

# **Radiation Effects on MHD Free Convective Flow Past an Oscillating Vertical Porous Plate with Periodic Heat Flux**

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## **ABSTRACT**

Radiation effects on MHD free convective flow of a viscous incompressible electrically conducting fluid past an oscillating vertical porous plate with uniform suction or blowing at the plate in the presence of a uniform magnetic field have been studied. The governing equations are solved analytically. It is observed that the velocity decreases near the plate and it increases away from the plate with an increase in either magnetic parameter or radiation parameter or Prandtl number or suction parameter or frequency parameter or phase angle. The fluid velocity increases with an increase in Grashof number. The solution exists for the blowing at the plate. It is seen that the fluid temperature decreases near the plate and it increases away from the plate with an increase in either radiation parameter or Prandtl number or suction parameter or frequency parameter. Further, it is seen that the amplitude of the shear stress and the tangent of the phase angle at the plate decrease with an increase in either radiation parameter or Prandtl number or suction parameter.

**Keywords::** MHD free convection, radiation, porous plate, heat flux, suction/blowing and phase angle.

## **1. INTRODUCTION**

Free convective flow occurs frequently in an environment where differences between land and air temperature can give rise to complicated flow patterns. The subject of magnetohydrodynamics has attracted the attention of a large number of scholars due to its diverse application. In astrophysics and geophysics, it is applied to study the stellar and solar structures, interstellar matter, radio propagation through the ionosphere etc. In engineering, it finds its application in MHD pumps, MHD bearing etc. The study of effects of magnetic field on free convection flow is important in liquid metals, electrolytes and ionized gases. At the high temperature attained in some engineering devices, gas, for example, can be ionized and so becomes an electrical conductor. The ionized gas or plasma can be made to interact with the magnetic field and alter heat transfer and friction characteristic. Recently, it is of great interest to study the effect of magnetic field on the temperature distribution and heat transfer when the fluid is not only an electrical conductor but also when it is capable of emitting and absorbing thermal radiation. Heat transfer by thermal radiation is becoming of greater importance when we are concerned with space applications, higher operating temperatures and also power engineering. The radiative free convective flows are encountered in countless industrial and environment processes e.g. heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-

entry. The radiative heat transfer plays an important role in manufacturing industries for the design of reliable equipment. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering applications. Such a flow past an infinite vertical plate oscillating in its own plane was first studied by Soundalgekar [1] in case of an isothermal plate. M.A. Mansour [2] studied the interaction of free convection with thermal radiation of the oscillatory flow past a vertical plate. Zhang et al [3] studied the free convection effects on a heated vertical plate subjected to a periodic oscillation. The effects of thermal radiation on flow past an oscillating plate with variable temperature were studied by Pathak et al. [4]. Free convection effects on a vertical oscillating porous plate with constant heating was studied by Toki [5]. Chandrakala [6] investigated the radiation effects on flow past an impulsively started vertical oscillating plate with uniform heat flux. In many industrial applications, hydromagnetic flows also occur at very high temperatures in which thermal radiation effects become significant. Radiation magnetohydrodynamic convection flows are also important in astrophysical and geophysical regimes. Soundalgekar [7] worked in hydro-magnetic natural convection flow past a vertical surface. Helmy [8] investigated MHD unsteady free convective flow past a vertical porous plate. Hossain et al. [9] investigated heat transfer response of MHD free convective flow along a vertical plate to surface temperature oscillation. Kim [10] founded an unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. The hydrodynamic free convective flow of an optically thin gray gas in the presence of radiation, when the induced magnetic field is taken into account was studied by Raptis et al. [11]. Chandrakala and Antony Raj [12] studied the effects of thermal radiation on the flow past a semi infinite vertical isothermal plate with uniform heat flux in the presence of transversely applied magnetic field. Chandrakala and Bhaskar [13] studied the effects of thermal radiation on the flow past an infinite vertical oscillating isothermal plate in the presence of transversely applied magnetic field. Abd El-Naby et al. [14] numerically investigated magnetohydrodynamic (MHD) transient natural convection-radiation boundary layer flow with variable surface temperature, showing that velocity, temperature, and skin friction are enhanced with a rise in radiation parameter whereas Nusselt number is reduced. MHD flow over a moving infinite vertical porous plate with uniform heat flux in the presence of thermal radiation has been investigated by Rani and Murthy [15]. Das [16] analyzed the exact solution of MHD free convection flow and mass transfer near a moving vertical plate in presence of thermal radiation. Effects of radiation on unsteady MHD free convective flow past an oscillating vertical porous plate

embedded in a porous medium with oscillatory heat flux have been studied by Manna et al. [17].

In this paper, we study the radiation effects on MHD free convective flow of a viscous incompressible electrically conducting fluid past an oscillating vertical porous plate with uniform suction or blowing at the plate. The plate is oscillating in its own plane with a velocity  $u_0 \cos \omega t$ ,  $\omega$  being the frequency of the oscillations and  $u_0$  is a positive constant. A uniform transverse magnetic field of strength  $B_0$  is imposed perpendicular to the plate. The governing equations along with the boundary conditions are solved analytically. It is observed that the velocity  $u_1$  decreases near the plate and it increases away from the plate with an increase in either magnetic parameter  $M^2$  or radiation parameter  $R$  or Prandtl number  $Pr$  or suction parameter  $S$  or frequency parameter  $n$  or phase angle  $n\tau$ . The fluid velocity  $u_1$  increases with an increase in Grashof number  $Gr$ . The solution also exists for the blowing at the plate. It is seen that the fluid temperature  $\theta$  decreases near the plate and it increases away from the plate with an increase in either  $R$  or  $Pr$  or  $S$  or  $n$ . Further, it is seen that the amplitude of the shear stress  $R_0$  and the tangent of the phase angle  $\phi$  at the plate decreases with an increase in either  $R$  or  $Pr$  or  $S$ .

