Role of Superior Iterates in Optimizing the Dynamic Noise

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ABSTRACT
The intent of this paper is to present a review of literature on perturbations of the Mandelbrot map in fractal analysis in the recent years. In this paper we have studied the work of various researchers in the field of effect of various noise on Mandelbrot set.

Keywords
Superior Mandelbrot Set, Complex Dynamics, Relative Superior Julia Set, Ishikawa Iteration.

1. INTRODUCTION
In 1975 Benoit Mandelbrot investigates the term fractals are a geometrical shape that appears similar at all scales of magnification having infinitely complex nature [6, 16]. Mandelbrot shows that the fractals are common in every ware in nature from small to large structures, in human body, trees, sea, mountains, clouds, bacteria, and small cells etc, these structures are appear in nature having differ in detail and a repeating patterns. It combines the mathematic with strong structure and computers, computer graphic and computer experiments are essential to working and understand the fractals and dynamical systems [17]. The fractal is deal with the dynamical systems means any process that moves or changes in time, some of dynamical systems are clearly predictable such are the motion of planet, chemical reactions, but some are not predictable such as stock market, weather etc, because of there are many variables which is very difficult to find, these multiple variable at any one time, so that's why it is chaos.

On other hand, one of the most remarkable simple dynamical systems that depend on only one variable can behave Just as unpredictably strange and complicated way that type of system is called iterated functions systems, iterated functions means that iterate himself again and again just like magnification in fractal geometrical shape[6]. In the field of fractal theory there are many researchers worked but there is a milestone name Gaston Julia (1918), a French mathematician investigated the iteration process of complex function intensively and attained a Julia set [15], which is a landmark in the field of fractal theory one of the most complicated and beautiful object, these beautiful object are generated by process called iterations. The complex number (x + iy) is the combination of real and imaginary number, in that case imaginary number (i) is a special because when we squared they give a negative result, normally it not happens when squared positive or negative numbers. Computer use these complex number to generate fractals because of complex numbers have ability to put two coordinate into one number with two parts. The function used for Julia set is $f(z)=z^2+c$ where c is a complex number, and it can have different c values produce different Julia sets.

2. PRELIMINARIES
2.1 Definition: Ishikawa Iteration [12]
Let X be a subset of real or complex numbers and $f: X \to X$ for $x_0 \in X$, we have the sequences $\{x_n\}$ and $\{y_n\}$ in X in the following manner:

$$y_n = s'_n f(x_n) + (1 - s'_n)x_n$$

$$x_{n+1} = s_n f(y_n) + (1 - s'_n)x_n$$

Where $0 \le s'_n \le 1, 0 \le s_n \le 1$ and $s'_n \,\&\, s_n$ are both convergent to non zero number.

2.2 Definition [18]
The sequences x_n and y_n constructed above is called Ishikawa sequences of Iterations or Relative Superior sequences of iterate. We denote it by $RSO(x_0, s_n, s'_n, t)$.

Notice that $RSO(x_0, s_n, s'_n, t)$ with $s'_n = 1$ is $SO(x_0, s_n, t)$ i.e. Mann's orbit [11] and if we place $s_n = s'_n = 1$ then $RSO(x_0, s_n, s'_n, t)$ reduces to $O(x_0, t)$ .We remark that Ishikawa orbit $RSO(x_0, s_n, s'_n, t)$ with $s'_n = \frac{1}{2}$ is relative superior orbit.

Now we define Mandelbrot sets with respect to Ishikawa iterates is named as Relative Superior Mandelbrot set by Negi and Chauhan [18].

2.3 Definition [18]
Relative Superior Mandelbrot set RSM for the function of the form $Q_c(Z) = z^n + c$, where $n = 1, 2, 3, ...$ is defined as the collection of $c \in C$ for which the orbit of 0 is bounded *i.e.* $RSM = \{c \in C : Q^k{}_c(0) : k = 0, 1, 2, ...\}$ is bounded. In functional dynamics, we have existence of two different types of points. Points that leave the interval after a finite number are in stable set of infinity. Points that never leave the interval after any number of iterations have bounded orbits. So, an orbit is bounded if there exists a positive real number.

