Orthogonal Phase Coded Waveforms for MIMO Radars

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ABSTRACT
Modern radar systems employ multiple transmit antennas on transmit and multiple receive antennas on receive to improve many aspects of the system performance including improved target detection performance, improved angle estimation accuracy, decreased minimum detectable velocity. However these advantages are obtained only when orthogonal probing waveforms are emitted from each of the transmit antenna elements. These waveforms further need to have good autocorrelation and crosscorrelation properties for good range resolution and multiple target return separability. Many previous works presented noise-like pseudo random phase coded waveforms with good correlation properties. However these waveforms suffered from limited family size, zero-lag non-orthogonality and hence non uniform distribution of transmit power in space. This paper presents the design of phase coded pulse waveforms with good correlation properties, zero-lag orthogonality property, delay and add property that support Doppler compensation, large family size, simpler generation, constant modulus etc.

Keywords
Multiple Input Multiple Output Radar, four phase waveforms, orthogonal transmit waveforms, uniform transmit beampattern

1. INTRODUCTION
Radar systems transmit electromagnetic energy into free space and use the reflected energy to detect and locate desired targets of interest. Modern radar systems employ antenna array based transmit system and antenna array based receive system to improve many aspects of system performance. Multiple transmit antenna elements allow directive radiation of transmit power and multiple receive antenna elements allow directive reception of reflected echoes. The use of array based antenna systems allows angular parameters of the target to be determined accurately. Conventional phased array radars transmit fully coherent waveforms (possibly scaled by a complex constant) from their M different transmit antenna elements forming a strong transmit beam in the desired direction. Beamforming is performed only by the receive array (containing N antenna elements) to estimate the angular parameters of the target. Thus the transmit degrees of freedom are limited to one and receive degrees of freedom are N. However multiple input multiple output (MIMO) radars transmit diverse waveforms from their different transmit antenna elements and use joint processing of the received signals from the different receive array elements. While phased array radars employ only spatial diversity, MIMO radars employ both spatial and waveform diversity to improve many aspects of system performance. MIMO radars can employ widely spaced antennas [1] or collocated antennas [2]. While the former configuration offers improved spatial diversity to improve target detection capabilities the latter configuration improves the spatial resolution, parameter identifiability and interference rejection capability. However the above advantages are achieved only with orthogonal waveforms with good autocorrelation and crosscorrelation properties at all time lags.

Deng [3], Liu [4] have initially proposed polyphase sets based on genetic algorithm and simulated annealing respectively. However the size of the sequence sets is small and do not satisfy the orthogonality requirement. They also suffer from Doppler degradation. Hao and Stoica [5] have proposed unimodular sequence sets based on cyclic algorithm having continuous phases over the range $[0,2\pi]$. This makes generation of signals at the transmitter and design of matched filters at the receiver difficult. Hammad [6] addressed the Doppler problems of Deng sequences but still the size of the sequence family is limited. Singh [7] proposed a simulated annealing algorithm combined with hamming scan algorithm for designing eight phase sequence sets. However the size the sequence sets is small and using eight phase sequence sets introduces complex multiplications in the digital implementation of matched filters at the receiver. It is also commented in [3] that using polyphase sequences with number of phases $K > 4$ does not yield a significant improvement.

MIMO radars allow phase shifts to be obtained for each transmit-receive antenna pair thereby increasing the degrees of freedom to MN. However these phase shifts could be obtained only by transmitting noncoherent probing signals from each transmit antenna. These probing signals should have zero-lag orthogonality property (for uniform illumination of space), good autocorrelation properties (for high range resolution), good crosscorrelation (for multiple target return separability and low interference at matched filter), constant modulus (for high transmit power efficiency), large sequence length (for high transmit energy), large family size (for immunity from jamming attacks), simpler generation and high degree of randomness. This paper presents the design of four phase pulse coded waveforms with large family size, simpler generation and good aperiodic correlation properties. The correlation properties of these sequences for MIMO radar systems have been studied in [9]. The scope of this paper is to present the construction and code optimization of these sequences for achieving zero-lag orthogonality property required for uniform illumination of transmit power in space. Section 2 presents the MIMO radar signal model and structure of phase coded pulse waveforms. Section 3 presents the construction of four phase pulse coded waveforms with good aperiodic correlation properties. Section 4 presents the code optimization procedure for achieving orthogonal pulse coded waveforms and also the numerical results. Section 5 concludes the paper.