## 2. FORMULATION OF THE PROBLEM AND ITS SOLUTION

Consider the unsteady flow of a viscous incompressible electrically conducting fluid past an oscillating vertical porous plate with uniform suction or blowing at the plate. The plate oscillates in its own plane with the velocity  $u_0 \cos \omega t$  in a given direction. We choose a cartesian coordinates with the  $x$ -axis along the plate,  $y$ -axis perpendicular to the plate. A uniform magnetic field of strength  $B_0$  is imposed perpendicular to the plate [See Fig.1] and the plate is taken electrically non-conducting. Thermal radiation acts as a unidirectional flux in the  $y$ -direction. The fluid is gray and absorbing-emitting but non-scattering. Effects of Hall and ion-slip currents are also neglected. The electromotive force generated by a magnetic field is a function of the speed of the fluid and the magnetic field strength. The velocity components are  $(u, v)$  relative to a frame of reference. Since the plate lying on the plane  $y=0$  is infinitely long, all the physical quantities will be the function of  $y$  and  $t$

only. The equation of continuity  $\nabla \cdot \vec{q} = 0$  gives  $\frac{\partial v}{\partial y} = 0$  which on integration yields  $v = -v_0(\text{constant})$ , where  $\vec{q} \equiv (u, v)$ . The constant  $v_0$  which denotes the normal velocity at the plate is positive for suction and negative for blowing. We assume that the magnetic Reynolds number for the flow is small so that the induced magnetic field can be neglected. This assumption is justified since the magnetic Reynolds number is generally very small for partially ionized gases. The solenoidal relation  $\nabla \cdot \vec{B} = 0$  for the magnetic field gives  $B_y = B_0 = \text{constant}$  everywhere in the fluid where  $\vec{B} \equiv (B_x, B_y)$ .

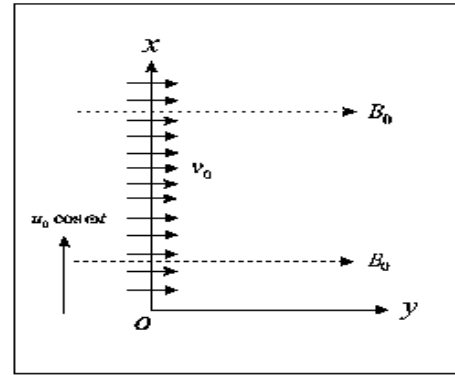


Fig.1. Geometry of the problem

Then the fully developed flow of a radiating gas is governed by the following equations

$$\frac{\partial u}{\partial t} - v_0 \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) - \frac{\sigma B_0^2}{\rho} u, \quad (1)$$

$$\rho c_p \left( \frac{\partial T}{\partial t} - v_0 \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y}, \quad (2)$$

where  $u$  is the fluid velocity in the  $y$ -direction,  $g$  the acceleration due to gravity,  $\nu$  the kinematic viscosity,  $\sigma$  the electrical conductivity of fluid,  $\rho$  the fluid density,  $k$  the thermal conductivity,  $c_p$  the specific heat at constant pressure and  $q_r$  the radiative heat flux.

The boundary conditions of the problem are

$$u = u_0 \cos \omega t, \quad \frac{\partial T}{\partial y} = -\frac{q}{k} \cos \omega t \quad \text{at } y = 0, \\ u \rightarrow 0, \quad T \rightarrow 0 \quad \text{as } y \rightarrow \infty, \quad (3)$$

where  $u_0$  is a constant.

The radiative heat flux can be found from Rosseland approximation [17] and its formula is derived from the diffusion concept of radiative heat transfer in the following way

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}, \quad (4)$$

where  $\sigma^*$  is the Stefan-Boltzman constant and  $k^*$  the spectral mean absorption coefficient of the medium. It should be noted that by using the Rosseland approximation we limit our analysis to optically thick fluids. If the temperature differences within the flow are sufficiently small, then equation (4) can be linearized by expanding  $T^4$  into the Taylor series about  $T_\infty$  and neglecting higher order terms to give:

$$T^4 = 4T_\infty^3 T - 3T_\infty^4. \quad (5)$$

It is emphasized here that equation (5) is widely used in computational fluid dynamics involving radiation absorption problems [18] in expressing the term  $T^4$  as a linear function.