The collection of points that are bounded, *i.e.* there exists M, such that $|Q^n(z)| \le M$, for all n, is called as a prisoner set while the collection of points that are in the stable set of infinity is called the escape set. Hence, the boundary of the prisoner set is simultaneously the boundary of escape set and that is Julia set for Q.

Mandelbrot set M for the quadratic $Q_c(z) = z^n + c$ is defined as the collection of all $c \in C$ for which the orbit of point 0 is bounded, i.e.

$$M = \{c \in C : \{Q_c{}^n(0)\}; n = 0, 1, 2, 3 ... \text{ is bounded}\}.$$

An equivalent formulation is

$M = \{c \in C \{Q_c^{\,n}(0)\ \textit{does not tends to}\ \infty\ \textit{as}\ n \to \infty\}\}$

we choose the initial point 0, as 0 is the only critical point of Q_c [7].

2.4 Definition [18]

The Mandelbrot map denoted by $Q_c(z)$ which is also called the complex logistic map [1, 15], is formulated as. $x_{n+1} = x^2{}_n - y^2{}_n + c_1,\ y_{n+1} = 2x_n + c_2$, where $c_1, c_2 \in R$. Noise is a type of random variable. It can be divided into additive noise and multiplicative noise according to its appearances style in dynamical equations. Argyris et.al; [1-5] have studied additive noise on Mandelbrot set given by $x_{n+1} = x^2{}_n - y^2{}_n + c_1 + m_1 w_n,\quad y_{n+1} = 2x_n y_n + c_2 + m_2 w_n$, where w_n is a vector of noise, and $m_1, m_2 \in R$ are parameters designating the strength of noise, and the other hand multiplicative noise studied in [2,3] is given by $x_{n+1} = (1 + k_1 w_n) x^2{}_n - (1 + k_1 w_n) y^2{}_n + c_1$, $y_{n+1} = (2 + k_2 w_n) x_n y_n + c_{2,}$ where $k_1, k_2 \in R$ are parameter designating the strength of noise, Argyris et al;[3] classification of dynamic systems subject of noise perturbation are $Q_c^{\,a,m}$ as following $x_{n+1} = (1 + m_1 w_n)(x_n)^2 - (1 + m_1 w_n)(y_n)^2 + c_1 + a_1 w_n$ $y_{n+1} = (2 + m_2 w_n) x_n y_n + c_2 + a_2 w_n$ .Where w_n is a noise, input and $a_1, a_2, m_1, m_2 \in R$ define the strength of the additive and multiplicative dynamic noise, noise was simulated using random numbers.

3. GENERATING PROCESS

The basic principle of generating fractals employs the iterative formula $z_{n+1} \leftarrow f(z_n)$ where z_0 is the initial value of z and z_i is the value of the complex quantity z at the i^{th} iteration for example the Mandelbrot self squared function for generating fractals is $f(z) = z^2 + c$ where z and c are both complex quantities [9] .our intent to study the various researchers in recent year in noise on Mandelbrot set. The author Argyris [1] have investigate the Perturbations on Mandelbrot set and find additive dynamic and output noise and its effect on the Mandelbrot set, the low strength of noise of the Mandelbrot set is found to be stable and when gradually increase the noise, the deterioration in shape of the Mandelbrot set is significant. By the definition of Mandelbrot set [15] associated with the map defined by $x_{n+1} = x^2{}_n - y^2{}_n + c_1\ \textit{and}\ y_{n+1} = 2x_n y_n + c_2$, and find the two parameter family of deformations $Q_c^{\,\alpha,\beta}$, $x_{n+1} = x^2{}_n - y^2{}_n + \alpha x_n + c_1\ \textit{and}\ y_{n+1} = 2x_n y_n + \beta y_n + c_2$, make a numerical results Fig(1-2) shows analytic and non analytic perturbations of the Mandelbrot set.