2. MIMO RADAR SIGNAL MODEL

Consider a monostatic MIMO radar that contains M transmitters with the antenna elements configured as uniform linear arrays. We assume a point target and also that the target and transmitters lie in the same 2-D plane (see Fig. 1).

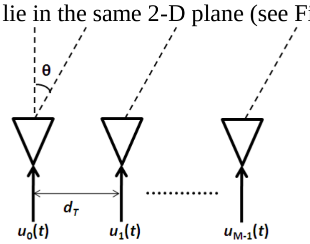

Figure 1: Transmitter Model

Let d_T represent the spacing between consecutive transmitters. Let θ be the target angle with respect to the broadside direction and λ is the carrier wavelength of the transmitted waveforms. Let $\{u_m(t)\}$, $m \in \{0,1,\dots,M-1\}$ represent the M transmitter waveforms. All the transmit antennas transmit waveforms simultaneously in time. We further assume that the transmitter waveforms are narrowband and the baseband signal waveforms are not modified because of Doppler effect [16]. The correlation between two transmit waveforms $u_m(t)$ and $u_{m'}(t)$ at zero time-lag is defined as

$$r_{m,m'} = \int_0^{T_0} u_m(t)u^*_{m'}(t)dt \qquad (1)$$

and $\mathbf{R} = [r_{m,m'}]_{M\times M}$ represents the zero-lag correlation matrix of the M transmit waveforms.

2.1 Phase Coded Pulse Waveforms

The phase coded pulse waveform emitted by the m^{th} transmitter can be represented as

$$u_m(t) = \sum_{l=0}^{L-1} \phi_m(t - T_l), \qquad (2)$$

where

$$\phi_m(t) = \frac{1}{\sqrt{T_p}} \sum_{q=0}^{Q-1} c_{m,q}\, s\left(\frac{t - q\Delta t}{\Delta t}\right) \qquad (3)$$

$$s(t) = \begin{cases} 1 & \text{if } 0 < t < 1 \\ 0 & \text{otherwise} \end{cases} \qquad (4)$$

Here L represents the number of pulses emitted by each transmitter. Here, $c_{m,q}$ is the $(m,q)^{\text{th}}$ element of the code matrix $[\mathbf{C}]_{M\times Q}$ and it can assume a value from the set $\left\{e^{j\frac{2\pi}{K}0}, e^{j\frac{2\pi}{K}1}, \dots, e^{j\frac{2\pi}{K}(K-1)}\right\}$. $T_p = Q\Delta t$ is the duration of each pulse and Δt is the duration of each subpulse. K is the phase number and represents the number of phases allowed by each polyphase waveform. Each row of the code matrix $\mathbf{C}$ represents the phase code associated with each transmitted waveform. Each column of the code matrix corresponds to the phase code transmitted by each of the M transmitters during the q^{th} subpulse. As shown in Fig. 2, each transmitter waveform $u_m(t)$ consists of a stream of L identical pulses$\phi_m(t)$. Each pulse in turn contains Q phase coded subpulses each having width Δt. For each of the transmitter waveforms $u_m(t)$ to be orthogonal (at zero Doppler and zero delay mismatch) i.e.,

$$\int_{-\infty}^{\infty} u_m(t)u^*_{m'}(t)dt = 0, \qquad \forall m \neq m' \qquad (5)$$

we require

$$\mathbf{C}\mathbf{C}^T = \mathrm{I}_{M\times M} \qquad (6)$$

Orthogonal waveforms result in uniform illumination in all directions. For fixed Δt, these waveforms can be completely described by the code matrix $\mathbf{C} = [c_{m,q}]_{M\times Q}$ and the pulse spacings $(T_0,\ T_1,\ \dots\ T_{L-1})$.