On the use of (4) and (5), the equation (2) becomes

$$\rho c_p \left( \frac{\partial T}{\partial t} - v_0 \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y^2}. \quad (6)$$

Introducing the non-dimensional variables

$$\eta = \frac{u_0}{\nu} y, u_1 = \frac{u}{u_0}, \theta = \frac{(T - T_\infty) k u_0}{q \nu}, \tau = \frac{u_0^2}{\nu} t, n = \frac{\nu \omega}{u_0^2}, \quad (7)$$

equations (1) and (6) become

$$\frac{\partial u_1}{\partial \tau} - S \frac{\partial u_1}{\partial \eta} = Gr \theta + \frac{\partial^2 u_1}{\partial \eta^2} - M^2 u_1, \quad (8)$$

$$\lambda \left( \frac{\partial \theta}{\partial \tau} - S \frac{\partial \theta}{\partial \eta} \right) = \frac{\partial^2 \theta}{\partial \eta^2}, \quad (9)$$

where  $M^2 = \frac{\sigma B_0^2 \nu}{\rho u_0^2}$  is the magnetic parameter,  $R = \frac{k k^*}{4 \sigma^* T_\infty^3}$  the

radiation parameter,  $Pr = \frac{\rho \nu c_p}{k}$  the Prandtl number,

$Gr = \frac{g \beta q \nu^2}{k u_0^4}$  the Grashof number,  $S = \frac{\nu_0}{u_0}$  the suction

parameter and  $\lambda = \frac{3RPr}{3R+4}$ .

On the use of (5), the boundary conditions (3) become

$$u_1 = \frac{1}{2} (e^{in\tau} + e^{-in\tau}), \quad \frac{\partial \theta}{\partial \eta} = -\frac{1}{2} (e^{in\tau} + e^{-in\tau}) \text{ at } \eta = 0, \quad (10)$$

$$u_1 \rightarrow 0, \quad \theta \rightarrow 0 \text{ as } \eta \rightarrow \infty.$$

To solve equations (8) and (9) subject to the boundary conditions (10), we assume the solution in the following form

$$u_1(\eta, \tau) = f_1(\eta) e^{in\tau} + \bar{f}_1(\eta) e^{-in\tau}, \quad (11)$$

$$\theta(\eta, \tau) = g_1(\eta) e^{in\tau} + \bar{g}_1(\eta) e^{-in\tau}. \quad (12)$$

Substituting (11) and (12) in equations (8) and (9) we find that

$f_1(\eta)$ ,  $\bar{f}_1(\eta)$ ,  $g_1(\eta)$  and  $\bar{g}_1(\eta)$  satisfy the following equations

$$\lambda [in g_1(\eta) - S g_1'(\eta)] = g_1''(\eta), \quad (13)$$

$$-\lambda [in \bar{g}_1(\eta) + S \bar{g}_1'(\eta)] = \bar{g}_1''(\eta), \quad (14)$$

$$in f_1(\eta) - S f_1'(\eta) = Gr g_1(\eta) + f_1''(\eta) - M^2 f_1(\eta), \quad (15)$$

$$-in \bar{f}_1(\eta) - S \bar{f}_1'(\eta) = Gr \bar{g}_1(\eta) + \bar{f}_1''(\eta) - M^2 \bar{f}_1(\eta), \quad (16)$$

where primes denote differentiation with respect to  $\eta$ .

The corresponding boundary conditions for  $f_1(\eta)$ ,  $\bar{f}_1(\eta)$ ,  $g_1(\eta)$  and  $\bar{g}_1(\eta)$  are

$$g_1'(0) = \bar{g}_1'(0) = -\frac{1}{2}, \quad f_1(0) = \frac{1}{2}, \quad \bar{f}_1(0) = \frac{1}{2},$$

$$g_1 \rightarrow 0, \quad \bar{g}_1 \rightarrow 0, \quad f_1 \rightarrow 0, \quad \bar{f}_1 \rightarrow 0 \text{ as } \eta \rightarrow \infty. \quad (17)$$

Solutions of equations (15) and (16) subject to the boundary conditions (17) are

$$g_1(\eta) = \frac{1}{2(\alpha + i\beta)} e^{-(\alpha + i\beta)\eta}, \quad \bar{g}_1(\eta) = \frac{1}{2(\alpha - i\beta)} e^{-(\alpha - i\beta)\eta}, \quad (18)$$

$$f_1(\eta) = \frac{1}{2} e^{-(\alpha_1 + i\beta_1)\eta} + \frac{Gr}{2} (A + iB) \left[ e^{-(\alpha_1 + i\beta_1)\eta} - e^{-(\alpha - i\beta)\eta} \right], \quad (19)$$

$$\bar{f}_1(\eta) = \frac{1}{2} e^{-(\alpha_1 - i\beta_1)\eta} + \frac{Gr}{2} (A - iB) \left[ e^{-(\alpha_1 - i\beta_1)\eta} - e^{-(\alpha - i\beta)\eta} \right], \quad (20)$$

where

$$\alpha = \frac{S\lambda}{2} + \frac{1}{\sqrt{2}} \left[ \left\{ \left( \frac{S^2 \lambda^2}{4} \right)^2 + n^2 \lambda^2 \right\}^{\frac{1}{2}} + \frac{S^2 \lambda^2}{4} \right]^{\frac{1}{2}},$$

$$\beta = \frac{1}{\sqrt{2}} \left[ \left\{ \left( \frac{S^2 \lambda^2}{4} \right)^2 + n^2 \lambda^2 \right\}^{\frac{1}{2}} - \frac{S^2 \lambda^2}{4} \right]^{\frac{1}{2}}, \quad (21)$$

$$\alpha_1, \beta_1 = \frac{S}{2} + \frac{1}{\sqrt{2}} \left[ \left\{ \left( \frac{S^2}{4} + M^2 \right)^2 + n^2 \right\}^{\frac{1}{2}} \pm \left( \frac{S^2}{4} + M^2 \right) \right]^{\frac{1}{2}},$$

