Coupled Fixed Point Theorem in Intuitionistic Fuzzy Metric Space using E. A. Property

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ABSTRACT
The present study introduce the notion of weak compatibility and E. A. property for mixed g-monotone mappings in setting of intuitionistic fuzzy metric space and utilize these perceptions to prove a coupled fixed point theorem for such nonlinear contractive mappings. More to the point together with several recent developments, the efforts of this work can be used to explore a large category of problems. An example is also established for the support of our result.

MSC: 47H10, 54H25

KEYWORDS:
Intuitionistic Fuzzy metric space; Coupled coincidence point; Mixed g-monotone property; Weakly compatible mappings; E.A. property.

1. INTRODUCTION
The concept of intuitionistic fuzzy metric space was introduced and studied by J. H. Park in [26] that generalizes the concept of fuzzy metric space due to George and Veeramani. Initially the concept was motivated by physics point of view in the context of two-slit experiment as the foundation of E-infinity of high energy physics by EI Naschie in ([19] – [24]). Alaca et al. [2] have established intuitionistic fuzzy versions of Banach contraction principle and Edelstein fixed point theorem. No wonder that intuitionistic fuzzy fixed point theory and its application has become an area of interest for researchers for mere review one can refer ([2], [3], [10], [28] – [30], [32]). Pant [35] initially investigated common fixed points of non-compatible mappings on metric space. Aamri and El Moutawakil [1] defined a property (E.A) which generalizes the concept of non-compatible mappings and gave some common fixed point theorems under strict contractive conditions.

Specifically, Bhaskar and Lakshmikanthan [5] established coupled fixed point for mixed monotone operator in partially ordered metric spaces. Afterward, Lakshmikanthan and Ciric [7] extended the results of [5] by furnishing coupled coincidence and coupled fixed point theorem for two commuting mappings having mixed g-monotone property. In a subsequent series, Choudhary and Kundu [6] introduced the concept of compatibility and proved the result of [7] under different set of condition. Recently, Sedghi et al. [25] introduced the concept of coupled fixed point theorem for contraction in fuzzy metric spaces.

Motivated by Bhaskar and Lakshmikanthan [5] and Choudhary and Kundu [6], the purpose of present study is to investigate coupled common fixed point for mappings satisfying E.A. property and possessing monotonicity type properties, in the context of intuitionistic fuzzy metric space which combine method of contraction principle with method of monotone iterations.

In what follows, we collect some relevant definitions, results, examples for our further use.

Definition 1.1[33] A fuzzy set A in X is a function with domain X and Values in [0, 1].

Definition 1.2 A continuous t-norm (in sense of Schweizer and Sklar [31]) is a binary operation $*$ on [0, 1] satisfying the following conditions:

(i) $*$ is a commutative and associative;

(ii) $a * 1 = a$ for all $a \in [0,1]$;

(iii) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ $(a, b, c, d \in [0,1])$;

(iv) $*$ is continuous.

Definition 1.3 A continuous t-conorm (in sense of Schweizer and Sklar [31]) is a binary operation $\diamond$ on [0, 1] satisfying the following conditions:

(i) $\diamond$ is a commutative and associative;

(ii) $a \diamond 0 = 0$ for all $a \in [0,1]$;

(iii) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$ $(a, b, c, d \in [0,1])$;

(iv) $\diamond$ is continuous.

Note 1.4 The concepts of triangular norms (t-norms) and triangular conorms (t-conorms) are known as the axiomatic skeletons that we use for characterizing fuzzy intersections and unions, respectively. These concepts were originally introduced by Menger [17] in his study of statistical metric spaces. Several examples for these concepts were proposed by many authors (see [8], [11], [13]).

