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Fuzzy Multi Objective Assignment Linear Programming Problem based on L-R fuzzy Numbers

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ABSTRACT
Transportation and assignment models play significant role in logistics and supply chain management for reducing cost and time, for better service. In this paper, a fuzzy multi objective assignment problem using linear programming model is developed. The reference functions of L-R fuzzy numbers of fuzzy multi objective assignment problem are considered being linear and non-linear functions. This paper develops a procedure to derive the fuzzy objective value of the fuzzy multi objective assignment problem, in that the fuzzy cost coefficients, the fuzzy time and fuzzy quality are L-R fuzzy numbers. The method is illustrated with an example by various cases.

Keywords
Multi objective assignment; Yager's ranking index; L-R fuzzy numbers; linear programming.

1. INTRODUCTION
Assignment problem is used worldwide in solving real world problems. It plays an important role in industry and is used very often in solving problems of engineering, management science and it has many other applications. Project management is designed to control organization resources on a given set of activities, within time, cost and quality. Therefore, the limited resources must be utilized efficiently such that the optimal available resources can be assigned to the most needed tasks so as to maximize and minimize the profit and cost respectively. The assignment problem is one of the most important problem in mathematical programming in which a number of jobs (tasks or works) assigned to an equal number of machines (persons), so as to perform the jobs depend on their efficiency. It can be viewed as a balanced transportation problem in which all supplies and demands equal to 1, and the number of rows and columns in the transportation matrix are identical.

Hiller and Libermann[1], Taha[2], Murthy[3] and Swarup et al.[4] discussed a single objective function in crisp environment for different type of assignment problems. Ravindran et al. [5] solved the assignment problem by using the transportation simplex method, but due to the high degree of degeneracy in the problem, it is often inefficient and not recommended to attempt to solve it by simplex method. Ravindran et al. [6] utilized another technique called Hungarian method to solve the minimizing assignment problem. The multi-objective assignment problem in crisp environment studied by Bao et al. [7]. Labeling algorithm to solve assignment problem with fuzzy interval cost proposed by Lin and Wen [8]. Chen [9] proposed fuzzy assignment model by considering that all the individuals involved have the same skills and Wang[10] solved a similar model by graphical approach. Mukherjee and Basu [11] resolved an assignment problem with fuzzy cost by Yager's ranking method [12] which transforms the fuzzy assignment problem into a crisp assignment problem. Geetha et al. [13] expressed the cost-time minimizing assignment as the multicriteria problem. The fuzzy programming technique with linear membership function applied to solve the multi-objective transportation problem by Bit et al. [14]. Tsai et al [15] solved a balanced multi-objective decision making problem which is related with cost, time and quality in fuzzy environment. The multi-objective assignment problem as a vector minimum problem was resolved by Kagade and Bajaj [16] using linear and non-linear membership functions. In the paper [17] the solution procedure to the multi-objective assignment problem where the cost coefficients of the objective functions are interval values and the equivalent transformed problem explained using fuzzy programming techniques. Verma et al. [18] worked out a multi-objective transportation problem by some non-linear (hyperbolic and exponential) membership functions using fuzzy programming method. Dhingra et al. [19] defined other types of the non-linear membership functions and relate them to an optimal design problem.

In this paper, we consider multi-objective assignment problem with fuzzy parameters for the case of construction process. Here, we let $\tilde{c}_{ij}$ to be fuzzy payment to i^{th} person for doing j^{th} work, $\tilde{t}_{ij}$ to be fuzzy time for i^{th} person for doing j^{th} work and $\tilde{q}_{ij}$ to be fuzzy quality of i^{th} person for doing j^{th} work. Here, the fuzzy cost coefficients, the fuzzy time and fuzzy quality are L-R fuzzy numbers. Due to different unit of fuzzy cost, fuzzy time, and fuzzy quality, it is not possible to merge each other until we normalize them. Therefore, for normalization purpose divide fuzzy cost, fuzzy time and fuzzy quality by their corresponding maximum ranking index. By assigning a weight to the objectives according to their priorities the single objective function is obtained. Then, by ranking method, transform a newly formed single objective fuzzy assignment problem to a crisp assignment problem in linear programming problem form and it can be solved by any conventional method.

The rest of the paper is organized as follows : In section 2, preliminaries of L-R fuzzy numbers, λ-cut of L-R fuzzy number, reference functions and Yager's ranking approach for various linear and non-linear functions using L-R fuzzy numbers are presented. In section 3, the proposed linear programming model for fuzzy multi objective assignment problem for various linear and non-linear functions, where the fuzzy cost coefficients, the fuzzy time and fuzzy quality are L-R fuzzy numbers has been given. In section 4, the proposed method to find the total optimal fuzzy solution for fuzzy multi-objective assignment problem has been given. In section 5, we discuss the application of proposed method with a numerical example and total optimal fuzzy cost, fuzzy time and fuzzy quality for various cases are given. Finally, the conclusion is given in section 6.

2. Preliminaries

In this section, L-R fuzzy numbers, λ-cut of L-R fuzzy number, reference functions and Yager's ranking approach for various linear and non-linear functions using L-R fuzzy numbers are presented.

2.1 L-R fuzzy numbers and reference functions

In this section, L-R fuzzy number, λ-cut of L-R fuzzy number, and reference functions are reviewed [20].

Definition 1 A fuzzy number $\tilde{A} = (m,n,\alpha,\beta)_{L-R}$ is said to be an L-R fuzzy number if

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\dfrac{m-x}{\alpha}\right), & x \le m,\ \alpha > 0, \\ R\left(\dfrac{x-n}{\beta}\right), & x \ge n,\ \beta > 0, \\ 1, & otherwise \end{cases}$$

where L and R are continuous, non-increasing functions that define the left and right shapes of $\mu_{\tilde{A}}(x)$ respectively and L(0) = R(0) =1.