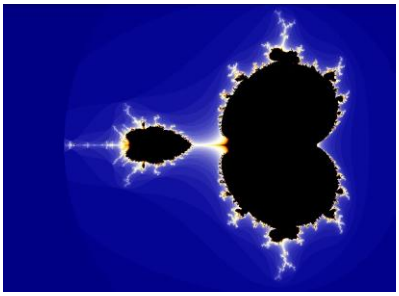

Fig 1:Mandelbrot set non analytic perturbation $\alpha = 0.3, \beta = 0$

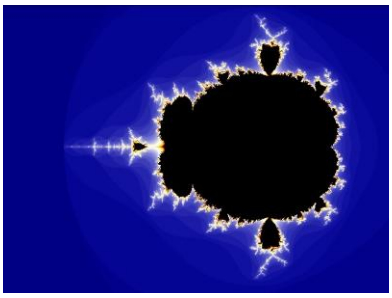

Fig 2:Mandelbrot set Analytic perturbation $\alpha = 0, \beta = 0.3$

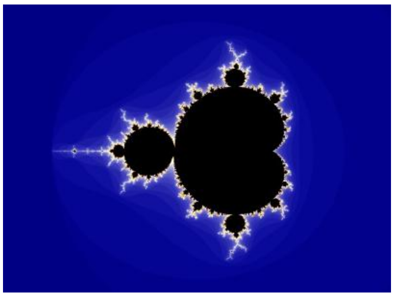

Fig 3:The influence of an additive noise with strength $m_1 = m_2 = 0.01$ **applied to Mandelbrot set.**

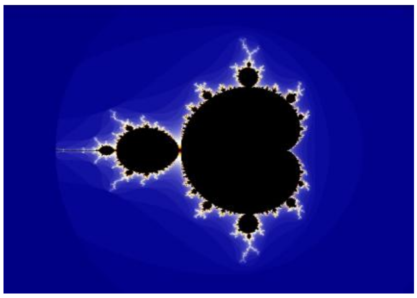

Fig 4:The influence of Multiplicative noise with strength $k_1 = k_2 = 0.1$ **applied to Mandelbrot set.**

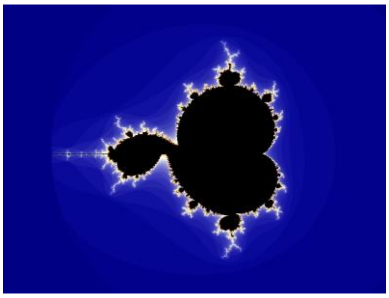

Fig 5: The perturbed Mandelbrot set under the effect of dynamic additive noise with strength $m_1 = m_2 = 0.1$.

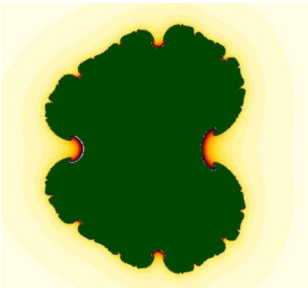

Fig 6: The Julia set $J\left(Q_c^{0.3,0}\right)$ **with** $c_2 = 0$ **and** $c_1 = 0.12253905$

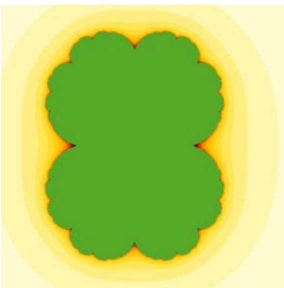

Fig 7: The Julia set $J\left(Q_c^{0.3}\right)$ **with** $c_2 = 0$ **and** $c_1 = 0.1225380$

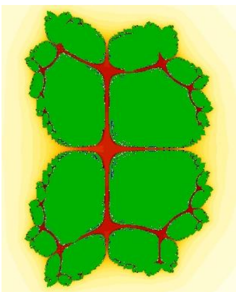

Fig 8: The Julia set $J\left(Q_c^{0,0.3}\right)$ **with** $c_2 = 0$ **and** $c_1 = 0.271$

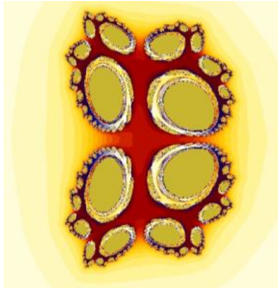

Fig 9: The Julia set $J\left(Q_c^{0,0.3}\right)$ **with** $c_2 = 0$ **and** $c_1 = 0.290$