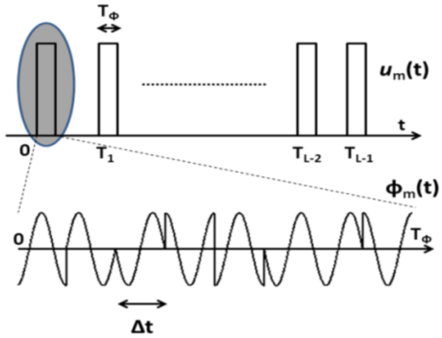

Figure 2: Structure of Phase-Coded Pulse waveforms

2.2 Correlation Metrics

The aperiodic cross-correlation $C_{m,n}(l)$ at a discrete shift l between the m^{th} phase code sequence $\{c_{m,q}, 1 \le q \le Q\}$ and the n^{th} phase code sequence $\{c_{n,q}, 1 \le q \le Q\}$ is defined as

$$C_{mn}(l) = \begin{cases} \frac{1}{Q}\sum_{i=0}^{Q-1-l} c_{m,i}c^*_{n,(i+l)} & 0 \le l \le (Q-1) \\ \frac{1}{Q}\sum_{i=0}^{Q-1+l} c_{m,i-l}c^*_{n,i} & (1-Q) \le l \le 0 \\ 0 & |l| \ge Q \end{cases} \qquad (7)$$

where $(\,)^*$ denotes complex conjugate of the argument $(\,)$. The aperiodic autocorrelation $C_m(l)$ of $\{c_{m,q}, 1 \le q \le Q\}$ at shift l is the aperiodic cross-correlation of $\{c_{m,q}\}$ with itself $C_m(l)$.

2.3 Problem Formulation

The goal of orthogonal polyphase signal design problem is to design the $M \times Q$ polyphase code set matrix $\mathbf{C}$

$$\mathbf{C}(M,Q,K) = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1Q} \\ c_{21} & c_{22} & & c_{2Q} \\ \vdots & & \ddots & \vdots \\ c_{M1} & c_{M2} & \cdots & c_{MQ} \end{bmatrix}$$

where each row represents an individual polyphase sequence used to transmit the phase coded pulse waveform $\phi_m(t)$. Each column j represents the complex symbol transmitted in the j^{th} subpulse, subject to minimization of the following criteria (in addition to satisfying the requirements (5) and (6)).

a) maximum AC peak sidelobe level

$$\sum_{m=1}^{M} \max_{l \neq 0} |C_{mm}(l)| \qquad (8)$$

b) maximum CC peak sidelobe level

$$\sum_{m=1}^{M-1}\sum_{n=1}^{p+1} \max_{l} |C_{mn}(l)| \qquad (9)$$

2.4 Transmit Beampattern

The baseband signal at the target location can be described by the expression

$$\sum_{m=0}^{M-1} e^{-j2\pi fm} u_m(t) \triangleq \mathbf{a}^H(f)\mathbf{u}(t) \qquad (10)$$

where $f = d_T \sin(\theta)/\lambda$ is the spatial frequency of the target,

$$\mathbf{u}(t) = [u_0(t)\, u_1(t)\, u_2(t) \ldots u_{M-1}(t)]^T \qquad (11)$$

is the vector of M transmit waveforms and $\mathbf{a}(f)$ is the array steering vector given by

$$\mathbf{a}(f) = [e^{j2\pi f.0} \quad e^{2\pi f.1} \quad \cdots \quad e^{j2\pi f(M-1)}]^T \qquad (12)$$

With typical transmitter spacing of $d_T = \lambda/2$, the spatial frequency f is in $[-½, ½]$. The spatial distribution of power of the transmit signals is called the *transmit beampattern* and is given by [8],

$$P(f) = \mathbf{E}[\mathbf{a}^H(f)\mathbf{u}(t)\mathbf{u}^H(t)\mathbf{a}(f)] = \mathbf{a}^H(f)\mathbf{R}\mathbf{a}(f) = \sum_{m=0}^{M-1}\sum_{m'=0}^{M-1} r_{m,m'} e^{j2\pi f(m-m')} \qquad (13)$$