Linear reference functions and nonlinear reference functions with their inverses are presented in Table I.

Definition 2 Let $\tilde{A} = (m,n,\alpha,\beta)_{L-R}$ be an L-R fuzzy number and λ be a real number in the interval [0,1]. Then the crisp set $A_\lambda = \{x \in X : \mu_{\tilde{A}}(x) \ge \lambda\} = [m - \alpha\, L^{-1}(\lambda), n + \beta\, R^{-1}(\lambda)]$ is said to be λ-cut of $\tilde{A}$.

2.2 Yager's ranking approach to various linear and non-linear functions for L-R fuzzy numbers

In this section, Yager's method [12] is presented for the ranking of L-R fuzzy numbers. This method involves a procedure for ordering fuzzy sets in which a ranking approach R($\tilde{A}$) is calculated for the fuzzy number $\tilde{A} = (m,n,\alpha,\beta)_{L-R}$ from its λ-cut $A_\lambda = [m - \alpha\, L^{-1}(\lambda), n + \beta\, R^{-1}(\lambda)]$ according to the following formula:

$$R(\tilde{A}) = \frac{1}{2}\left(\int_0^1 \left(m - \alpha\, L^{-1}(\lambda)\right) d\lambda + \int_0^1 \left(n + \beta\, R^{-1}(\lambda)\right) d\lambda\right).$$

The ranking indexes for L-R fuzzy number $\tilde{A} = (m,n,\alpha,\beta)_{L-R}$ with various linear and non-linear functions are:

Case (i) L (x) = R (x) = max $\{0, 1 - |x|\}$

$$R(\tilde{A}) = \frac{1}{2}\left[m + n - \frac{\alpha}{2} + \frac{\beta}{2}\right]$$

Case (ii) L (x) = R (x) = e^{-x}

$$R(\tilde{A}) = \frac{1}{2}\left[m + n - \alpha + \beta\right]$$

Case (iii) L (x) = max $\{0, 1 - |x|\}$ **and**
R (x) = e^{-x}

$$R(\tilde{A}) = \frac{1}{2}\left[m + n - \frac{\alpha}{2} + \beta\right]$$

Case (iv) L (x) = e^{-x} **and**
R (x) = max $\{0, 1 - |x|\}$

$$R(\tilde{A}) = \frac{1}{2}\left[m + n + \alpha + \frac{\beta}{2}\right]$$

Case (v) L (x) = e^{-px} **and**
R (x) = max $\{0, 1 - |x^p|\}$

$$R(\tilde{A}) = \frac{1}{2}\left[m + n - \frac{\alpha}{p} + \frac{\beta p}{p+1}\right]$$

Case (vi) L (x) = max $\{0, 1 - |x^p|\}$ **and**
R (x) = e^{-px}

$$R(\tilde{A}) = \frac{1}{2}\left[m + n + \frac{\alpha p}{p+1} + \frac{\beta}{p}\right]$$

Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers. Then

(i) $\tilde{A} > \tilde{B}$ if $R(\tilde{A}) > R(\tilde{B})$.

(ii) $\tilde{A} = \tilde{B}$ if $R(\tilde{A}) = R(\tilde{B})$

(iii) $\tilde{A} \ge \tilde{B}$ if $R(\tilde{A}) \ge R(\tilde{B})$

3. Linear programming model for fuzzy multi objective assignment problem to various linear and non-linear functions with L-R fuzzy numbers

In a general assignment problem, n works to be performed by n persons depending on their efficiency to do the job in one to one basis such that the assignment cost is minimum or maximum. If the objective of an assignment problem is to minimize fuzzy cost, fuzzy time and fuzzy quality, then this type of fuzzy problem is treated as a fuzzy multi objective assignment problem. Here, we consider fuzzy assignment problem with three objectives in the following form of n × n fuzzy matrix (Table II) where each cell having a fuzzy cost ($\tilde{c}_{ij}$), fuzzy time ($\tilde{t}_{ij}$) and fuzzy quality ($\tilde{q}_{ij}$) .

Mathematical formulation of fuzzy multi-objective assignment problem

Mathematically, the fuzzy multi-objective assignment problem in table II can be stated as:

Minimize $\tilde{z}_k = \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\tilde{p}_{ij}^{k}\right) x_{ij},\ k = 1,2,\ldots,n$

subject to $x_{ij} = \begin{cases} 1 & if\ i^{th}\ person is\ assigned j^{th}\ work \\ 0 & otherwise \end{cases}$

$$\sum_{i=1}^{n} x_{ij} = 1, \qquad j = 1,2,..., n.$$

$$\sum_{j=1}^{n} x_{ij} = 1, \qquad i = 1,2,..., n.$$

where $\tilde{z}_k$ = { $\tilde{z}_1, \tilde{z}_2, ..., \tilde{z}_k$ } is a vector of k-objective functions. If the objective function $\tilde{z}_1$ denotes the fuzzy cost function,

Minimize $\tilde{z}_1 = \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{c}_{ij} x_{ij}$,

If the objective function $\tilde{z}_2$ denotes the fuzzy time function,

Minimize $\tilde{z}_2 = \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{t}_{ij} x_{ij}$,

If the objective function $\tilde{z}_3$ denotes the fuzzy quality function,

Minimize $\tilde{z}_3 = \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{q}_{ij} x_{ij}$,

Then it is a three objective fuzzy assignment problem. To convert this three objective fuzzy assignment problem into a single fuzzy objective problem, we first normalize the fuzzy cost $(\tilde{c}_{ij})$, fuzzy time $(\tilde{t}_{ij})$ and fuzzy quality $(\tilde{q}_{ij})$. The normalized data will not affect the assignment problem. Use weights to consider the priorities of the objective.

