrix} (10,15,25) \\ (20,30,40) \end{pmatrix}$$

$$A = \begin{pmatrix} 15 & 5 \\ 10 & 25 \end{pmatrix}, \quad M = \begin{pmatrix} 1 & 2 \\ 5 & 3 \end{pmatrix}, \quad N = \begin{pmatrix} 4 & 9 \\ 6 & 4 \end{pmatrix}$$

$$b = \begin{pmatrix} 10 \\ 20 \end{pmatrix}, \quad h = \begin{pmatrix} 15 \\ 30 \end{pmatrix}, \quad g = \begin{pmatrix} 25 \\ 40 \end{pmatrix}$$

From equation 3.2

$$\begin{pmatrix} 15 & 5 \\ 10 & 25 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 10 \\ 20 \end{pmatrix}$$

The solution for x is $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0.461538 \\ 0.6153846 \end{pmatrix}$

Next,

$$\begin{pmatrix} 15 & 5 \\ 10 & 25 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 15 \\ 30 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} 0.461538 \\ 0.6153846 \end{pmatrix}$$

The solution for y is $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0.626035 \\ 0.783432 \end{pmatrix}$

Next,

$$\begin{pmatrix} 15 & 5 \\ 10 & 25 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} 25 \\ 40 \end{pmatrix} - \begin{pmatrix} 49 \\ 64 \end{pmatrix} \begin{pmatrix} 0.461538 \\ 0.6153846 \end{pmatrix}$$

The solution for z is $\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} 0.8201185 \\ 1.0627219 \end{pmatrix}$

The solution of FFLS is $\tilde{x}_i = (x_i, y_i, z_i)$ for all i = 1, 2, ..., n.

Therefore,

$$\tilde{x}_1 = (0.461538, 0.626035, 0.8201185)$$

and $\tilde{x}_2 = (0.6153846, 0.783432, 1.0627219)$

5. CONCLUSION

In this paper, arithmetic operation on fuzzy numbers introduced by Kauffman is used [1]. The fuzzy linear system of equations (i.e., fully fuzzy linear system) with fuzzy coefficients involving fuzzy variables are investigated and matrix inversion method is applied for solving these system of equations. The method is illustrated with numerical example.

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