Consider $P(f)$ for a phased array radar case. The $M \times 1$ transmit signal vector $\mathbf{u}(t)$ is given by $\mathbf{u}(t) = \mathbf{a}(f_0)u(t)$ where $f_0 = d_T \sin(\theta_0)/\lambda$ with $\theta_0$denoting the steered direction. Then, $\mathbf{R} = \mathbf{a}(f_0)\mathbf{a}^{\boldsymbol{H}}(f_0)$ assuming unit power signal $u(t)$ and

$$P(f) = \mathbf{a}^{\boldsymbol{H}}(f)\mathbf{a}(f_0)\mathbf{a}^{\boldsymbol{H}}(f_0)\mathbf{a}^{\boldsymbol{H}}(f) = \left|\mathbf{a}^{\boldsymbol{H}}(f)\mathbf{a}(f_0)\right|^2 \qquad (14)$$

Note that the transmit gain attains maximum value in the direction θ_0 and is decreased at $\theta \neq \theta_0$. Now, consider $P(f)$ with orthogonal signals. Then, $\mathbf{R} = \mathbf{I}$, and

$$P(f) = \mathbf{a}^{\boldsymbol{H}}(f)\mathbf{a}(f) = M \qquad (15)$$

This implies that the beampattern is omnidirectional. Thus, the traditional beamforming results in a focused beampattern while the beampattern of MIMO with orthogonal signals is uniform in all directions.

3. FOUR PHASE CODE GENERATION

Sequences with good autocorrelation properties can be generated by using pseudo noise generators called maximal length sequences. When it is desired to generate a binary m-sequence of period $2^r - 1$, one looks up a table of binary irreducible polynomials of degree r and then selects from amongst the table, an irreducible polynomial that is also primitive. The procedure for generation of four-phase sequences is very similar. Given that it is desired to generate the family with size $M = 2^r + 1$ of four phase sequences of period $Q = 2^r - 1$, one proceeds as follows. First, identify polynomials with coefficients in $Z_4 \in \{0, 1, 2, 3\}$ that are irreducible as binary polynomials when their coefficients are reduced modulo 2 (i.e. irreducible over Z_2). It is easily shown that these polynomials are also irreducible over Z_4. Next, from amongst these pick a primitive polynomial $f(x)$; An irreducible polynomial of degree r is said to be primitive if the smallest exponent Q for which the polynomial $f(x)$ divides $x^Q - 1$ is $Q = 2^r - 1$. Complete listings of all primitive polynomials having degree ≤ 10 are listed in [11]. Table-I below provides partial listing of primitive polynomials given in [11] for reference. Note: For degree 3, the entry 1213 represents the polynomial $x^3 + 2x^2 + x + 3$.

Table 1: Partial Listing of Characteristic Polynomials for Linear Recurrence

Degree	3	1213, 1323
Degree	4	10231, 13201
Degree	5	100323, 113013, 113123, 121003, 123133
Degree	6	1002031, 1110231, 1211031, 1301121
Degree	7	10020013, 10030203, 10201003
Degree	8	100103121, 100301231, 102231321
Degree	9	1000030203, 1001011333, 1001233203

3.1 Signal Generation

Given a primitive polynomial

$$f(x) = f_0 + f_1 x + f_0 x^2 + \cdots + f_{r-1}x^{r-1} + x^r \qquad (16)$$

with coefficients $f_i \in Z_4$, the m^{th} sequence $\{s_m(q), 1 \leq q \leq Q\}\ 1 \leq m \leq M$ in the family A can be generated from the linear recursion associated with $f(x)$ given by

$$s_m(q) = -f_{r-1}s_m(q-1) - f_{r-2}s_m(q-2) + \cdots - f_0 s_m(q-r) \qquad (17)$$

over Z_4. By initializing the linear recursion with different initial states and evaluating the linear recursion we get $M = 2^r - 1$ cyclically distinct sequences in family A each of length $Q = 2^r - 1$. The linear recurrence described above can be efficiently implemented using the shift register configuration shown in Fig. 3.