Minimize $\tilde{z}$ =

$$w_1 \sum_{i=1}^{n}\sum_{j=1}^{n} (\tilde{c}_{ij}) x_{ij} + w_2 \sum_{i=1}^{n}\sum_{j=1}^{n} (\tilde{t}_{ij}) x_{ij} + w_3 \sum_{i=1}^{n}\sum_{j=1}^{n} (\tilde{q}_{ij}) x_{ij}$$

subject to $x_{ij} = \begin{cases} 1 & if\ i^{th}\ person is\ assigned j^{th}\ work \\ 0 & otherwise \end{cases}$

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1,2,..., n.$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1,2,..., n.$$

and $w_1 + w_2 + w_3 = 1$.

where $\tilde{c}_{ij} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{L-R}$: Fuzzy payment to i^{th} person for doing j^{th} work, $\tilde{t}_{ij} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{L-R}$: Fuzzy time for i^{th} person for doing j^{th} work, $\tilde{q}_{ij} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{L-R}$: Fuzzy quality of i^{th} person for doing j^{th} work.

(i) L(x) = R(x) = max $\{0, 1 - |x|\}$, (ii) L(x) = R(x) = e^{-x},

(iii) L(x) = max $\{0, 1 - |x|\}$ and R(x) = e^{-x}, (iv) L(x) = e^{-x} and R(x)=max $\{0, 1 - |x|\}$, (v)L(x)= e^{-px} and R (x) = max $\{0, 1 - |x^p|\}$,

(vi) L (x) = max $\{0, 1 - |x^p|\}$ and R (x) = e^{-px}.

L(x) = left shape function; R(x) = right shape function.

All $\tilde{c}_{ij}$, $\tilde{t}_{ij}$, $\tilde{q}_{ij}$, L(x) and R(x) denotes a non-negative L-R fuzzy numbers.

$\sum_{i=1}^{n}\sum_{j=1}^{n} (\tilde{c}_{ij}) x_{ij}$: Total fuzzy cost for performing all the works.

$\sum_{i=1}^{n}\sum_{j=1}^{n} (\tilde{t}_{ij}) x_{ij}$: Total fuzzy time for performing all the works

$\sum_{i=1}^{n}\sum_{j=1}^{n} (\tilde{q}_{ij}) x_{ij}$: Total fuzzy quality for performing all the works.

4. The total optimal fuzzy solution for fuzzy multi-objective assignment problem

The proposed method is to solve the total optimal fuzzy cost, fuzzy time and fuzzy quality for fuzzy multi-objective assignment problem using linear programming as follows:

Step 1: First test whether the given fuzzy multi-objective assignment matrix is a balanced one or not. If it is a balanced one (i.e, number of persons is equal to the number of works) then go to step 3. If it is an unbalanced one (i.e., number of persons is not equal to the number of works) then go to step 2.

Step 2: Introduce dummy rows and/or columns with zero fuzzy costs, time and quality so as to form a balanced one.

Step 3: Consider the fuzzy linear programming model as proposed in section 3.

Step 4: Convert the fuzzy multi-objective assignment problem into the following crisp linear programming problem

Minimize z =

$$w_1 \sum_{i=1}^{n}\sum_{j=1}^{n} R(\tilde{c}_{ij}) x_{ij} + w_2 \sum_{i=1}^{n}\sum_{j=1}^{n} R(\tilde{t}_{ij}) x_{ij} + w_3 \sum_{i=1}^{n}\sum_{j=1}^{n} R(\tilde{q}_{ij}) x_{ij}$$

subject to $x_{ij} = \begin{cases} 1 & if\ i^{th}\ person is\ assigned j^{th}\ work \\ 0 & otherwise \end{cases}$

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1,2,..., n.$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1,2,..., n.$$

and $w_1 + w_2 + w_3 = 1$.

Step 5: Based on the case chosen in section 2.2**,** calculate the values of $R(\tilde{c}_{ij})$, $R(\tilde{t}_{ij})$ and $R(\tilde{q}_{ij})$, $\forall i, j$ by using the ranking procedure as mentioned in definition 2 and section 2.2 for the chosen fuzzy multi-objective assignment problem.

Step 6: For the values obtained in step 5, choose the maximum cost, time and quality. To normalize operation cost, time and quality, let the maximum cost = $1/k_1$ (say), the maximum time = $1/k_2$ (say), the maximum quality = $1/k_3$ (say).The objective of this assignment problem is to minimize cost, time, quality. Let $a = w_1 k_1$, $b = w_2 k_2$ and $c = w_3 k_3$.

Step 7: Using the values of a, b and c obtained in step 6, the fuzzy multi-objective assignment problem is converted into the following crisp linear programming problem

Minimize z =

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a\ R(\tilde{c}_{ij})\ x_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{n} b\ R(\tilde{t}_{ij})\ x_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{n} c\ R(\tilde{q}_{ij})\ x_{ij}$$

subject to $x_{ij} = \begin{cases} 1 & if\ i^{th}\ person is\ assigned j^{th}\ work \\ 0 & otherwise \end{cases}$

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1,2,..., n.$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1,2,..., n.$$

Step 8: Solve the crisp linear programming problem obtained in step 7 to find the optimal solution $\{x_{ij}\}$.

Step 9: Find the optimal total fuzzy assignment cost, total fuzzy assignment time and total fuzzy assignment quality by substituting the optimal solution obtained in step 8 in the objective function of step 3.

5. Numerical Example

To illustrate the proposed model, consider a case of construction process with six persons and six works as a fuzzy multi objective assignment problem so as to minimize the fuzzy cost, fuzzy time and fuzzy quality. The fuzzy cost coefficients, the fuzzy time and the fuzzy quality in fuzzy multi objective assignment problem are considered as L-R fuzzy numbers for allocating each person. The fuzzy multi objective assignment problem with fuzzy cost, fuzzy the time and the fuzzy quality is shown in table III and it is solved by using various cases.