$$A_1 = \alpha^2 - \beta^2 - S\alpha - M^2, \quad B_1 = 2\alpha\beta - S\beta - n,$$

$$A_2 = \alpha A_1 - \beta B_1, \quad B_2 = \alpha B_1 + \beta A_1,$$

$$A = \frac{A_2}{A_2^2 + B_2^2}, \quad B = -\frac{B_2}{A_2^2 + B_2^2}.$$

On the use of (18)-(20), equations (12) and (11) yield

$$\theta(\eta, \tau) = \frac{e^{-\alpha\eta}}{\alpha^2 + \beta^2} [\alpha \cos(n\tau - \beta\eta) + \beta \sin(n\tau - \beta\eta)], \quad (22)$$

$$u_1(\eta, \tau) = e^{-\alpha_1\eta} [(1 + GrA) \cos(n\tau - \beta_1\eta) - GrB \sin(n\tau - \beta_1\eta)] - e^{-\alpha\eta} [A \cos(n\tau - \beta\eta) - B \sin(n\tau - \beta\eta)]. \quad (23)$$

Solutions (22) and (23) are valid for both suction and blowing at the plate. Equations (14) and (15) of Rani and Murty [15] are incorrect as they are independent of time  $t$ . This is due to error in boundary condition (4) of Rani and Murty [15]. The correct boundary condition is given by equation (3) and hence the equations (22) and (23) are not identical with the equations (14) and (15) of Rani and Murty [15] in the absence of suction/blowing ( $S = 0$ ).

### 3. RESULTS AND DISCUSSION

We have presented the non-dimensional velocity  $u_1$  and the temperature  $\theta$  for several values of magnetic parameter  $M^2$ , radiation parameter  $R$ , Prandtl number  $Pr$ , Grashof number  $Gr$ , suction parameter  $S$ , frequency parameter  $n$  and phase angle  $n\tau$  in Figs.2-13. Fig.2 shows that the velocity  $u_1$  decreases near the plate and it increases away from the plate with an increase in parameter  $M^2$ . The presence of a magnetic field in an electrically-conducting fluid introduces a force called Lorentz force which acts against the flow if the magnetic field is applied in the normal direction as considered in the present problem. This type of resistive force tends to slow down the flow field. Since the magnetic field has a stabilizing effect, the maximum velocity overshoot is observed for the conducting air while minimum overshoot takes place for the water. It is observed from Fig.3 that the fluid velocity  $u_1$  decreases near the plate and it increases away from the plate with an increase in radiation parameter  $R$ . The radiation parameter arises only in the energy equation in the thermal diffusion term and via coupling of the temperature field with the buoyancy terms in the momentum equation, the fluid velocity is indirectly influenced by thermal radiation effects. An increase in radiation parameter  $R$  clearly reduces substantially the fluid velocity in the boundary layer i.e. decelerates the flow.

Fig.4 reveals that the fluid velocity  $u_1$  decreases near the plate and it increases away from the plate with an increase in Prandtl number  $Pr$ . Physically, this is true because the increase in the Prandtl number is due to increase in the viscosity of the fluid which makes the fluid thick and hence causes a decrease in the velocity of the fluid. It is seen from Fig.5 that the velocity  $u_1$  increases near the plate but on moving away from the plate, the opposite trend is observed with an increase in Grashof number  $Gr$ . An increase in Grashof number leads to increase the fluid velocity, this is because, an increase in Grashof number means more heating and less density. Fig.6 displays that the fluid velocity  $u_1$  decreases near the plate and it increases away from the plate with an increase in suction parameter  $S$ . This means that the suction at the plate have a retarding influence on the flow field. It is seen from Figs.7-8 that the fluid velocity  $u_1$  decreases near the plate and it increases away from the plate with an increase in either frequency parameter  $n$  or phase angle  $n\tau$ . This means that the frequency parameter or the phase angle have a retarding influence on the flow field. It is observed from Fig.9 that the temperature  $\theta(\eta)$  decreases near the plate and it increases away from the plate with an increase in radiation parameter  $R$ . Increasing radiation parameter clearly depressed the fluid temperature in presence of conducting air ( $Pr=0.71$ ) and magnetohydrodynamic flow. Also, the plate temperature decreases with an increase in radiation parameter  $R$ . Fig.10 reveals that the temperature  $\theta(\eta)$  decreases near the plate and it increases away from the plate with an increase in Prandtl number  $Pr$ . Prandtl number controls the relative thickness of the momentum and thermal boundary layers. When  $Pr$  is of low value, heat diffusion exceeds momentum diffusion. For  $Pr < 1$ , the thickness of the thermal boundary layer therefore exceeds the thickness of the velocity boundary layer that is, temperatures will be greater. In Fig.10, temperatures are seen to decrease considerably with an increase in  $Pr$  values as we progress into the boundary layer regime; profiles also decay much more sharply for higher  $Pr$  values since momentum diffusion exceeds energy diffusion for  $Pr > 1$ . It is found from Figs.11-12 that the temperature  $\theta(\eta)$  decreases near the plate and it increases away from the plate with an increase in either suction parameter  $S$  or frequency parameter  $n$ . Fig.13 demonstrates that the fluid temperature  $\theta(\eta)$  increases near the plate and it decreases away from the plate with an increase in phase angle  $n\tau$ . It is seen that there exists a boundary layer of thickness of the order  $O(\alpha_1^{-1})$  where  $\alpha_1$  is given by (21). It is seen that the thickness of this boundary layer decreases with an increase in either magnetic parameter  $M^2$  or suction parameter  $S$  or frequency parameter but it is independent of radiation parameter  $R$ .