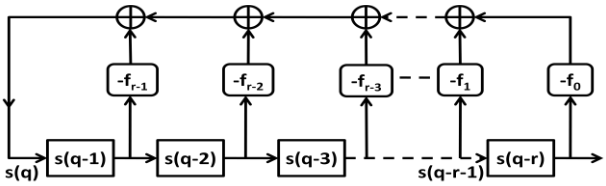

Figure 3: Shift Register implementation of family A as generated by characteristic polynomial $f(x) = f_0 + f_1 x + f_0 x^2 + \cdots + f_{r-1}x^{r-1} + x^r$. All addition, multiplication and negation operations follow modulo-4 arithmetic

3.2 Example

Consider the degree $r = 4$ polynomial $f(x) = x^4 + 2x^2 + 3x + 1$, primitive as a Z_4 polynomial. By reducing the coefficients modulo 2, one obtains the binary primitive polynomial $x^4 + x + 1$. We notice that $f_0 = 1, f_1 = 3, f_2 = 2$ and $f_3 = 0$. This implies $-f_0 = 3, -f_1 = 1,$and $-f_2 = 2$. The linear recurrence given by $f(x)$ is defined as

$$s_m(q) = -f_3 * s_m(q-1) + -f_2 * s_m(q-2) + -f_1 * s_m(q-3) - f_0 * s_m(q-4)$$

$$s_m(q) = 2 * s_m(q-2) + s_m(q-3) + 3 * s_m(q-4)$$

This linear recurrence can be implemented using the shift register configuration shown in Fig. 4. Cyclically distinct members of the family can be found by loading the shift register with 4-tuples not previously seen during the generation of prior sequences

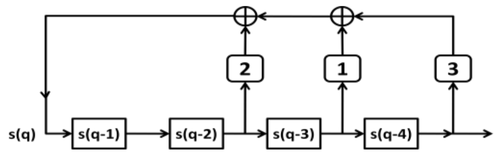

Figure 4: Shift Register implementation of family A as generated by characteristic polynomial $f(x) = x^4 + 2x^2 + 3x + 1$

The above recurrence yields the family of Z_4 sequences of size 17 (2^4+1) and signal length 15 (2^4-1).

$$\boldsymbol{S}_{M\times Q} = \begin{bmatrix} 111123231001022 \\ 111301033023020 \\ 112101302120213 \\ 112210021133012 \\ 113123013203000 \\ 113232132212203 \\ 121231223220133 \\ 121300302233332 \\ 122213330303320 \\ 123300120031310 \\ 131303211003200 \\ 132103120100033 \\ 133230310232023 \\ 222202022002000 \\ 222311101011203 \\ 233110323322300 \\ 333103011021020 \end{bmatrix}_{17\times 15}$$

Polyphase sequences **C** can now be obtained from the above sequences **S** by mapping the elements in Z_4 to $\{e^{j0}, e^{j\pi/2}. e^{j\pi}, e^{j3\pi/2}\}$.

4. CODE OPTIMIZATION AND NUMERICAL RESULTS

The four phase codes discussed in the preceding section are shown [9] to have good aperiodic autocorrelation and aperiodic crosscorrelation properties. However the sequences are do not satisfy the orthogonality property. This results in non-uniform illumination of power in space. The property of orthogonality between the every pair of sequences is one of the key requirements of MIMO radars.