The total optimal fuzzy cost, fuzzy time and fuzzy quality for fuzzy multi-objective assignment problem using fuzzy linear programming for various cases as follows:

Case (i) L (x) = R (x) = max $\{0,1-|x|\}$

Step 1: The given fuzzy multi-objective assignment problem is a balanced one.

Step 2: Using step3 of the proposed model, the given fuzzy multi-objective assignment problem is converted into a single fuzzy objective problem as follows

Minimize $\tilde{Z}$ =

$$0.5\sum_{i=1}^{6}\sum_{j=1}^{6}(\tilde{c}_{ij})x_{ij}+0.4\sum_{i=1}^{6}\sum_{j=1}^{6}(\tilde{t}_{ij})x_{ij}+0.1\sum_{i=1}^{6}\sum_{j=1}^{6}(\tilde{q}_{ij})x_{ij}$$

subject to $$x_{ij}=\begin{cases}1 & if\ i^{th}\ person is\ assigned j^{th}\ work\\ 0 & otherwise\end{cases}$$

$$\sum_{i=1}^{6}x_{ij}=1,\qquad j=1,2,...,6.$$

$$\sum_{j=1}^{6}x_{ij}=1,\qquad i=1,2,...,6.$$

Here, w_1 = 0.5, w_2 = 0.4, w_3 = 0.1.

Step 3: The fuzzy multi-objective assignment problem is converted into the following crisp linear programming problem

Minimize z =

$$0.5\sum_{i=1}^{6}\sum_{j=1}^{6}R(\tilde{c}_{ij})x_{ij}+0.4\sum_{i=1}^{6}\sum_{j=1}^{6}R(\tilde{t}_{ij})x_{ij}+0.1\sum_{i=1}^{6}\sum_{j=1}^{6}R(\tilde{q}_{ij})x_{ij}$$

subject to

$$x_{ij}=\begin{cases}1 & if\ i^{th}\ person is\ assigned j^{th}\ work\\ 0 & otherwise\end{cases}$$

$$\sum_{i=1}^{6}x_{ij}=1,\quad j=1,2,...,6.$$

$$\sum_{j=1}^{6}x_{ij}=1,\quad i=1,2,...,6.$$

Step 4: Using definition 2 and section 2.2, the values of $R(\tilde{c}_{ij})$, $R(\tilde{t}_{ij})$ and $R(\tilde{q}_{ij})$, $\forall i, j$ are calculated and given in Table IV.

Step 5: Using step 6 of the proposed model highest cost, highest time and highest quality from table IV respectively are 25, 20.5 and 0.33. For w_1= 0.5, w_2 = 0.4 and w_3 = 0.1, the values of *a*, *b*, and *c* are: *a* = 0.02, *b* = 0.01 and *c* = 0.303.

Step 6: Using step 7 of the proposed method convert the chosen fuzzy multi-objective assignment problem into the following crisp linear programming

Minimize: $(0.28)x_{11} + (0.25)x_{12} + (0.23)x_{13} + (0.31)x_{14} + (0.33)x_{15} + (0.25)x_{16} + (0.21)x_{21} + (0.33)x_{22} + (0.24)x_{23} + (0.26)x_{24} + (0.29)x_{25} + (0.31)x_{26} + (0.29)x_{31} + (0.30)x_{32} + (0.29)x_{33} + (0.28)x_{34} + (0.31)x_{35} + (0.22)x_{36} + (0.28)x_{41} + (0.31)x_{42} + (0.37)x_{43} + (0.26)x_{44} + (0.30)x_{45} + (0.32)x_{46} + (0.28)x_{51} + (0.56)x_{52} + (0.39)x_{53} + (0.26)x_{54} + (0.46)x_{55} + (0.39)x_{56} + (0.55)x_{61} + (0.67)x_{62} + (0.66)x_{63} + (0.68)x_{64} + (0.59)x_{65} + (0.33)x_{66}$

subject to:

$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} = 1$; $x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} = 1$; $x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} = 1$; $x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} = 1$; $x_{51} + x_{52} + x_{53} + x_{54} + x_{55} + x_{56} = 1$; $x_{61} + x_{62} + x_{63} + x_{64} + x_{65} + x_{66} = 1$; $x_{11} + x_{21} + x_{31} + x_{41} + x_{51} + x_{61} = 1$; $x_{11} + x_{22} + x_{32} + x_{42} + x_{52} + x_{62} = 1$; $x_{13} + x_{23} + x_{33} + x_{43} + x_{53} + x_{63} = 1$; $x_{14} + x_{24} + x_{34} + x_{44} + x_{54} + x_{65} = 1$; $x_{15} + x_{25} + x_{35} + x_{45} + x_{55} + x_{65} = 1$; $x_{16} + x_{26} + x_{36} + x_{46} + x_{56} + x_{66} = 1$. where $x_{ij} \geq 0$, for all i=1,2,3,4,5,6 and j=1,2,3,4,5,6.

Step 7: Solve the crisp linear programming problem, obtained in step 6, the optimal solution obtained is: $x_{13} = 1$, $x_{21} = 1$, $x_{32} = 1$, $x_{45} = 1$, $x_{54} = 1$, $x_{66} = 1$.

Step 8: Using step 9 of the proposed model, the minimum fuzzy assignment cost, the fuzzy assignment time and the fuzzy assignment quality respectively are, (45, 58, 12, 12) , (37, 48, 8, 9) and (0.57, 0.72, 0.13, 0.14).

Case (ii) L (x) = R (x) = e^{-x}

Step 1: The given fuzzy multi-objective assignment problem is a balanced one.