Definition 1.5([27]) A intuitionistic fuzzy metric space (in sense of George and Veeramani [14]) is a 5-tuple $(X, M, N, *, \diamond)$, where X is a nonempty set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and M, N is a fuzzy set on $X^2 \times (0, \infty)$ such that the following axioms holds:

(i) $M(x, y, t) > 0$ $(x, y \in X)$;

(ii) $M(x, y, t) = 1$ for all $t > 0$ iff $x = y$;

(iii) $M(x, y, t) = M(y, x, t)$ $(x, y \in X, t > 0)$;

(iv) $M(x, y, \cdot): [0, \infty) \to [0, 1]$ is continuous for all $x, y \in X$;

(v) $M(x, z, t + s) \geq M(x, y, t) * M(y, z, s)$ for all $x, y, z \in X$ and $s, t > 0$.

(vi) $N(x, y, t) \geq 0$ $(x, y \in X)$;

(vii) $N(x, y, t) = 0$ for all $t > 0$ iff $x = y$;

(viii) $N(x, y, t) = N(y, x, t)$ $(x, y \in X, t > 0)$;

(ix) $N(x, y, \cdot): (0, \infty) \to (0, 1]$ is continuous for all $x, y \in X$;

(x) $N(x, z, t + s) \leq N(x, y, t) * N(y, z, s)$ for all $x, y, z \in X$ and $s, t > 0$.

Notice that (M, N) is called an instuitionstic fuzzy metric on X. The value M(x, y, t) can be thought of as degree of nearness between x and y and N(x, y, t) as degree of non-nearness between x and y with respect to t respectively.

Definition 1.6([16]) An instuitionstic fuzzy metric $(X, M, N, *, \diamond)$ on X is said to be stationary if M and N does not depend on t, i.e., the function $M_{x,y}(t) = M(x, y, t)$ and $N_{x,y}(t) = N(x, y, t)$ is constant.

Example 1.7 Let $(X, M, N, *, \diamond)$ be intuitionstic fuzzy metric space and $g : R^+ \to R^+$ is an increasing continuous function. For $m > 0$, we define the function M, N by

$$(1.1)\quad M(x,y,t) = \frac{g(t)}{g(t) + m \cdot d(x,y)} \text{ and } N(x,y,t) = \frac{m \cdot d(x,y)}{g(t) + m \cdot d(x,y)}$$

Then for $a * b = a \cdot b$ and $a \diamond b = \min\{1, a + b\}$, $(X, M, N, *, \diamond)$ is an instuitionstic fuzzy metric on X.

As a particular case if we take $g(t) = t^n$ where $n \in I^+$ and $m = 1$. Then (1.1) becomes

$$(1.2)\quad M(x, y, t) = \frac{t^n}{t^n + d(x,y)} \text{ and } N(x, y, t) = \frac{d(x,y)}{t^n + d(x,y)}$$

Then for $a * b = \min\{a, b\}$ and $a \diamond b = \max\{a, b\}$, $(X, M, N, *, \diamond)$ is an instuitionstic fuzzy metric on X.

If we take $n = 1$ in (1.2), the well-known instuitionstic fuzzy metric obtained.

On the other hand, if we take g as a constant function in (1.1) i.e., $g(t) = k > 0$ and $m = 1$, we obtain

$$M(x, y, t) = \frac{k}{k + d(x,y)} \text{ and } N(x, y, t) = \frac{d(x,y)}{k + d(x,y)}$$

And so $(X, M, N, *, \diamond)$ is an instuitionstic fuzzy metric space for $a * b = a \cdot b$ and $a \diamond b = \min\{1, a + b\}$ but, in general, $(X, M, N, *, \diamond)$ is not for $a * b = \min\{a, b\}$ and $a \diamond b = \max\{a, b\}$.

Definition 1.8([5]) An element $(x, y) \in X \times X$, is called a coupled fixed point of mapping $F: X \times X \to X$ if $F(x, y) = x$ and $F(y, x) = y$.

Definition 1.9([5]) An element $(x, y) \in X \times X$, is called a coupled coincident point of mapping $F: X \times X \to X$ and $g: X \to X$ if $F(x, y) = g(x)$ and $F(y, x) = g(y)$.