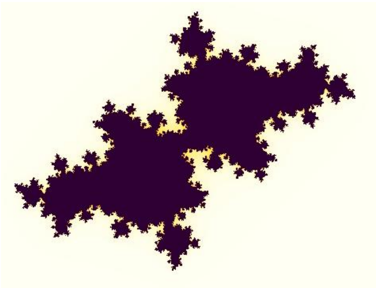

Fig 10:The Julia set $J\left(Q_c\right)$ **with** $c_1 = -0.3904$ **and** $c_2 = -0.58769$

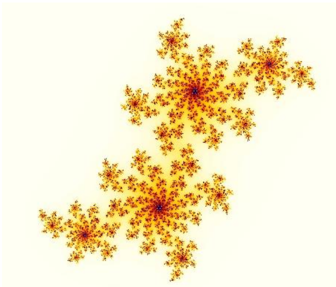

Fig 11:The Julia set with $k_1 = k_2 = 0.01$

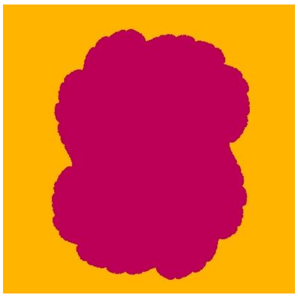

Fig 12: The Julia set of the Mandelbrot map subject to Multiplicative dynamic noise $c_2 = -0.065850$ **and** $c_1 = 0.195$

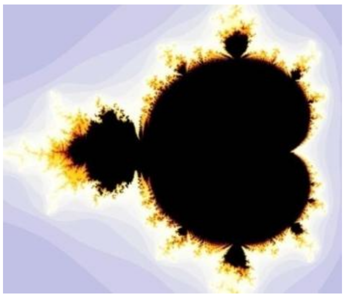

Fig 13: The Mandelbrot set for high additive and low multiplicative noise $(m_1, m_2, k_1, k_2) = (0.2, 0.2, 0.01, 0.01)$

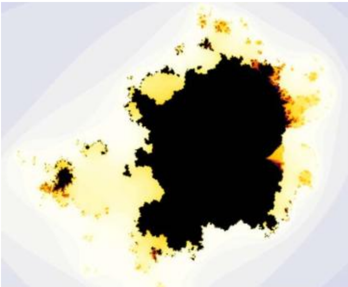

Fig 14: Mandelbrot set for low and high multiplicative noise $(m_1, m_2, k_1, k_2) = (0.01, 0.01, 0.3, 0.3)$

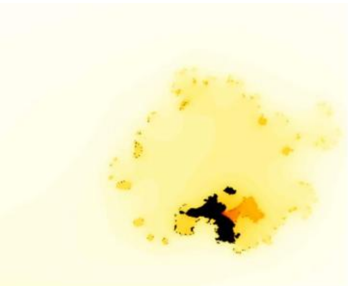

Fig 15: The Mandelbrot set for high additive and high multiplicative noise $(m_1, m_2, k_1, k_2) = (0.5, 0.5, 0.5, 0.5)$

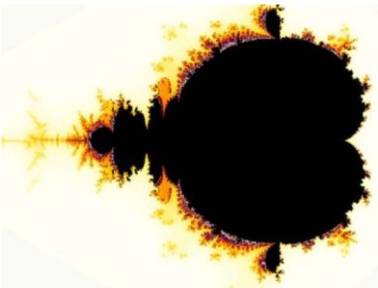

Fig 16: Superior Mandelbrot set for high additive and low multiplicative noise $(m_1, m_2, k_1, k_2) = (0.2, 0.2, 0.01, 0.01)$ **and** $s = 0.3$