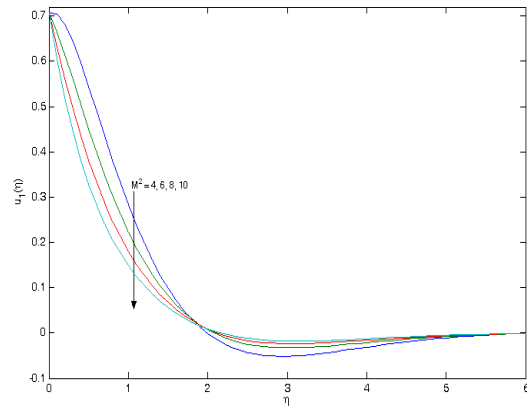


Fig.2: Velocity profiles for  $M^2$  when  $R=4$ ,  $S=0.5$ ,  $Pr=0.71$ ,  $Gr=5$ ,  $n=2$ ,  $\tau=0.5$  and  $n\tau = \frac{\pi}{4}$

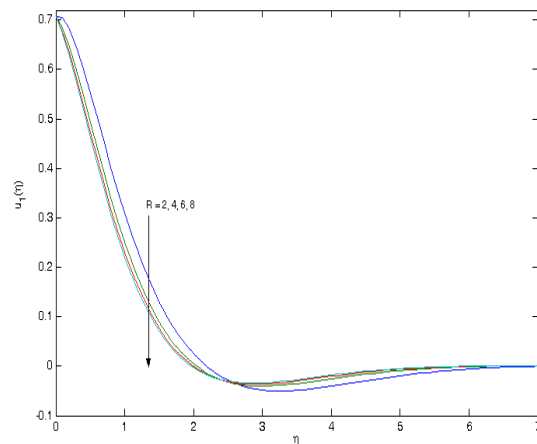


Fig.3: Velocity profiles for  $R$  when  $M^2=5$ ,  $S=0.5$ ,  $Pr=0.71$ ,  $Gr=5$ ,  $n=2$ ,  $\tau=0.5$  and  $n\tau = \frac{\pi}{4}$

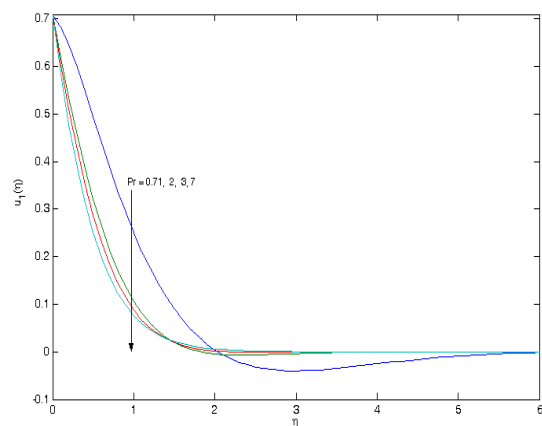
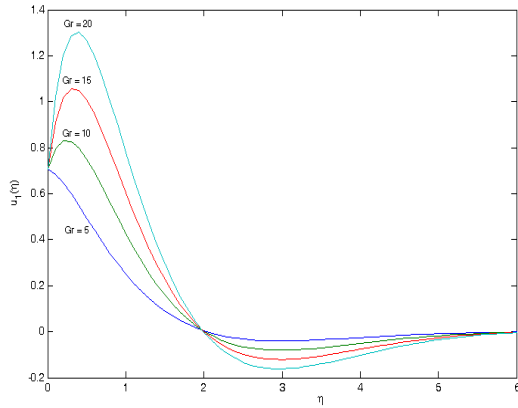
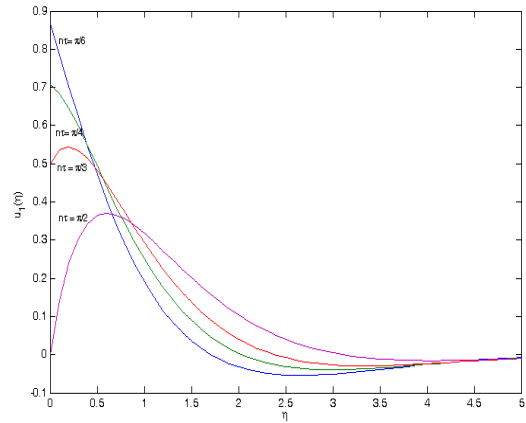


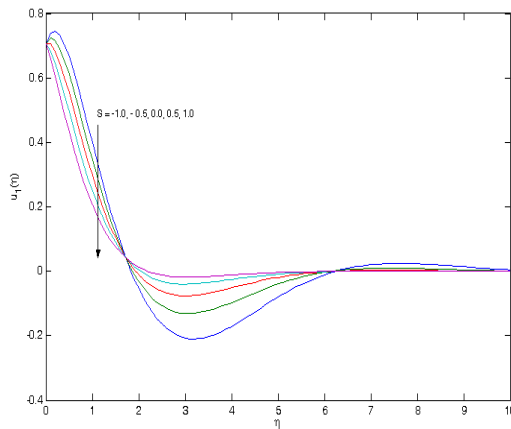
Fig.4: Velocity profiles for  $Pr$  when  $M^2=5$ ,  $S=0.5$ ,  $R=4$ ,  $Gr=5$ ,  $n=2$  and  $n\tau = \frac{\pi}{4}$