Definition 1.10([6]) The mappings F and g where $F: X \times X \to X$ and $g: X \to X$, are said to be compatible if

$$\lim_{n \to \infty} d(g(F(x_n, y_n)), F(g(x_n), g(y_n))) = 0$$

and

$$\lim_{n \to \infty} d(g(F(y_n, x_n)), F(g(y_n), g(x_n))) = 0.$$

whenever $\{x_n\}$ and $\{y_n\}$ are sequences in X, such that $\lim_{n \to \infty} F(x_n, y_n) = \lim_{n \to \infty} g(x_n) = x$ and $\lim_{n \to \infty} F(y_n, x_n) = \lim_{n \to \infty} g(y_n) = y$, for all $x, y \in X$ are satisfied.

Definition 1.11 The mappings F and g where $F: X \times X \to X$ and $g: X \to X$, of a instuitionstic fuzzy metric space $(X, M, N, *, \diamond)$ has g-mixed monotone property if F is monotone g-nondecreasing in fist argument and is monotone g-nonincreasing in second argument.

Now, we introduce the notion of compatibility, weakly compatible and E.A property. for g-mixed monotone mapping in instuitionstic fuzzy metric space.

Definition 1.12 The mappings F and g where $F: X \times X \to X$ and $g: X \to X$, over instuitionstic fuzzy metric space $(X, M, N, *, \diamond)$ are said to be compatible if

$$\lim_{n \to \infty} M(g(F(x_n, y_n)), F(g(x_n), g(y_n)), t) = 1,$$

$$\lim_{n \to \infty} M(g(F(y_n, x_n)), F(g(y_n), g(x_n)), t) = 1.$$

and

$$\lim_{n \to \infty} N(g(F(x_n, y_n)), F(g(x_n), g(y_n)), t) = 0,$$

$$\lim_{n \to \infty} N(g(F(y_n, x_n)), F(g(y_n), g(x_n)), t) = 0.$$

whenever $\{x_n\}$ and $\{y_n\}$ are sequences in X, such that $\lim_{n \to \infty} F(x_n, y_n) = \lim_{n \to \infty} g(x_n) = x$ and $\lim_{n \to \infty} F(y_n, x_n) = \lim_{n \to \infty} g(y_n) = y$, for all $x, y \in X$ are satisfied.

Definition 1.13 The bivariate self mapping, i.e., $F: X \times X \to X$ and self mapping $g: X \to X$ of a intuitionstic fuzzy metric space $(X, M, N, *, \diamond)$ are said to be weakly compatible if they commute at there coincidence points, that is, if for all $x, y \in X$ and $t > 0$

$F(x, y) = g(x)$ for some $x \in X$, then $F(g(x), g(y)) = g(F(x, y))$

and

$F(y, x) = g(y)$ for some $y \in X$, then $F(g(y), g(x)) = g(F(y,x))$.

Definition 1.14 The mappings F and g where $F: X \times X \to X$ and $g: X \to X$, of an intuitionstic fuzzy metric space $(X, M, N, *, \diamond)$ satisfy E.A. property, if there exist sequences $\{x_n\}$ and $\{y_n\}$ in X, such that $\lim_{n \to \infty} F(x_n, y_n) = \lim_{n \to \infty} g(x_n) = g(u)$ and $\lim_{n \to \infty} F(y_n, x_n) = \lim_{n \to \infty} g(y_n) = g(v)$ for $u, v \in X$ and $t > 0$.

Example 1.15 Let $X = [0, \infty)$. Consider $(X, M, N, *, \diamond)$ be an intuitionstic fuzzy metric spaces as in example 1.7. We define $F: X \times X \to X$ and $g: X \to X$ as

$$F(x, y) = \begin{cases} \frac{x-y}{5} & \text{if } x \geq y \\ 0 & \text{if } x < y \end{cases} \text{ and } g(x) = \frac{2x}{5}, \text{ for } x, y \in X.$$

F obeys mixed g-monotone property

Let $\{x_n\}$ and $\{y_n\}$ be two sequences in X defined as

$$x_n = \frac{1}{n} \text{ and } y_n = \frac{1}{2n}$$

then $\lim_{n\to\infty} F(x_n, y_n) = \lim_{n\to\infty} g(x_n) = g(0)$ and $\lim_{n\to\infty} F(y_n, x_n) = \lim_{n\to\infty} g(y_n) = g(0)$ for $0 \in X$ and $t > 0$, i.e., F and g satisfy E. A. property.