Popovic and Suehiro [10] showed that there exist a "circular phase shift" for each of these sequences that result in zero-delay crosscorrelation of any two distinct sequences in this set is -1. If a 1 is preprended to each sequence in this set, then the zero-delay crosscorrelation of any pair of distinct sequences in this set will be zero. The zero-delay autocorrelation of each of these sequences is however $Q + 1$ (where Q is the initial sequence length). The generated four phase sequences are circular shifted to obtain a zero-delay correlation of -1 with all the other sequences in the matrix. The example of the shifted versions of the input sequences generated has been shown in the second column of Table-2. These shifted sequences are prepended with a 0 in order to obtain the zero-lag correlation matrix as zero. So the zero-lag correlation matrix will have the diagonal elements to be equal to the number of elements in each sequence and the other elements in the matrix to be zeros. This is illustrated in the zero-lag correlation matrix column as shown in Table-2. Therefore the correlation between any two sequences in the matrix is now zero. Hence the property of orthogonality has been achieved along with the good autocorrelation and crosscorrelation properties. The transmit beampattern (5) of MIMO radar transmit array with M=16 elements (waveforms) as a function of spatial direction (frequency f) of initial sequences and code optimized orthogonal sequences is displayed in Fig. 5. We see that the transmit beampattern of waveforms using initial sequences generated using (17) is nonuniform whereas the code optimized sequences have a uniform beampattern which is desirable in MIMO radar systems.

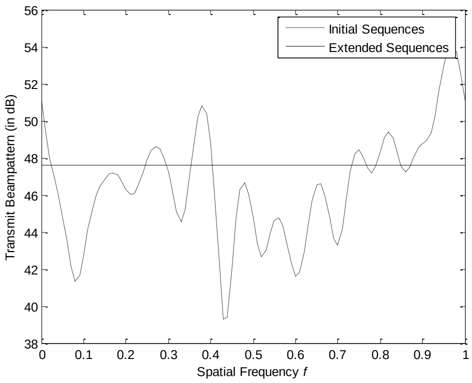

Figure 5: Transmit Beampattern

5. CONCLUSIONS

This paper presents the construction of four phase pulse coded waveforms for MIMO radar applications. The advantages of the proposed method includes large family size, constant modulus, near optimal auto and cross correlation properties, pair wise orthogonality of sequences for uniform transmit beampattern etc.

This work can be further extended to identify eight phase sequence sets based on ML sequence generators and finite field theory. Since MIMO radars suffer from directive gain loss each waveform emitted by each transmit antenna need to have long pulse width to maximize the transmit energy and further the received SNR. Sequences of any required length can be generated by selecting the primitive polynomial of appropriate order.

6. REFERENCES

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Table 2: a) Four Phase Sequences generated using the linear recursion of (6) b) Circular Shifted Sequences of (a) for optimal zero-delay crosscorrelation c) Orthogonal Sequences with a zero-delay cross correlation of 0 between any pair of distinct sequences d) Zero-Delay cross correlation matrix of each pair of sequences