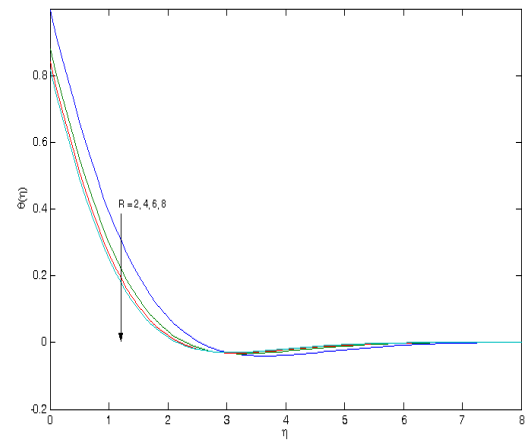
**Fig.5.** Velocity profiles for  $Gr$  when  $M^2 = 5$ ,  $S = 0.5$ ,  $R = 4$ ,  $Pr = 0.71$ ,  $n = 2$  and  $n\tau = \frac{\pi}{4}$



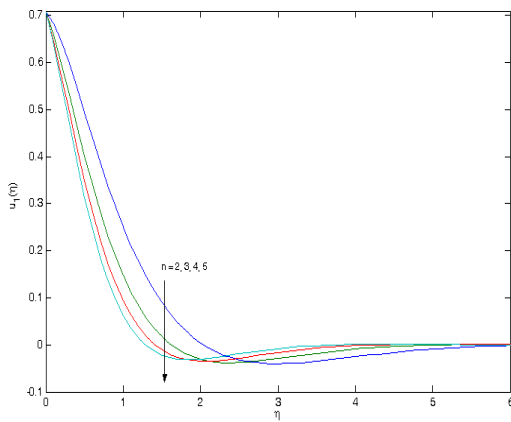
**Fig.8.** Velocity profiles for  $n\tau$  when  $M^2 = 5$ ,  $Gr = 5$ ,  $R = 4$ ,  $Pr = 0.71$ ,  $n = 2$  and  $S = 0.5$



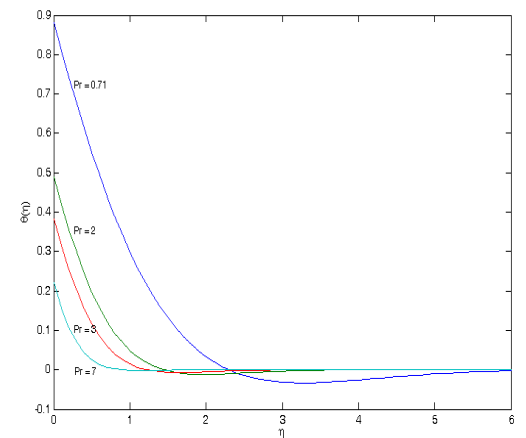
**Fig.6.** Velocity profiles for  $S$  when  $M^2 = 5$ ,  $Gr = 5$ ,  $R = 4$ ,  $Pr = 0.71$ ,  $n = 2$  and  $n\tau = \frac{\pi}{4}$



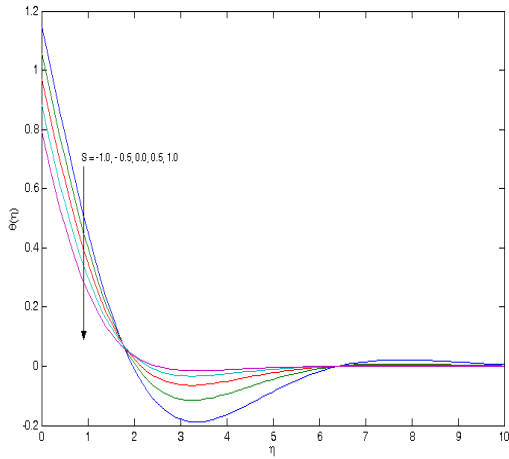
**Fig.9.** Temperature profiles for  $Ra$  when  $Pr = 0.71$ ,  $S = 0.5$ ,  $n = 2$  and  $n\tau = \frac{\pi}{4}$



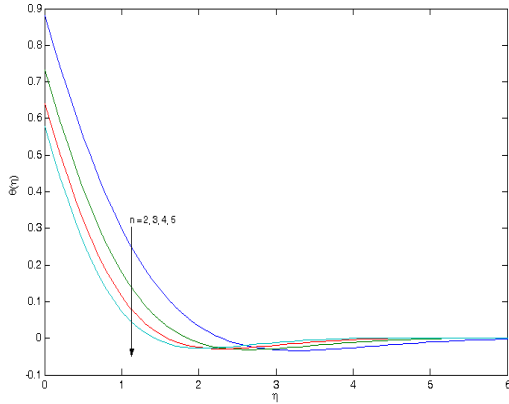
**Fig.7.** Velocity profiles for the variation of  $n$  when  $M^2 = 5$ ,  $Gr = 5$ ,  $R = 4$ ,  $Pr = 0.71$ ,  $n = 2$  and  $n\tau = \frac{\pi}{4}$