Step 2: Using step3 of the proposed model, the given fuzzy multi-objective assignment problem is converted into a single fuzzy objective problem as follows :

Minimize $\tilde{Z}$ =

$$0.5\sum_{i=1}^{6}\sum_{j=1}^{6}(\tilde{c}_{ij})x_{ij}+0.4\sum_{i=1}^{6}\sum_{j=1}^{6}(\tilde{t}_{ij})x_{ij}+0.1\sum_{i=1}^{6}\sum_{j=1}^{6}(\tilde{q}_{ij})x_{ij}$$

subject to $$x_{ij}=\begin{cases}1 & if\ i^{th}\ person is\ assigned j^{th}\ work\\ 0 & otherwise\end{cases}$$

$$\sum_{i=1}^{6}x_{ij}=1,\qquad j=1,2,...,6.$$

$$\sum_{j=1}^{6}x_{ij}=1,\qquad i=1,2,...,6.$$

Here, w_1 = 0.5, w_2 = 0.4, w_3 = 0.1.

Step 3: The fuzzy multi-objective assignment problem is converted into the following crisp linear programming problem

Minimize z =

$$0.5\sum_{i=1}^{6}\sum_{j=1}^{6}R(\tilde{c}_{ij})x_{ij}+0.4\sum_{i=1}^{6}\sum_{j=1}^{6}R(\tilde{t}_{ij})x_{ij}+0.1\sum_{i=1}^{6}\sum_{j=1}^{6}R(\tilde{q}_{ij})x_{ij}$$

subject to

$$x_{ij}=\begin{cases}1 & if\ i^{th}\ person is\ assigned j^{th}\ work\\ 0 & otherwise\end{cases}$$

$$\sum_{i=1}^{6} x_{ij} = 1, \quad j = 1,2,...,6.$$

$$\sum_{j=1}^{6} x_{ij} = 1, \quad i = 1,2,...,6.$$

Step 4: Using definition 2 and section 2.2, the values of $R(\tilde{c}_{ij})$, $R(\tilde{t}_{ij})$ and $R(\tilde{q}_{ij})$, $\forall i, j$ are calculated and given in Table V.

Step 5: Using step 6 of the proposed model highest cost, highest time and highest quality from table V respectively are 25, 20.5 and 0.33. For w_1= 0.5, w_2 = 0.4 and w_3 = 0.1, the values of a, b, and c are: a = 0.02, b = 0.01 and c = 0.303.

Step 6: Using step 7 of the proposed method convert the chosen fuzzy multi-objective assignment problem into the following crisp linear programming

Minimize z = $(0.28)x_{11}$ + $(0.25)x_{12}$ + $(0.23)x_{13}$ + $(0.31)x_{14}$ + $(0.33)x_{15}$ + $(0.25)x_{16}$ + $(0.22)x_{21}$ + $(0.34)x_{22}$ + $(0.25)x_{23}$ + $(0.26)x_{24}$ + $(0.30)x_{25}$ + $(0.30)x_{26}$ + $(0.29)x_{31}$ + $(0.30)x_{32}$ + $(0.30)x_{33}$ + $(0.29)x_{34}$ + $(0.31)x_{35}$ + $(0.23)x_{36}$ + $(0.27)x_{41}$ + $(0.32)x_{42}$ + $(0.37)x_{43}$ + $(0.26)x_{44}$ + $(0.30)x_{45}$ + $(0.33)x_{46}$ + $(0.28)x_{51}$ + $(0.55)x_{52}$ + $(0.40)x_{53}$ + $(0.26)x_{54}$ + $(0.47)x_{55}$ + $(0.39)x_{56}$ + $(0.55)x_{61}$ + $(0.67)x_{62}$ + $(0.66)x_{63}$ + $(0.68)x_{64}$ + $(0.59)x_{65}$ + $(0.33)x_{66}$

subject to: $x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} = 1$; $x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} = 1$; $x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} = 1$; $x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} = 1$; $x_{51} + x_{52} + x_{53} + x_{54} + x_{55} + x_{56} = 1$; $x_{61} + x_{62} + x_{63} + x_{64} + x_{65} + x_{66} = 1$; $x_{11} + x_{21} + x_{31} + x_{41} + x_{51} + x_{61} = 1$; $x_{11} + x_{22} + x_{32} + x_{42} + x_{52} + x_{62} = 1$; $x_{13} + x_{23} + x_{33} + x_{43} + x_{53} + x_{63} = 1$; $x_{14} + x_{24} + x_{34} + x_{44} + x_{54} + x_{65} = 1$; $x_{15} + x_{25} + x_{35} + x_{45} + x_{55} + x_{65} = 1$; $x_{16} + x_{26} + x_{36} + x_{46} + x_{56} + x_{66} = 1$. $x_{ij} \geq 0$, for all i=1,2,3,4,5,6 and j=1,2,3,4,5,6.

Step 7: Solving the crisp linear programming problem, obtained in step 6, the optimal solution obtained is: x_{13} = 1, x_{21} = 1, x_{32} = 1, x_{46} = 1, x_{54} = 1, x_{65} = 1.

Step 8: Using step 9 of the proposed model, the minimum fuzzy assignment cost, the fuzzy assignment time and the fuzzy assignment quality respectively are ,(56, 71, 11, 14) , (40, 50, 11, 13) and (0.54, 0.71, 0.16, 0.15).

Case (iii) L (x) = max $\{0, 1-|x|\}$ **and R (x) =** e^{-x}

Step 1: The given fuzzy multi-objective assignment problem is a balanced one.

Step 2: Using step3 of the proposed model the given fuzzy multi-objective assignment problem is converted into a single fuzzy objective problem as follows

Minimize $\tilde{z}$ =

$$0.5\sum_{i=1}^{6}\sum_{j=1}^{6}(\tilde{c}_{ij})x_{ij} + 0.4\sum_{i=1}^{6}\sum_{j=1}^{6}(\tilde{t}_{ij})x_{ij} + 0.1\sum_{i=1}^{6}\sum_{j=1}^{6}(\tilde{q}_{ij})x_{ij}$$

subject to $x_{ij} = \begin{cases} 1 & \text{if } i^{th} \text{ personis assigned} j^{th} \text{ work} \\ 0 & \text{otherwise} \end{cases}$

$$\sum_{i=1}^{6} x_{ij} = 1, \quad j = 1,2,...,6.$$

$$\sum_{j=1}^{6} x_{ij} = 1, \quad i = 1,2,...,6.$$

Here, w_1 = 0.5, w_2 = 0.4, w_3 = 0.1.