Let the class Φ of all mappings φ: $[0, 1] \to [0, 1]$ satisfying the following properties:

(i) φ is continuous and nondecreasing on [0, 1];

(ii) $\varphi(x) > x$ for all $x \in (0, 1)$.

We note that $\varphi \in \Phi$, then $\Phi(1) = 1$ and $\varphi(x) \geq x$ for all $x \in [0, 1]$.

Let Ψ be class of function ψ: $[0, +\infty) \to [0, +\infty)$ satisfying the following properties:

(iii) ψ is continuous and nondecreasing on $[0;+\infty)$;

(iv) $\psi(t) < t$ for each $t > 0$ and $\psi(t) = 0$ if $t = 0$

2. MAIN RESULTS.

In this section, we prove coupled common fixed point theorem for weakly compatible g-mixed monotone mapping satisfying E.A. property for φ-contraction, ψ -contraction. E.A. property buys containment of ranges without any continuity requirements; besides minimize the commutative conditions of the maps to commutativity at their points of coincidence. Moreover, E.A. property allows replacing the completeness requirement of the space with more natural condition of closeness of ranges.

Theorem 2.1 Let $(X, M, N, *, \diamond)$ be intuitionstic fuzzy metric space and F: $X \times X \to X$, g: $X \to X$ be weakly compatible maps of X such that, for some $\varphi \in \Phi$, $\psi \in \Psi$ and x, y, u, v $\in$ X, $t > 0$,

(2.1) $M(F(x, y), F(u, v), t) \geq \varphi(\min\{M(g(x), g(u), t), M(F(x, y), g(u), t), M(F(u, v), g(u), t), M(F(y, x), g(v), t), M(F(x, y), g(u), t)\})$.

(2.2) $N(F(x, y), F(u, v), t) \leq \psi(\max\{N(g(x), g(u), t), N(F(x, y), g(u), t), N(F(u, v), g(u), t), N(F(y, x), g(v), t), N(F(x, y), g(u), t)\})$.

If F and g satisfy E. A. property and g is a closed subspace of X, then F and g have a unique coupled common fixed point.

Proof: Since F and g satisfy E. A. property, therefore, we can find sequences $\{x_n\}$ and $\{y_n\}$ in X and the point u, v in X such that

$$\lim_{n\to\infty} F(x_n, y_n) = \lim_{n\to\infty} g(x_n) = g(u) \text{ and}$$

$$\lim_{n\to\infty} F(y_n, x_n) = \lim_{n\to\infty} g(y_n) = g(v).$$

Then, using (2.1) and (2.2), one obtain

$$M(F(x_n, y_n), F(u, v), t) \geq \varphi(\min\{M(g(x_n), g(u), t), M(F(x_n, y_n), g(u), t), M(F(u, v), g(u), t), M(F(y_n, x_n), g(v), t), M(F(x_n, y_n), g(u), t)\})$$

$$N(F(x_n, y_n), F(u, v), t) \leq \psi(\max\{N(g(x_n), g(u), t), N(F(x_n, y_n), g(u), t), N(F(u, v), g(u), t), N(F(y_n, x_n), g(v), t), N(F(x_n, y_n), g(u), t)\})$$

