Solving Two Stage Fuzzy Transportation Problem by Row Minima Method

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ABSTRACT

In this paper, two stage cost minimizing fuzzy transportation problem is solved in a feasible method. For this solution Row minimum method is used in which the supplies and demands are trapezoidal fuzzy numbers. A parametric approach is used to obtain a fuzzy solution. Here a Numerical example is solved to check the validity of the proposed method.

Keywords

Trapezoidal fuzzy number, Two stage fuzzy transportation problem, α -optimal solution, Row minima method.

1.INTRODUCTION

The transportation problem (TP) refers to a special class of linear programming problems. In a typical problem a product is to be transported from m sources to n designations and their capacities are $a_1.a_2......a_m$ and b_1, $b_2.....b_n$ respectively. In addition there is a penalty c_{ij} associated with transporting unit of product from source i to designations j. This penalty may be cost or delivery time or safety of delivery etc. A variable x_{ij} represents the unknown quantity to be shipped from source i to destination j.

In some circumstances due to storage constraints, designations are unable to receive the quantity in excess of their minimum demand. After consuming part of whole of this initial shipment, they are prepared to receive the excess quantity in the second stages. According to Sonia and Rita Malhotra [4], in such situations the product transported to the destination has two stages. Just enough of the product is shipped in stage-I so that the minimum requirements of the destinations are satisfied and having done this the surplus quantities (if any) at the sources are shipped to the destinations according to cost consideration. In both the stages the transportation of the product from sources to the destinations is done in parallel. The aim is to minimize the sum of the transportation costs in the two states.

2.FUZZY CONCEPTS

L.A Zadeh, advanced the fizzy theory in 1965. The theory proposes a mathematical technique for dealing with imprecise concepts and problems that have many possible solutions.

The concept of fuzzy mathematical programming on a general level was first proposed by Tanaka *et al* (1974) in the frame work of the fuzzy decision of Bellman and Zadeh [5] Now, we present some necessary definition [2.3].

2.1 Definition (Fuzzy Number)

A real fuzzy number $\bar{a}$ is a fuzzy subset of the real number R with membership function $\mu_{\bar{a}}$ satisfying the following conditions.

1. $\mu_{\bar{a}}$ is continuous from R to the closed interval [0, 1]
2. $\mu_{\bar{a}}$ is strictly increasing and continuous on $[a_1, a_2]$
3. $\mu_{\bar{a}}$ is strictly decreasing and continuous of $[a_3, a_4]$

where a_1, a_2, a_3, and a_4 are real numbers, and the fuzzy number denoted by a=[a_1, a_2, a_3, a_4] is called fuzzy trapezoidal number.

2.2 Definition

The fuzzy number $\bar{a}$ = [a_1, a_2, a_3, a_4] is a trapezoidal number, denoted by [a_1, a_2, a_3, a_4] its membership function μ_{a} is given by Fig 1.

