Graph Characterization based on DRI Values and Degree of Graph

Shreedevi V. Shindhe, PhD.
Department of Computer Science
Karnatak University,
Dharwad, India

ABSTRACT

In this paper we are characterizing the different types of graphs based on the value of DRI of each vertex and degree of graph. Here the degree of graph means the maximum degree. The DRI of each vertex has some specific properties for some basic class of graphs. For example if graph is of degree 2 and the DRI sequence $DRI(1, 0, \dots ,01)$ then the graph is a path. Using these observations we can determine the class of graph. Some classes that are covered in this paper are complete graphs, paths, cycles, star graphs, self centered and almost self centered graphs.

Keywords: DRI, diametral paths, diametral reachable index, diametral vertex.

1. INTRODUCTION:

1.1 Distance in graphs:

The basics and definitions are referred by [1, 2, and 3]. The distance $d(u, v)$ between two (not necessary distinct) vertices u and v in a graph G is the length of a shortest path between them. The subscript G is usually dropped when there is no danger of confusion. When u and v are identical, their distance is 0. When u and v are unreachable from each other, their distance is defined to be infinity ∞. The eccentricity $\varepsilon(v)$ of a vertex v in a graph G is the maximum distance from v to any other vertex. $e(v) = max\ \{d(u, v): u\ \epsilon\ V\}$.

The diameter diam(G) of a graph G is the maximum eccentricity over all vertices in a graph; and the radius rad(G), the minimum. When there are two components in G, diam(G) and rad(G) defined to be infinity ∞. Trivially, diam(G) ≤ 2 rad(G).

1.2 Diametral Reachable Index (DRI) of a vertex:

Every graph has one or more diametral path/s. A *diametral path (DP)* of a graph is a shortest path whose length is equal to the diameter of the graph [4, 5].

$$DP = diam(G) = dist(s, d)$$

Let dv be a diametral vertex. There may be one or more diametral paths originating from dv. We want to find all the diametral paths, originating from dv. The total number of diametral paths reachable from a vertex v is called the *Diametral Reachable Index* of that vertex, denoted $DRI(v)$.

For any vertex v, the $DRI(v) = 0$, if there are no diametral paths reachable from v, else we write $DRI(v) = t$, where t is the total number of diametral paths reachable from vertex v. In other words, the DRI of each vertex gives the maximum number of diametral paths reachable from that vertex. The further details regarding DRI can be obtained in [4].

Example 1: The diameter of the graph in fig 1(a) is 2. The diametral reachable index of each vertex is given in fig 1(b).

With these basic definitions and details in the next sectio we can see how to identify the graphs based on the DRI of each vertex for the given input graph. Throughout the paper, the graph is given as input to the algorithm in the form of adjacency matrix.

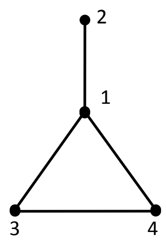

Figure 1.(a)

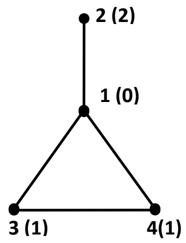

Figure 1.(b)

2. Characterization of Graphs based on DRI of vertices and degree of graph:

In this section we are going to study the DRI sequences of some basic graphs. We have to consider the degree of graphs to make the unambiguous characterization. For our study we are considering the graphs with maximum degree 0, 1 and 2. The *degree of graph* is nothing but the maximum of degrees of all the vertices in the graph. We use the notation $G(n, md, ds)$ to represent the graphs. Here n is the number of vertices, md is the maximum degree of graph and ds is the DRI sequence of the graph.

There are some special cases for the number of vertices i.e., $n = 1$ and $n = 2$. The basic class of graphs with maximum degree 2 are paths and cycles. In this paper we will characterise them using DRI sequence.

Case 1: Characterization of graphs with DRI sequence $\boldsymbol{DRI(1)}$ and maximum degree=0.

When the number of vertex is one, i.e., $n = 1$, the diameter of the graph is 0. The graph does not have any edges. Hence we can say that the DRI of that vertex is 1 as there can be one path with diameter zero (But some people may not agree with this, as the diameter is zero and hence no path).

Any graph with DRI sequence given as $DRI(1)$ is obviously the **trivial graph** [2] with maximum degree 0. The trivial graph is represented by $G(1,0,(1))$. That means any graph with $n=1$, maximum degree=0 and DRI sequence $DRI(1)$ is characterised as trivial graph.

Case 2: Characterization of graphs with DRI sequence $DRI(1,1)$ and maximum degree=1.

When there are two vertices the only connected simple graph that is possible is P_2. For this graph each vertex has the DRI=1, hence the DRI sequence is $DRI(1,1)$. Therefore we can characterise the graph with DRI sequence $DRI(1,1)$ and maximum degree 1 as P_2. The path P_2 is represented by $G(2,1,(1,1))$, i.e. any graph with $n=2$, maximum degree=1 and DRI sequence $DRI(1,1)$ is characterised as path P_2.(Note that path P_2 can also be a star graph with 2 spokes). Example: figure 2.