After that use the random number generator [10] for simulation of noise and taken additive dynamic noise for Mandelbrot map $x_{n+1} = x^2{}_n - y^2{}_n + c_1 + m_1 w_n,$ $y_{n+1} = 2x_n y_n + c_2 + m_2 w_n,$ where w_n vector of noise and $m_1, m_2 \in R$ are parameters [1] and for multiplicative noise $x_{n+1} = (1 + k_1 w_n) x^2{}_n - (1 + k_1 w_n) y^2{}_n + c_1,\ y_{n+1} = (2 + k_2 w_n) x_n y_n + c_2,$ where $k_1, k_2 \in R$ are parameters see fig(3-5) similarly for output noise take $X_n = x_n + M_1 w_n,\ Y_n = y_n + M_2 w_n$ where $M_1, M_2 \in R$ are parameters and for multiplicative noise $X_n = x_n K_1 w_n,\ Y_n = y_n K_2 w_n$ where $K_1, K_2 \in R$ are parameters. The author [2] further study the corresponding Julia set of a Mandelbrot map for noise, and found that the when the low strengths of noise the Julia set is stable and other hand when increase the noise the Julia set is deterioration of the shape. For this they use their previous numerical results [1], and also consider the generalized Mandelbrot map and stochastic perturbations of Mandelbrot map. For generalize Mandelbrot map follow the

$$x_{n+1} = (x_n)^2 - (y_n)^2 + \alpha x_n + c_1, \quad y_{n+1} = 2x_n y_n + \beta y_n + c_2.$$

where see fig (6, 8, 9) and for stochastic perturbations influence of additive dynamic noise see fig (7) $x_{n+1} = (x_n)^2 - (y_n)^2 + c_1 + m_1 w_n,\ y_{n+1} = 2x_n y_n + c_2 + m_2 w_n,$ wh-ere w vector of a noise is input and m_1, m_2 are parameters, for multiplicative dynamic noise see fig (11).

$$x_{n+1} = (1 + k_1 w_n)(x_n)^2 - (1 + k_1 w_n)(y_n)^2 + c_1,$$
$$y_{n+1} = (2 + k_2 w_n) x_n y_n + c_2 + m_2 w_n,$$

The author inspire by peinke et al [13] and study the influence of low–strength noise for Mandelbrot set, which follows the peinke et al [13] results for higher-strength noise of Julia set shows that the outer reason of Julia set is bifurcation phenomena. And also investigate the application of the smooth decomposition method in [1, 3] noise perturbed Mandelbrot maps for small strength noise, when variation in parameter c_1 and c_2 [13] the interior of Julia sets gives 2^n -type bifurcation appearance of empty space which occur interior towards the outer region and for higher-strength noise when applied to Mandelbrot map it loses it^s symmetry. When changes it^s parameter shows the 2^n -type bifurcation appearance of empty space which occur outer region towards the interior similar behavior is also observed in Mandelbrot map see fig (5).

The author Negi and Rani [14] in 2006 inspired by Argyris [1] study and introduce a new noise criterion and its effect on the superior Mandelbrot set and find the instructing results. For this they use a new parameter λ $0 \le \lambda \le 1$ for formulation of both additive and multiplicative noise and general noise on Mandelbrot set on the map. $x_{n+1} = \lambda x_a + (1 - \lambda) x_m$ and $y_{n+1} = \lambda y_a + (1 - \lambda) y_m$ where x_a, y_a satisfy the additive noise and x_m, y_m satisfy the multiplicative noise system. Afterwards they fix the $\lambda = \dfrac{1}{2}$ for general noise and observed the behavior of additive and multiplicative noise [14] see fig (13-15). Similarly use these cases on superior Mandelbrot set and find that when low additive and low multiplicative noise apply on superior Mandelbrot set there is a insignificant distortion and for low additive and high multiplicative noise observed stable behavior of superior Mandelbrot set there is a interesting phenomenon that when multiplicative noise is high s act as noise controlling factor and on a applying high additive and low multiplicative noise on superior Mandelbrot set observed that the distortion of bulbs attached to main body of the super Mandelbrot set see fig(16).

4. CONCLUSION

In the above study we conclude that the different perturbation brings different effects to the Mandelbrot and Julia sets. The perturbed by the additive noise and perturbed by the multiplicative noise is dominant in the whole practices and play crucial role in distortion. In their study we observed various remarkable facts and new scope of research.

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