Step 3: The fuzzy multi-objective assignment problem is converted into the following crisp linear programming problem

Minimize z =

$$0.5\sum_{i=1}^{6}\sum_{j=1}^{6}R(\tilde{c}_{ij})x_{ij} + 0.4\sum_{i=1}^{6}\sum_{j=1}^{6}R(\tilde{t}_{ij})x_{ij} + 0.1\sum_{i=1}^{6}\sum_{j=1}^{6}R(\tilde{q}_{ij})x_{ij}$$

subject to

$$x_{ij} = \begin{cases} 1 & \text{if } i^{th} \text{ personis assigned} j^{th} \text{ work} \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_{i=1}^{6} x_{ij} = 1, \quad j = 1,2,...,6.$$

$$\sum_{j=1}^{6} x_{ij} = 1, \quad i = 1,2,...,6.$$

Step 4: Using definition 2 and section 2.2, the values of $R(\tilde{c}_{ij})$, $R(\tilde{t}_{ij})$ and $R(\tilde{q}_{ij})$, $\forall i, j$ are calculated and given in Table VI.

Step 5: Using step 6 of the proposed model highest cost, highest time and highest quality from table VI respectively are 25.5, 21 and 0.34. For w_1= 0.5, w_2 = 0.4 and w_3 = 0.1, the values of a, b, and c are: a = 0.019, b = 0.018 and c = 0.2941.

Step 6: Using step 7 of the proposed method, convert the chosen fuzzy multi-objective assignment problem into the following crisp linear programming

Minimize z = $(0.37)x_{11}$ + $(0.34)x_{12}$ + $(0.28)x_{13}$ + $(0.41)x_{14}$ + $(0.42)x_{15}$ + $(0.36)x_{16}$ + $(0.29)x_{21}$ + $(0.46)x_{22}$ + $(0.31)x_{23}$ + $(0.35)x_{24}$ + $(0.38)x_{25}$ + $(0.39)x_{26}$ + $(0.33)x_{31}$ + $(0.36)x_{32}$ + $(0.37)x_{33}$ + $(0.36)x_{34}$ + $(0.38)x_{35}$ + $(0.27)x_{36}$ + $(0.35)x_{41}$ + $(0.38)x_{42}$ + $(0.45)x_{43}$ + $(0.32)x_{44}$ + $(0.35)x_{45}$ + $(0.43)x_{46}$ + $(0.39)x_{51}$ + $(0.58)x_{52}$ + $(0.50)x_{53}$ + $(0.31)x_{54}$ + $(0.54)x_{55}$ + $(0.54)x_{56}$ + $(0.70)x_{61}$ + $(0.71)x_{62}$ + $(0.82)x_{63}$ + $(0.82)x_{64}$ + $(0.67)x_{65}$ + $(0.43)x_{66}$

subject to: $x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} = 1$; $x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} = 1$; $x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} = 1$; $x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} = 1$; $x_{51} + x_{52} + x_{53} + x_{54} + x_{55} + x_{56} = 1$; $x_{61} + x_{62} + x_{63} + x_{64} + x_{65} + x_{66} = 1$; $x_{11} + x_{21} + x_{31} + x_{41} + x_{51} + x_{61} = 1$; $x_{11} + x_{22} + x_{32} + x_{42} + x_{52} + x_{62} = 1$; $x_{13} + x_{23} + x_{33} + x_{43} + x_{53} + x_{63} = 1$;$x_{14} + x_{24} + x_{34} + x_{44} + x_{54} + x_{65} = 1$; $x_{15} + x_{25} + x_{35} + x_{45} + x_{55} + x_{65} = 1$; $x_{16} + x_{26} + x_{36} + x_{46} + x_{56} + x_{66} = 1$. $x_{ij} \geq 0$, for all i=1,2,3,4,5,6 and j=1,2,3,4,5,6.

Step 7: Solving the crisp linear programming problem, obtained in step 3, the optimal solution obtained is x_{13} = 1, x_{21} = 1, x_{32} = 1, x_{45} = 1, x_{54} = 1, x_{66} = 1.

Step 8: Using step 9 of the proposed model, the minimum fuzzy assignment cost, the fuzzy assignment time and the fuzzy assignment quality respectively are, (45, 58, 12, 12) , (37, 48, 8, 9) and (0.57, 0.72, 0.13, 0.14)

Case (iv) L (x) = e^{-x} **and R (x) = max** $\{0, 1-|x|\}$.

Step 1: The given fuzzy multi-objective assignment problem is a balanced one.

Step 2: Using step3 of the proposed model the given fuzzy multi-objective assignment problem is converted into a single fuzzy objective problem as follows

Minimize $\tilde{z}$ =

$$0.5\sum_{i=1}^{6}\sum_{j=1}^{6}(\tilde{c}_{ij})x_{ij}+0.4\sum_{i=1}^{6}\sum_{j=1}^{6}(\tilde{t}_{ij})x_{ij}+0.1\sum_{i=1}^{6}\sum_{j=1}^{6}(\tilde{q}_{ij})x_{ij}$$

subject to $x_{ij}=\begin{cases}1 & if\ i^{th}\ person is\ assigned j^{th}\ work\\ 0 & otherwise\end{cases}$

$$\sum_{i=1}^{6}x_{ij}=1,\quad j=1,2,...,6.$$

$$\sum_{j=1}^{6}x_{ij}=1,\quad i=1,2,...,6.$$

Here, w_1 = 0.5, w_2 = 0.4, w_3 = 0.1.