Taking the limit as n tends to infinity in the above inequality,

$$M(g(u), F(u, v), t) \geq \varphi(\min\{M(g(u), g(u), t), M(g(u), g(u), t), M(F(u, v), g(u), t), M(g(v), g(v), t), M(g(u), g(u), t)\})$$

$$\geq \varphi(\min\{1, 1, M(F(u, v), g(u), t), 1, 1\}$$

$$= \varphi(M(F(u, v), g(u), t))$$

and

$$N(g(u), F(u, v), t) \leq \psi(\max\{N(g(u), g(u), t), N(g(u), g(u), t), N(F(u, v), g(u), t), N(g(v), g(v), t), N(g(u), g(u), t)\})$$

$$\leq \psi(\max\{0, 0, N(F(u, v), g(u), t), 0, 0\}$$

$$= \psi(N(F(u, v), g(u), t))$$

Now, if $F(u, v) \neq g(u)$, then $0 < M(F(u, v), g(u), t) < 1$ and $N(F(u, v), g(u), t) > 0$, that is,

$$\varphi(M(F(u, v), g(u), t)) > M(F(u, v), g(u), t),$$

and $\quad \psi(N(F(u, v), g(u), t)) < N(F(u, v), g(u), t),$

contradicting the above inequality. This proves that $M(g(u), F(u, v), t) = 1$ and $N(g(u), F(u, v), t) = 0$, which implies due to (ii) and (vii) of definition 1.5, $F(u, v) = g(u)$. Similarly, it can be proved that $F(v, u) = g(v)$.

By denoting $F(u, v)$ $(=g(u)) = z_1$ and $F(v, u)$ $(=g(v)) = z_2$, Since F and g are weakly compatible then one obtain that $F(z_1, z_2) = gz_1$ and $F(z_2, z_1) = gz_2$. Let us prove that $z_1 = F(z_1, z_2)$.

Indeed, we obtain by (2.1) and (2.2)

$$M(F(z_1, z_2), z_1, t) = M(F(z_1, z_2), F(u, v), t)$$

$$\geq \varphi(\min\{M(g(z_1), g(u), t), M(F(z_1, z_2), g(u), t), M(F(u, v),g(u),t), M(F(v, u),g(v),t), M(F(z_1, z_2),g(u),t)\}).$$

$$= \varphi(\min\{M(F(z_1, z_2), z_1, t), M(F(z_1, z_2), z_1, t), M(z_1, z_1, t), M(z_2, z_2, t), M(F(z_1, z_1), z_1, t)\}).$$

$$= \varphi(M(F(z_1, z_2), z_1, t)).$$

and

$$N(F(z_1, z_2), z_1, t) = N(F(z_1, z_2), F(u, v), t)$$

$$\leq \psi(\max\{N(g(z_1), g(u), t), N(F(z_1, z_2), g(u), t), N(F(u, v),g(u),t), N(F(v, u),g(v),t), N(F(z_1, z_2),g(u), t)\}).$$

$$= \psi(\max\{N(F(z_1, z_2), z_1, t), N(F(z_1, z_2), z_1, t), N(z_1, z_1, t), N(z_2, z_2, t), N(F(z_1, z_1), z_1, t)\}).$$

$$= \psi(N(F(z_1, z_2), z_1, t)).$$

If $F(z_1, z_2) \neq z_1$ then, from (ii) and (vii) of definition 1.5, $0 < M(F(z_1, z_2), z_1, t) < 1$ and $N(F(z_1, z_2), z_1, t) \geq 0$ for all $t > 0$ and therefore

$$\varphi(M(F(z_1, z_2), z_1, t)) > M(F(z_1, z_2), z_1, t),$$

and

$$\psi(N(F(z_1, z_2), z_1, t)) < N(F(z_1, z_2), z_1, t),$$

which contradicts the above inequality. Thus we obtain that $F(z_1, z_2) = z_1$, Hence z_1 is coupled common fixed point of F and g. Similarly, it can be proved that z_2 is common fixed point of F and g.