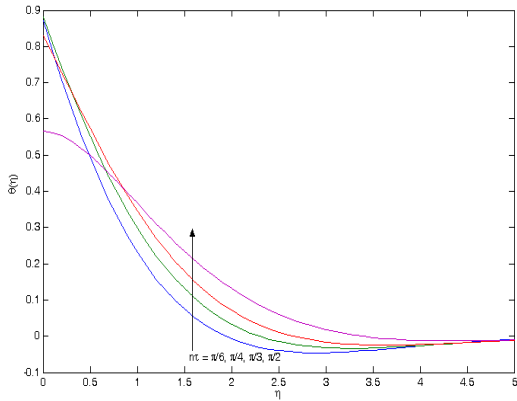
**Fig.10.** Temperature profiles for  $Pr$  when  $R = 4$ ,  $S = 0.5$ ,  $n = 2$  and  $n\tau = \frac{\pi}{4}$



**Fig.11.** Temperature profiles for  $S$  when  $Pr = 0.71$ ,  $R = 4$ ,  $n = 2$  and  $n\tau = \frac{\pi}{4}$



**Fig.12.** Temperature profiles for  $n$  when  $Pr = 0.71$ ,  $R = 4$ ,  $S = 0.5$  and  $n\tau = \frac{\pi}{4}$



**Fig.13.** Temperature profiles for  $n\tau$  when  $R = 4$ ,  $Pr = 0.71$ ,  $S = 0.5$  and  $n = 2$

The non-dimensional shear stress at the plate  $\eta = 0$  is given by

$$\tau_x = \left( \frac{\partial u_1}{\partial \eta} \right)_{\eta=0} = -R_0 \cos(n\tau + \phi), \quad (24)$$

where

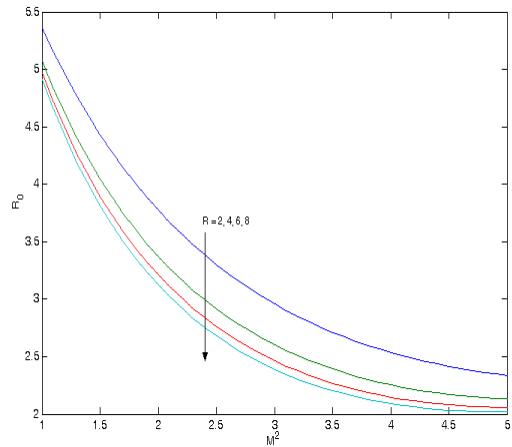
$$R_0 = \left[ \left\{ \alpha_1(1+GrA) + B\beta - \alpha A - GrB\beta \right\}^2 + \left\{ \beta_1(1+GrA) + A\beta + \alpha B + GrB\alpha_1 \right\}^2 \right]^{\frac{1}{2}} \quad (25)$$

$$\tan \phi = \frac{\beta_1(1+GrA) + A\beta + \alpha B + GrB\alpha_1}{\alpha_1(1+GrA) + B\beta - \alpha A - GrB\beta}, \quad (26)$$

where  $\alpha$ ,  $\beta$ ,  $\alpha_1$ ,  $\beta_1$ ,  $A$  and  $B$  are given by (21).

Variations of the amplitude of the shear stress  $R_0$  and the tangent of the phase angle of the shear stress  $\tan \phi$  respectively against  $M^2$  for several values of  $R$ ,  $Pr$ ,  $S$  and  $n$  with  $n\tau = \frac{\pi}{4}$

are shown in Figs.14-20. It is observed from Figs.14-16 that the amplitude  $R_0$  decreases with an increase in either radiation parameter  $R$  or Prandtl number  $Pr$  or suction parameter  $S$ . Fig.17 shows that the amplitude  $R_0$  first increases, reaches a maximum and then decreases with an increase in frequency parameter  $n$ . It is observed from Figs.18-20 that the tangent of the phase angle  $\tan \phi$  decreases with an increase in either radiation parameter  $R$  or Prandtl number  $Pr$  or suction parameter  $S$ .



**Fig.14.** Amplitude  $R_0$  for  $R$  when  $S = 0.5$ ,  $Pr = 0.71$ ,  $S = 0.5$  and  $n = 2$

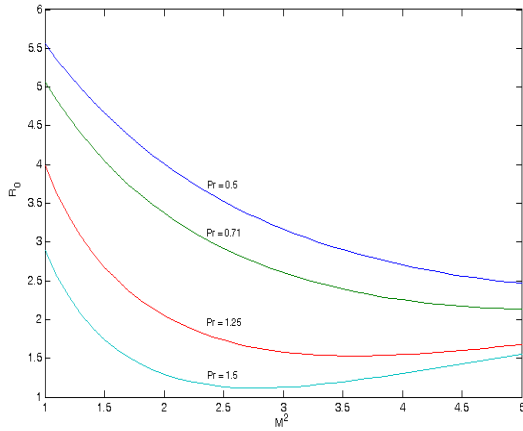


Fig.15. Amplitude  $R_0$  for  $Pr$  when  $R = 4$ ,  $S = 0.5$  and  $n = 2$

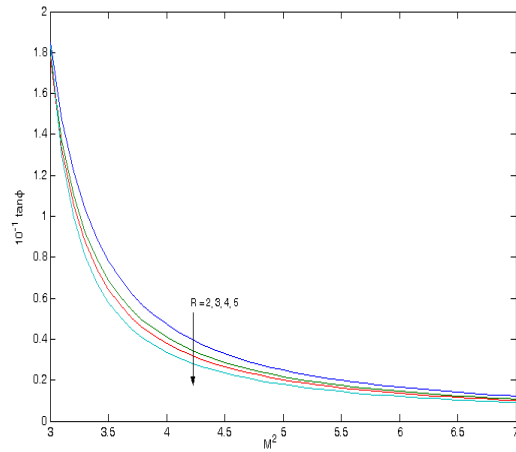


Fig.18. The tangent of phase angle  $\tan \phi$  for  $R$  when  $S = 0.5$ ,  $Pr = 0.71$  and  $n = 2$