Step 3: The fuzzy multi-objective assignment problem is converted into the following crisp linear programming problem

Minimize z =

$$0.5\sum_{i=1}^{6}\sum_{j=1}^{6}R(\tilde{c}_{ij})x_{ij}+0.4\sum_{i=1}^{6}\sum_{j=1}^{6}R(\tilde{t}_{ij})x_{ij}+0.1\sum_{i=1}^{6}\sum_{j=1}^{6}R(\tilde{q}_{ij})x_{ij}$$

subject to

$x_{ij}=\begin{cases}1 & if\ i^{th}\ person is\ assigned j^{th}\ work\\ 0 & otherwise\end{cases}$

$$\sum_{i=1}^{6}x_{ij}=1,\quad j=1,2,...,6.$$

$$\sum_{j=1}^{6}x_{ij}=1,\quad i=1,2,...,6.$$

Step 4: Using definition 2 and section 2.2, the values of $R(\tilde{c}_{ij})$, $R(\tilde{t}_{ij})$ and $R(\tilde{q}_{ij})$, $\forall i,j$ are calculated and given in Table VII.

Step 5: Using step 6 of the proposed model highest cost, highest time and highest quality from table VII respectively are 27.5, 22 and 0.35. For w_1= 0.5, w_2 = 0.4 and w_3 = 0.1, the values of a, b, and c are: a = 0.018, b = 0.018 and c = 0.285.

Step 6: Using step 7 of the proposed method convert the chosen fuzzy multi-objective assignment problem into the following crisp linear programming

Minimize z = $(0.40)x_{11} + (0.38)x_{12} + (0.32)x_{13} + (0.46)x_{14} + (0.42)x_{15} + (0.39)x_{16} + (0.30)x_{21} + (0.49)x_{22} + (0.33)x_{23} + (0.37)x_{24} + (0.38)x_{25} + (0.44)x_{26} + (0.35)x_{31} + (0.37)x_{32} + (0.37)x_{33} + (0.37)x_{34} + (0.38)x_{35} + (0.28)x_{36} + (0.41)x_{41} + (0.39)x_{42} + (0.48)x_{43} + (0.35)x_{44} + (0.38)x_{45} + (0.47)x_{46} + (0.43)x_{51} + (0.62)x_{52} + (0.51)x_{53} + (0.31)x_{54} + (0.56)x_{55} + (0.57)x_{56} + (0.73)x_{61} + (0.72)x_{62} + (0.80)x_{63} + (0.84)x_{64} + (0.68)x_{65} + (0.46)x_{66}$

subject to: $x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} = 1$; $x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} = 1$; $x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} = 1$; $x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} = 1$; $x_{51} + x_{52} + x_{53} + x_{54} + x_{55} + x_{56} = 1$; $x_{61} + x_{62} + x_{63} + x_{64} + x_{65} + x_{66} = 1$; $x_{11} + x_{21} + x_{31} + x_{41} + x_{51} + x_{61} = 1$; $x_{11} + x_{22} + x_{32} + x_{42} + x_{52} + x_{62} = 1$; $x_{13} + x_{23} + x_{33} + x_{43} + x_{53} + x_{63} = 1$; $x_{14} + x_{24} + x_{34} + x_{44} + x_{54} + x_{65} = 1$; $x_{15} + x_{25} + x_{35} + x_{45} + x_{55} + x_{65} = 1$; $x_{16} + x_{26} + x_{36} + x_{46} + x_{56} + x_{66} = 1$, $x_{ij} \geq 0$, for all i=1,2,3,4,5,6 and j=1,2,3,4,5,6.

Step 7: Solving the crisp linear programming problem, obtained in step3, the optimal solution obtained is: x_{13} = 1, x_{21} = 1, x_{32} = 1, x_{45} = 1, x_{54} = 1, x_{66} = 1.

Step 8: Using step 9 of the proposed model, the minimum fuzzy assignment cost, the fuzzy assignment time and the fuzzy assignment quality respectively are, (45, 58, 12, 12) , (37, 48, 8, 9) and (0.57, 0.72, 0.13, 0.14).

Case (v) L (x) = e^{-px} and R (x) = max $\{0,1-|x^p|\}$

Step 1: The given fuzzy multi-objective assignment problem is a balanced one.

Step 2: Using step3 of the proposed model the given fuzzy multi-objective assignment problem is converted into a single fuzzy objective problem as follows

Minimize $\tilde{Z}$ =

$$0.5\sum_{i=1}^{6}\sum_{j=1}^{6}(\tilde{c}_{ij})x_{ij}+0.4\sum_{i=1}^{6}\sum_{j=1}^{6}(\tilde{t}_{ij})x_{ij}+0.1\sum_{i=1}^{6}\sum_{j=1}^{6}(\tilde{q}_{ij})x_{ij}$$

subject to $x_{ij}=\begin{cases}1 & if\ i^{th}\ person is\ assigned j^{th}\ work\\ 0 & otherwise\end{cases}$

$$\sum_{i=1}^{6}x_{ij}=1,\quad j=1,2,...,6.$$

$$\sum_{j=1}^{6}x_{ij}=1,\quad i=1,2,...,6.$$

Here w_1 = 0.5, w_2 = 0.4, w_3 = 0.1.

Step 3: The fuzzy multi-objective assignment problem is converted into the following crisp linear programming problem. Minimize Z=

$$0.5\sum_{i=1}^{6}\sum_{j=1}^{6}R(\tilde{c}_{ij})x_{ij}+0.4\sum_{i=1}^{6}\sum_{j=1}^{6}R(\tilde{t}_{ij})x_{ij}+0.1\sum_{i=1}^{6}\sum_{j=1}^{6}R(\tilde{q}_{ij})x_{ij}$$

subject to $x_{ij}=\begin{cases}1 & if\ i^{th}\ person is\ assigned j^{th}\ work\\ 0 & otherwise\end{cases}$

$$\sum_{i=1}^{6}x_{ij}=1,\quad j=1,2,...,6.$$

$$\sum_{j=1}^{6}x_{ij}=1,\quad i=1,2,...,6.$$

Step 4: Using definition 2 and section 2.2, the values of $R(\tilde{c}_{ij})$, $R(\tilde{t}_{ij})$ and $R(\tilde{q}_{ij})$, $\forall i,j$ are calculated and given in Table VIII.