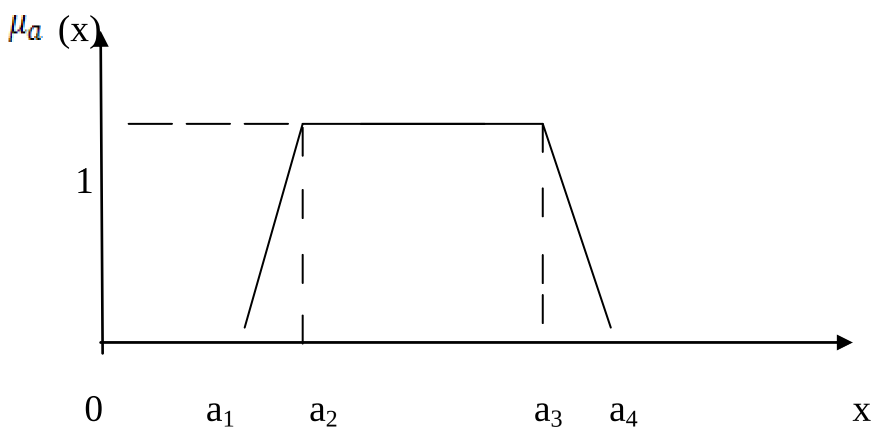

Fig.1. Membership function of a fuzzy number $\bar{a}$

2.3 Definition ($\propto$-level set)

The $\propto$-level set of the fuzzy number $\bar{a}$ and $\bar{b}$ is defined as the ordinary set $L_{\propto}$ ($\bar{a}$, $\bar{b}$) for which the degree of their membership function exceeds the level $\propto \in [0,1]$.

$L\propto(\bar{a}, \bar{b})=\{a,b \in R^m / \mu_{\bar{a}}(a_i, b_j) \geq ($, i= 1,2.........m, j=1,2.....n}

3 Theoretical development

Let $\bar{b}_j$ be the minimum fuzzy requirement of a homogeneous product at the destination j and $\bar{a}_i$ the fuzzy availability of the same at source i. The Two stage Fuzzy Cost Minimization Transportation Problem (FCMTP) deals with supplying the destinations their minimum requirements in stage-I and the quantity $\sum_i \bar{a}_i - \sum_j \bar{b}_j$, is supplied to the destinations in stages-II from the sources which have surplus quantity left after the completion of stage-I. Mathematically stated, the stage-I problem is

$$\min_{x \in s_1} [c_1(x)] = \min_{x \in s_1} \left[\max_{|x|}\left(c_{ij}(x_{ij})\right)\right]$$

where the set s_1 is given by

$$s_1 = \begin{cases} \sum_{j=1}^{n} X_{ij} \leq \bar{a}i & i=1,2\ldots\ldots m \\ \sum_{i=1}^{m} X_{ij} = \bar{b}j & j=1,2\ldots\ldots n \end{cases}$$

$x_{ij} \geq 0 \ \forall$ (i,j) corresponding to a feasible solution $X=(x_{ij})$ of the stage-1 problem, the set $S_2= \{\bar{X} = (\bar{X}_{ij})\}$ of feasible solutions of the stage-II problem is given by

$$s_2 = \begin{cases} \sum_{j=1}^{n} X_{ij} \leq \bar{a}i & i=1,2\ldots\ldots m \\ \sum_{i=1}^{m} X_{ij} \geq \bar{b}j & j=1,2\ldots\ldots n \end{cases}$$

$\bar{X}_{ij} \geq o, \forall(i,j)$, where $\bar{a}'_i$ is the quantity available at the i^{th} source on completion so the stage –I, that is $\bar{a}'_i = \bar{a}_i - \sum_j X_{ij}$ Clearly $\sum_i \bar{a}'_i = \sum_i \bar{a}_i - \sum_j \bar{b}_j$. Thus the stage –II problem would be mathematically formulated as:

$$\min_{\bar{X} \in s_2} [c_2(x)] = \min_{\bar{X} \in s_2} \left[\max_{|x|}\left(c_{ij}(\bar{X}_{ij})\right)\right]$$

We aim at finding that feasible schedule $X=(X_{ij})$ of the stage-1 problem corresponding to which the optimal cost for stage-II is such that the sum of the shipment is the least. The Two Stage fuzzy cost minimizing Transportation problem can, therefore, be stated as,

$$\min_{X \in s_1}\left[C_1(X) + \left(\min_{\bar{X} \in s_2} C_2(\bar{X})\right)\right] \qquad (3)$$

Also from a feasible solution of the problem (3) can be obtained. Further the problem (3) can be solved by solving the following fuzzy cost minimizing Transportation problem.