Finally, we prove that common fixed point is unique i.e., $z_1 = z_2$. Suppose that it not true. Then $0 < M(z_1, z_2, t) < 0$ and $N(z_1, z_2, t) > 0$ for all $t > 0$ and thus

$$\varphi(M(z_1, z_2, t)) > M(z_1, z_2, t) \text{ for } t > 0,$$

and

$$\psi(N(z_1, z_2, t)) < N(z_1, z_2, t) \text{ for } t > 0,$$

On the other hand, using (2.1) and (2.2), one obtain

$$\begin{aligned} M(z_1, z_2, t) &= M(F(z_1, z_2), F(z_2, z_1), t) \\ &\geq \varphi(\min\{M(g(z_1), g(z_2), t), M(F(z_1, z_2), g(z_2), t), \\ &\quad M(F(z_2,z_1),g(z_2),t), M(F(z_1,z_2),g(z_1),t), M(F(z_1,z_2), g(z_2),t)\}). \\ &= \varphi(\min\{ M(z_1, z_2, t), M(z_1, z_2, t), M(z_2, z_2, t), M(z_1, z_1, t), \\ &\quad M(z_1, z_2, t)\}). \\ &= \varphi(M(z_1, z_2, t)), \end{aligned}$$

and

$$\begin{aligned} N(z_1, z_2, t) &= N (F(z_1, z_2), F(z_2, z_1), t) \\ &\leq \psi(\max\{N(g(z_1), g(z_2), t), N(F(z_1, z_2), g(z_2), t), \\ &\quad N(F(z_2,z_1),g(z_2),t), N(F(z_1,z_2),g(z_1),t), N(F(z_1,z_2),g(z_2),t)\}). \\ &= \psi(\max\{N(z_1, z_2, t), N(z_1, z_2, t), N(z_2, z_2, t), N(z_1, z_1, t), \\ &\quad N(z_1, z_2, t)\}). \\ &= \psi(N(z_1, z_2, t)). \end{aligned}$$

which is a contradiction this concludes the proof.

Corollary 2.2 Let $(X, M, N, *, \diamond)$ be intuitionstic fuzzy metric space and $F: X \times X \to X$, $g: X \to X$ be weakly compatible maps of X such that, for some $\varphi \in \Phi$ and $x, y, u, v \in X$, $t > 0$,

$$\text{(2.2)} \quad M(F(x, y), F(u, v), t) \geq \varphi(\min\{M(g(x), g(u), t)\}),$$

and

$$N(F(x, y), F(u, v), t) \leq \psi(\max\{N(g(x), g(u), t)\})$$

If F and g satisfy E. A. property and g is a closed subspace of X, then F and g have a unique coupled common fixed point.

Remark 2.3 The results of [15] are deduced from the results discussed here, by choosing $f(x) = F(x, y)$ and $f(y) = F(y, x)$ and setting $u = y$ and $v = x$.

Example 2.4 Let $(X, M, N, *, \diamond)$ be intuitionstic fuzzy metric space, where $X = [0, 1]$ and $a*b = ab$ and $a\diamond b = \min\{1, a + b\}$. Let

$$M(x, y, t) = \frac{t}{t + |x-y|} \text{ and}$$

$$N(x, y, t) = \frac{|x-y|}{t + |x-y|} \quad \text{for } x, y \in X \text{ and } t > 0.$$

Let the mapping $g: X \to X$ be defined as

$$g(x) = x^2, \text{ for all } x, y \in X.$$

Let $F: X \times X \to X$ be defined as

$$F(x, y) = \begin{cases} (x-y)^2 & \text{if } x, y \in X \text{ and } x \neq y \\ 0 & \text{if } x = y \end{cases}$$

It is obvious that F obeys mixed g-monotone property and if $\varphi: [0, 1] \to [0, 1]$, $\varphi = \sqrt[5]{t}$ and $\psi: [0,+\infty) \to [0,+\infty)$, $\psi(t) = t^2$ then it is easy that all the conditions of preceding theorem are satisfied. The coupled common fixed point of F and g is (1, 0).

3. CONCLUSION

In this paper, common coupled fixed point theorem has been proved, which reduces in special choice to common fixed point theorems in intuitionstic fuzzy metric.

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