a) Four phase coded Input Sequences	b) Circular Shifted Sequences	c) Orthogonal Four Phase Sequences	d) Zero-Lag Correlation Matrix
M=8, Q=7	**M=8, Q=7**	**M=8, Q=8**	**$R=I_{8X8}$**
1110302	1110302	01110302	80000000
1121223	3112122	03112122	08000000
1132100	1132100	01132100	00800000
1230033	3312300	03312300	00080000
1310120	1310120	01310120	00008000
1332322	1332322	01332322	00000800
2333010	3330102	03330102	00000080
3130320	3130320	03130320	00000008
M=16, Q=15	**M=16, Q=15**	**M=15, Q=16**	**$R=I_{15X15}$**
111123231001022	111123231001022	0111123231001022	1500000000000000
111301033023020	111301033023020	0111301033023020	0150000000000000
112101302120213	131121013021202	0131121013021202	0015000000000000
112210021133012	113301211221002	0113301211221002	0001500000000000
113123013203000	113123013203000	0113123013203000	0000150000000000
113232132212203	311323213221220	0311323213221220	0000015000000000
121231223220133	133121231223220	0133121231223220	0000001500000000
121300302233332	333321231223222	0333321231223222	0000000150000000
122213330303320	133303033201222	0133303033201222	0000000015000000
123300120031310	313101233001200	0313101233001200	0000000001500000
131303211003200	131303211003200	0131303211003200	0000000000150000
132103120100033	331321031201000	0331321031201000	0000000000015000
133230310232023	313323031023202	0313323031023202	0000000000001500
222311101011203	311101011203222	0311101011203222	0000000000000150
233110323322300	331103233223002	0331103233223002	0000000000000015
333103011021020	333103011021020	0333103011021020	
M=31, Q=31	**M=31, Q=31**	**M=31, Q=32**	**$R=I_{31X31}$**
1111120330320122021210133013000	1111120330320122021210133013000	01111120330320122021210133013000	31000000000000000000000000000000
1111322312122322003032131211002	1111322312122322003032131211002	01111322312122322003032131211002	03100000000000000000000000000000
1112112001333300312300102021012	1333300312300102021012111211200	01333300312300102021012111211200	00310000000000000000000000000000
1112231010210002103231303102013	1311122310102100021032313031020	01311122310102100021032313031020	00031000000000000000000000000000
1112310023131100330122100223010	3131100330122100223010111231002	03131100330122100223010111231002	00003100000000000000000000000000
1113100112302122203030111033020	1113100112302122203030111033020	01113100112302122203030111033020	00000310000000000000000000000000
1113302130100322221212113231022	1113302130100322221212113231022	01113302130100322221212113231022	00000031000000000000000000000000
1121203022210101310332221331122	1331122112120302221010131033222	01331122112120302221010131033222	00000003100000000000000000000000
1121322031131203101223022012123	3113120310122302201212311213220	03113120310122302201212311213220	00000000310000000000000000000000
1122231133223323201022230303130	3113322332320102223030313011222	03113322332320102223030313011222	00000000031000000000000000000000
1123223200232101132112203311102	3311102112322320023210113211220	03311102112322320023210113211220	00000000003100000000000000000000
1123302213113203323003000032103	1311320332300300003210311233022	01311320332300300003210311233022	00000000000310000000000000000000
1131322110100120203010313213200	1131322110100120203010313213200	01131322110100120203010313213200	00000000000031000000000000000000
1133100310320320003012333031222	1133100310320320003010313213222	01133100310320320003010313213222	00000000000003100000000000000000
1133302332122120021230331233220	1133302332122120021230331233220	01133302332122120021230331233220	00000000000000310000000000000000
1213101302031333221303023000230	3133322130302300023012131013020	03133322130302300023012131013020	00000000000000031000000000000000
1213303320233133203121021202232	3313320312102120223212133033202	03313320312102120223212133033202	00000000000000003100000000000000
1221323221220010123132330023333	3333122132322122001012313233002	03333122132322122001012313233002	00000000000000000310000000000000
1222232323312130223331102310300	3331102310300122223232331213022	03331102310300122223232331213022	00000000000000000031000000000000
1223303003202010301312312003313	3313122330300320201030131231200	03313122330300320201030131231200	00000000000000000003100000000000
1232113033000313130231232030020	3131302312320300201232113033000	03131302312320300201232113033000	00000000000000000000310000000000
1233101100013131021321201002032	1313102132120100203212331011000	01313102132120100203212331011000	00000000000000000000031000000000
1313300110322300221030333213002	1313300110322300203210333213002	01313300110322300203210333213002	00000000000000000000003100000000
1322233113001301201200010321110	3311300130120100221032111013222	03311300130120100221032111013222	00000000000000000000000310000000
1331201121001000012301332110223	3133120112100100001230133211022	03133120112100100001230133211022	00000000000000000000000031000000
1331320130322102203232133231220	1331320130322102203232133231220	01331320130322102203232133231220	00000000000000000000000003100000
1333102330102302003230113013202	1333102330102302003230113013202	01333102330102302003230113013202	00000000000000000000000000310000
2311130211030210200121033103120	3111302110302102001210331031202	03111302110302102001210331031202	00000000000000000000000000031000
2322201010333011020333130033210	3331300332102322201010333011020	03331300332102322201010333011020	00000000000000000000000000003100
2333233200311110013210030202303	3111100132100302023032333233200	03111100132100302023032333233200	00000000000000000000000000000310
3333201101203220232303110310003	3333320110120322023230311031000	03333320110120322023230311031000	00000000000000000000000000000031

Note: Polyphase sequences C can now be obtained from the above sequences *S* by mapping the elements in Z_4 to $\{e^{j0}, e^{j\pi/2}. e^{j\pi}, e^{j3\pi/2}\}$