$$\min_{X \in S_2}[C(X)] = \min_{X \in S_2}\left[\max_{|X|}(C_{ij}(X_{ij}))\right]$$

where S_2, the set of feasible solutions of (3), is defined as follows

$$S_2 = \begin{cases} \sum_{j=1}^{n} X'_{ij} = \bar{a}_i & i = 1,2\ldots m \\ \sum_{i=1}^{m} X'_{ij} = b_j & j=1,2,\ldots\ldots n \end{cases}$$

$$x'_{ij} \geq 0 \forall (i,j)$$

where $\bar{a}_i$ and $\bar{b}_i$, represent fuzzy involved in the constraints with their membership functions for, $\mu_{\bar{a}}$ a certain degree α together with the concept of α level set of the fuzzy numbers $\bar{a}_i$, $\bar{b}_j$, therefore problem Two stage FCMTP can be understood as following non fuzzy a- general Two stage transportation problem(a- two stage FCMTP),

$$S = \begin{cases} \sum_{j=1}^{n} X_{ij} = \bar{a}_i & i=1,2,\ldots\ldots m \\ \sum_{i=1}^{m} X_{ij} = b_j & j=1,2,\ldots\ldots n \end{cases}$$

$a_i, b_j \in L\alpha(\bar{a}_i, \bar{b}_j)$

Where $L\alpha(\bar{a}_i, \bar{b}_j)$are the α level set of the fuzzy number $\bar{a}_i, \bar{b}_j$. let $x(\bar{a}_i, \bar{b}_j)$denote the constraint set of problem and supposed to be non empty. On the basis of the a-level sets of the fuzzy numbers, we give the concept of a -optimal solution in the following definition.

3.1 Definition (α -optimal solution)

A point $x^* \in X(\bar{a}_i, \bar{b}_j)$ is said to be α -optimal solution (α -Two stage FCMTP). if and only if there does not exist another X,Y $\in X(a,b)$, a,b $\in L_\alpha(\bar{a}i, \bar{b}j)$, such that C_{ij},X_{ij}, C_{ij},X_{ij} with strict inequality holding for at least one

C_{ij} where for corresponding values of parameters ($\bar{a}, \bar{b}$) are called α -level optimal parameters.

The problem (α -Two stage FCMTP) can be rewritten in the following equivalent form (α -Two stage FCMTP)

$$S = \begin{cases} \sum_{j=1}^{n} X_{ij} = a_i & i=1,2,......m \\ \sum_{i=1}^{m} X_{ij} = b_j & j=1,2,......n \end{cases}$$

$$h_i^0 \le a_i \le H_i^0 h_j^0 \le b_j \le H_j^0$$

$$X_{ij} \ge 0 \;\forall\, i,j$$

It should be noted that the constraint $(a_i, b_j \in L_1\alpha\,(\bar{a}_1(i,)\, \bar{b}_1 j$))has been replaced by the constraints $h_i^0 \le a_i \le H_i^0$ and $h_j^0 \le b_j \le H_j^0$,where h_i^0 and H_i^0 and h_j^0 and H_j^0 are lower and upper bounds and a_i, b_j are constants.

The parametric study of the problem (α' - Two stage FCMTP) where h_i^0 , H_i^0 and h_j^0, H_j^0 are assumed to be parameters rather than constants and (renamed h_i ,H_i and h_j,H_j) can be understood as follows

Let X(h,H) denotes the decision space of problem (α' - Two Stage FCMTP). defined by

$$X(h,H) = (X_{ij}, a_i, b_j) \in R^{n(n+1)} \left| a_i - \sum_j X_{ij} \ge 0 \right., \; b_j - \sum_i X_{ij} \ge 0, \; H_i - a_i \ge 0, \; H_j - b_j \ge 0,$$

$$a_i - h_i \ge 0, \; b_j - h_j \ge 0, \quad X_{ij} \ge 0, i \in I, j \in J$$

4. Row minima method algorithm

Step 1: Allocate as much as possible in the Lowest cost cell of the first row until the capacity of the source 1 (First row) is exhausted or the requirement at j^{th} distribution centre is satisfied or both three cases aries:[8]

If the capacity of the source 1 is completely exhausted, cross off the first row and proceed to second row.

i. If the requirement at j^{th} destination is satisfied, cross off the j^{th} column and reconsider the first row with the remaining capacity.

ii. If the capacity of the source 1 as well as the requirement at j^{th} destination is completely satisfied, make a zero allocation in the second lowest cost cell of the first row. Cross off the row as well as the j^{th} column and move down the second row.