Figure 2: Path P_2

Based on these basic observations a simple algorithm is developed as follows:

ALGORITHM:

1. Read the number of vertices i.e. n.
2. //**Case 1:**
   If (n=1)

   Display "The graph has only vertex and the DRI of the vertex is 1"

3. **//Case 2:**

If(n==2)

a. Read the adjacency matrix for the graph.
b. Find DRI for each vertex using the algorithm given in [4].

For i=1 to 2

If (DRI[i]==1)

Pcount++;

Else break;

If(Pcount==2)

Display " The graph is P_2"

Else

Display " The graph is not P_2"

2.1 Characterization of graphs with DRI sequence $DRI(1,0,\ldots,0,1)$ and maximum degree=2.

For any graph which is a path, DRI for end vertices will be equal to 1 and for other vertices on the path the DRI is equal to 0. The path itself is the only diametral path, and there are no other diametral paths reachable from any vertex of the path, except the end vertices of the path. Therefore, it can be represented collectively as $DRI(v_1,v_2,\ldots,v_n)=(1,0,0,\ldots,0,0,1)$ [4]. Example: figure 3.

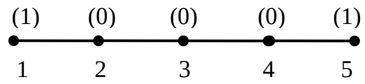

Figure 3: Path graph P_5 and DRI sequence of P_5

Any graph that has DRI sequence, $DRI(1,0,\ldots,01)$ and maximum degree=2, then the graph is a path P_n (for $n>2$). The path is represented as $G(n,2,(1,0,\ldots,0,1))$ where $n>2$. Here considering maximum degree of graph is very important, as there are some other graphs with same DRI sequence but with maximum degree greater than 2.

With the observation that only the end vertices have DRI=1 and all the other vertices have DRI=0, the algorithm is as follows.

1. Read the number of vertices i.e. n.
2. Read the adjacency matrix for the graph.
3. Find the maximum degree of graph.
4. If (maxdegree=2 And n>2)

   Go to step 5

   Else

   Display ("The input graph is not of max degree 2 or $n \ngtr 2$")

   Display("Enter the graph again")

   Go to step 1

5. Find DRI for each vertex using the algorithm given in [4].
6.

EV_ct=0;

NonEV_ct=0;

For i=1 to n

If (DRI[i]==0)

NonEv_ct++;

Else If (DRI[i]==1)

EV_ct++;

Else

Break;

If(EV_ct==2 AND NonEV_ct==n-2)

Display "The graph is Path"

Else

Display "The graph is not Path".

2.2 Characterization of graphs with DRI sequence $DRI(2,2,\ldots,2,2)$ and maximum degree=2.

For cycles we have $DRI(v_1, v_2, \ldots, v_n) = (2,2,\ldots,2,2,2)$ [4]. That is all the vertices have $DRI = 2$. The maximum degree of cycles is 2 (even the minimum degree is also 2). Therefore the graphs with maximum degree 2 and DRI sequence $DRI(2,2,\ldots,2,2)$ can be classified as cycles.

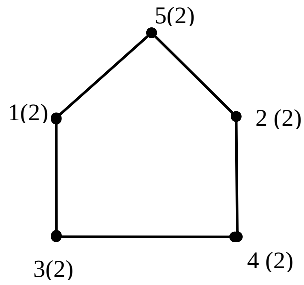

Figure 4: DRI of vertices of cycle C_5.

The algorithm to determine whether the given graph is cycle or not based on DRI sequence is given as follows.

1. Read the number of vertices i.e. n.
2. Read the adjacency matrix for the graph.
3. Find the maximum degree of graph.
4. If (maxdegree=2 And n>2)

   Go to step 5

   Else

   Display ("The input graph is not of max degree 2 or $n \ngtr 2$")

   Display ("Enter the graph again")

   Go to step 1
5. Find DRI for each vertex using the algorithm given in [4].
6.

```
vertex_ct=0;
For i=1 to n
    If (DRI[i]==2)
        vertex_ct++;
    Else
        Break;
If(vertex_ct==n)
    Display "The graph is a cycle"
Else
    Display "The graph is not cycle".
```

2.3. Characterization of graphs with DRI sequence only.

For the graphs that have maximum degree more than 2, characterising graphs based on DRI is not an easier task. But there are some class of graphs for which the DRI of each vertex will be of specific nature. Some of such classes are complete graphs and star graphs which can be characterised based on the DRI sequence. In this paper we have dealt with these two graphs only. *For these graphs we don't need to consider the maximum degree of graph*. For such graphs the graph representation is $G(n, ds)$, where n is the number of vertices and ds is DRI sequence.