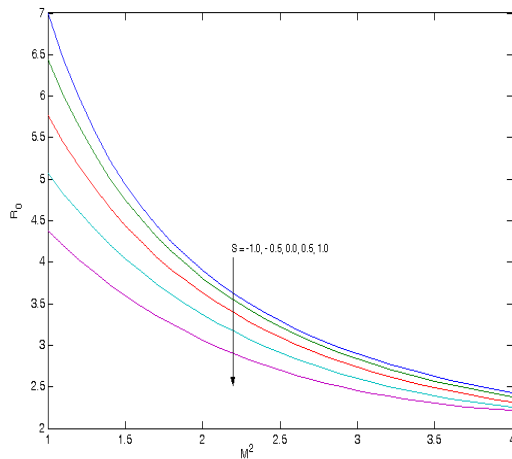


Fig.16. Amplitude  $R_0$  for  $S$  when  $R = 4$ ,  $Pr = 0.71$  and  $n = 2$

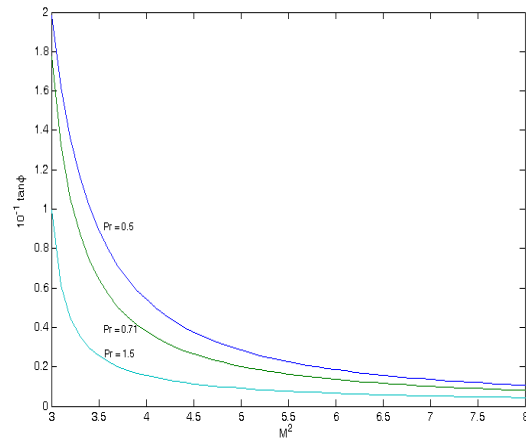


Fig.19. The tangent of phase angle  $\tan \phi$  for  $Pr$  when  $R = 4$ ,  $S = 0.5$  and  $n = 2$

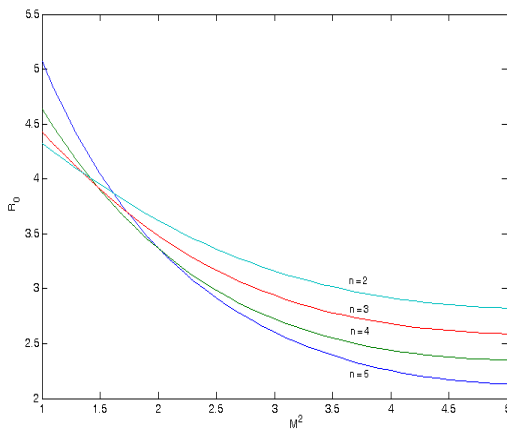


Fig.17. Amplitude  $R_0$  for  $n$  when  $R = 4$ ,  $S = 0.5$  and  $Pr = 0.71$

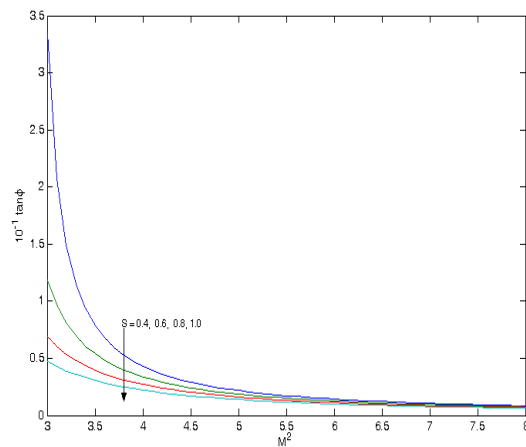


Fig.20. The tangent of phase angle  $\tan \phi$  for  $S$  when  $R = 4$ ,  $Pr = 0.71$  and  $n = 2$

#### 4. CONCLUSION

We have studied the radiation effects on an MHD free convection flow of a viscous incompressible electrically conducting fluid past an oscillating vertical porous plate with a uniform suction or blowing at the plate. It is observed that the fluid velocity  $u_1$  decreases near the plate and it increases away from the plate with an increase in either magnetic parameter  $M^2$  or radiation parameter  $R$  or Prandtl number  $Pr$  or suction parameter  $S$  or frequency parameter  $n$  or phase angle  $n\tau$ . The fluid velocity  $u_1$  increases with an increase in Grashof number  $Gr$ . It also is observed that the solution also exists for the blowing at the plate. The fluid temperature  $\theta$  decreases near the plate and it increases away from the plate with an increase in either  $R$  or  $Pr$  or  $S$  or  $n$ . Further, it is seen that the amplitude of the shear stress  $R_0$  and the tangent of the phase angle  $\phi$  at the plate decrease with an increase in either  $R$  or  $Pr$  or  $S$ .

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