Step 2 : Continue the process for the resulting reduced transportation table until the rim conditions are satisfied

5.Solution algorithm

Step 1 : Construct the Problem 4

Step 2 : Determine the Points (a_1, a_2,a_3,a_4) and (b_1,b_2,b_3,b_4) for the fuzzy number in the formulation problem (Two stage FCMTP).

Step 3 : Convert the problem (α-Two stage FCMTP) in the form of the problem (α '-Two stage FCMTP).

Step 4 : Formulate the Problem(α '-Two stage FCMTP) in the parametric form.

Step5 : Apply Row minima to get the basic feasible solution

Step6 : Declare min (C1+C2) as the optimal value of the objective function of the problem

6. Example

Consider the following 4X6 Two Stage cost Minimizing Transportation problem.

	D_1	D_2	D_3	D_4	D_5	D_6	a_i
S_1	2	3	5	11	4	2	(4,5,7,8)
S_2	4	7	9	5	10	4	(6,7,8,9)
S_3	12	25	9	6	26	12	(5,6,7,8)
S_4	8	7	9	24	10	8	(4,6,8,9)
b_j	(1,2, 4,5)	(4,5, 6,7)	(3,4,5, 7)	(4,5,6 7)	(2,3,4, 5)	(3,4,5, 6)	

Consider α- level set be α=0.75, then we get
$4{,}5\le a_1\le 7.5, 6.5\le a_2\le 8.5, 5.5\le a_3\le 7.5,$
$5.0\le a_4 \le 8.5,$
$1.5\le b_1\le 4.5, 4.5\le b_2\le 6.5, 3.5\le b_3\le 6.0, 4.5\le b_4\le 6.5, 2.5\le b_5\le 4.5, 3.5 \le b_6\le 5.5.$

The α-optimal parameters are

$a_1=6,\ a_2 =8,\ a_3= 7,\ a_4=7$

$b_1=3,\ b_2=5,\ b_3=5,\ b_4=6,\ b_5=4,\ b_6=5$

Stage 1

We take $a_1=3,\ a_2=4,\ a_3=3,\ a_4=3$

$b_1=1, b_2=2, b_3=3, b_4=3, b_5=2, b_6=2$

2	3	5	11	4	2	3
4	7	9	5	10	4	4
12	25	9	6	26	12	3
8	7	9	24	10	8	3
1	2	3	3	2	2	

After applying the Row minima

$X_{11}=1, X_{16}=2, X_{22}=1, X_{24}=3, X_{33}=3, X_{42}=1, X_{45}=2$ & minimize =82

Stage II

We take $a_1= 3$, $a_2=4$, $a_3=3$, $a_4=4$

$b_1=2$, $b_2=3$, $b_3=2$, $b_4=3$, $b_5=2$, $b_6=3$

2	3	5	11	4	2	3
4	7	9	5	10	4	4
12	25	9	6	26	12	3
8	7	9	24	10	8	3
2	3	2	3	2	3	

After applying the Row Minima method we get

$x_{11}=2$, $x_{16}=1$, $x_{24}=2$, $x_{26}=2$, $x_{33}=2$, $x_{34}=1$, $x_{42}=3$ & minimum=69

Therefore the optimal value of the objective function of the problem (3) is given by minimum (82+69) =151

7. Conclusion

Transportation models have wide applications in logistics and supply chain for reducing cost. In this paper row minima method is used to reducing the transportation cost for two stage fuzzy transportation problem, in which supplies, demands are fuzzy trapezoidal numbers is defined.

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