2.3.1 Complete Graph:

For complete graphs every vertex has $DRI(v) = n - 1$. [4]. The complete graph is represented as $G(n, (n-1, n-1, \ldots, n-1, n-1))$. The simple algorithm that can be used for determining whether the graph is complete or not is as follows and it makes use of DRI value of each vertex.

1. Read the number of vertices i.e. n.
2. Read the adjacency matrix for the graph.
3. Find DRI for each vertex using the algorithm given in [4].
4.

```
Complete_ct=0;
For i=1 to n
    If (DRI[i]==n-1)
        Complete_ct++;
    Else
        Break;
If(Complete_ct==n)
    Display "The graph is Complete"
Else
    Display "The graph is not Complete"
```

2.3.2 Star Graph:

For star graphs we have $DRI(v_1, v_2, \ldots, v_n) = (0, k-1, k-1, \ldots, k-1)$, where $n = k + 1$ and k is the number of spokes [4]. We observe that $k = n - 1$ and therefore $k - 1 = n - 2$. For star graphs also we don't need to find the maximum degree. The star graph is represented as $G(n, (0, n-2, n-2, \ldots, n-2))$.

The algorithm to determine whether the given graph is a star or not, is as follows.

1. Read the number of vertices i.e. n.
2. Read the adjacency matrix for the graph.
3. Find DRI for each vertex using the algorithm given in [4].
4.

```
Spoke_ct=0;
Center_ct=0;
For i=1 to n
    If (DRI[i]==0)
        Center_ct++;
    Else If ( DRI[i]==n-2)
        Spoke_ct++;
    Else
```

Break;

If(Center_ct==1 AND Spoke_ct==n-1)

Display “The graph is Star graph”

Else

Display “The graph is not star graph”.

3. Characterizing Self Centered Graphs and Almost self Centered Graphs:

This section consists of the relation between DRI, Self Centered Graphs and Almost self Centered graphs [6]. It shows that these graphs can be categorized based on the number of vertices that have $DRI(v) \geq 1$ and $DRI(v) = 0$. Some propositions of [6] are

i. The graphs for which all the vertices have $DRI\ (v) \geq 1$ are called as Self Centered Graphs.
ii. The graphs which have only 2 vertices for which $DRI\ (v) \geq 1$ and if remaining all vertices are central then it’s a almost self centered graph of type $(r+1, r)$.
iii. The graphs that have only one vertex such that its DRI=0 and remaining all vertices have $DRI\ (v) \geq 1$, are called almost self centered graphs of type $(r, r+1)$.

4. CONCLUSIONS:

The paper covers the algorithm for some of the basic class of graphs. For some of the graphs we have to consider the degree of graph to remove the ambiguity of DRI sequences. For some of the classes just the DRI sequence is sufficient to characterise the graphs. The graph characterization based on the number of vertices and DRI values can also be done.

The algorithms given in above sections take time complexity of $O(n^2)$ as we have used adjacency matrix. The algorithm to find DRI sequence of a graph needs $O(n^3)$ time. Therefore by considering this we can say that maximum time taken is $O(n^3)$.

Further the characterization and algorithms can be developed for some more class of graphs. The characterization can be tried with some more class of graphs such as wind mills, geodetic graphs etc. The algorithmic concepts are referred in [3, 8, and 9].

5. ACKNOWLEDGEMENTS:

My sincere thanks to Dr. H. B. Walikar, Professor, Department of Computer Science, Karnatak University, Dharwad, for his valuable suggestions.

6. REFERENCES

[1] Distance in graphs - Fred Buckley, Frank Harary, Addison-Wesley Pub. Co., ©1990 .

[2] Harary. F, Graph Theory, Addison Wesley.

[3] Anany Levitin, Introduction to the design and analysis of Algorithms, 2nd Ed, 2008.

[4] H. B. Walikar, Shreedevi V. Shindhe- *Diametral Reachable Index (DRI) of a vertex*, International Journal of Computer Applications (43-47) Volume 49– No.16, July 2012.

[5] Shreedevi V. Shindhe, Ph. D thesis, “Algorithmic aspects of distance based parameters”, Karnatak University, Dharwad, 2012.

[6] H. B. Walikar, Shreedevi V. Shindhe , Ishwar Baidari - *DRI of a vertex, Self Centered Graphs and Almost Self Centered Graphs - An algorithmic approach*,(152-156) International Journal of Computer Applications, Issue. 2, Vol. 4 August 2012.

[7] Sandi KLAVZAR, Kishori P. NARAYANKAR and H. B. WALIKAR -Almost Self-Centered Graphs, Acta Mathematica Sinica, English Series, Dec., 2011, Vol. 27, No. 12, pp. 2343–2350.

[8] Coremen. T. H, Leiserson, C. E. Rivest R. L, and C Stein, Introduction to Algorithms, 2nd Ed. MIT press, Cambridge, MA, 2001.

[9] Data Structures, Algorithms, and Applications in C++, Sartaj Sahni. McGraw-Hill Education, 1998.