Step 5: Using step 6 of the proposed model highest cost, highest time and highest quality from table VIII respectively are 25.25, 20.75 and 0.33. For w_1= 0.5, w_2 = 0.4 and w_3 = 0.1, the values of a, b, and c are: a = 0.0195, b = 0.0192 and c = 0.2857.

Step 6: Using step 7 of the proposed method convert the chosen fuzzy multi-objective assignment problem into the following crisp linear programming

Minimize z = $(0.37)x_{11} + (0.34)x_{12} + (0.28)x_{13} + (0.42)x_{14} + (0.42)x_{15} + (0.36)x_{16} + (0.30)x_{21} + (0.46)x_{22} + (0.31)x_{23} + (0.36)x_{24} + (0.38)x_{25} + (0.40)x_{26} + (0.33)x_{31} + (0.36)x_{32} + (0.37)x_{33} + (0.36)x_{34} + (0.38)x_{35} + (0.27)x_{36} + (0.35)x_{41} + (0.38)x_{42} + (0.46)x_{43} + (0.31)x_{44} + (0.36)x_{45} + (0.42)x_{46} + (0.40)x_{51} + (0.59)x_{52} + (0.51)x_{53} + (0.31)x_{54} + (0.55)x_{55} +$

$(0.55)x_{56}$ + $(0.73)x_{61}$ + $(0.72)x_{62}$ + $(0.85)x_{63}$ + $(0.85)x_{64}$ + $(0.68)x_{65}$ + $(0.44)x_{66}$
subject to: $x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} = 1$; $x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} = 1$; $x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} = 1$; $x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} = 1$; $x_{51} + x_{52} + x_{53} + x_{54} + x_{55} + x_{56} = 1$; $x_{61} + x_{62} + x_{63} + x_{64} + x_{65} + x_{66} = 1$; $x_{11} + x_{21} + x_{31} + x_{41} + x_{51} + x_{61} = 1$; $x_{11} + x_{22} + x_{32} + x_{42} + x_{52} + x_{62} = 1$; $x_{13} + x_{23} + x_{33} + x_{43} + x_{53} + x_{63} = 1$; $x_{14} + x_{24} + x_{34} + x_{44} + x_{54} + x_{65} = 1$; $x_{15} + x_{25} + x_{35} + x_{45} + x_{55} + x_{65} = 1$; $x_{16} + x_{26} + x_{36} + x_{46} + x_{56} + x_{66} = 1$, $x_{ij} \geq 0$, for all i=1,2,3,4,5,6 and j=1,2,3,4,5,6.

Step 7: Solving the crisp linear programming problem, obtained in step3, the optimal solution obtained is: $x_{13} = 1$, $x_{21} = 1$, $x_{32} = 1$, $x_{45} = 1$, $x_{54} = 1$, $x_{66} = 1$.

Step 8: Using step 9 of the proposed model, the minimum fuzzy assignment cost, the fuzzy assignment time and the fuzzy assignment quality respectively are, (45, 58, 12, 12) , (37, 48, 8, 9) and (0.57, 0.72, 0.13, 0.14).

Case (vi) L (x) = max $\{0, 1-|x^p|\}$ **and R (x)** $= e^{-px}$

Step 1: The given fuzzy multi-objective assignment problem is a balanced one.

Step 2: Using step3 of the proposed model the given fuzzy multi-objective assignment problem is converted into a single fuzzy objective problem as follows

Minimize $\tilde{Z}$ =

$$0.5\sum_{i=1}^{6}\sum_{j=1}^{6}(\tilde{c}_{ij})x_{ij} + 0.4\sum_{i=1}^{6}\sum_{j=1}^{6}(\tilde{t}_{ij})x_{ij} + 0.1\sum_{i=1}^{6}\sum_{j=1}^{6}(\tilde{q}_{ij})x_{ij}$$

subject to $x_{ij} = \begin{cases} 1 & if\ i^{th}\ person is\ assigned j^{th}\ work \\ 0 & otherwise \end{cases}$

$$\sum_{i=1}^{6} x_{ij} = 1, \quad j = 1,2,...,6.$$

$$\sum_{j=1}^{6} x_{ij} = 1, \quad i = 1,2,...,6.$$

Here, $w_1 = 0.5$, $w_2 = 0.4$, $w_3 = 0.1$.

Step 3: The fuzzy multi-objective assignment problem is converted into the following crisp linear programming problem

Minimize z =

$$0.5\sum_{i=1}^{6}\sum_{j=1}^{6}R(\tilde{c}_{ij})x_{ij} + 0.4\sum_{i=1}^{6}\sum_{j=1}^{6}R(\tilde{t}_{ij})x_{ij} + 0.1\sum_{i=1}^{6}\sum_{j=1}^{6}R(\tilde{q}_{ij})x_{ij}$$

subject to

$x_{ij} = \begin{cases} 1 & if\ i^{th}\ person is\ assigned j^{th}\ work \\ 0 & otherwise \end{cases}$

$$\sum_{i=1}^{6} x_{ij} = 1, \quad j = 1,2,..., n.$$

$$\sum_{j=1}^{6} x_{ij} = 1, \quad i = 1,2,..., n.$$

Step 4: Using definition 2 and section 2.2, the values of $R(\tilde{c}_{ij})$, $R(\tilde{t}_{ij})$ and $R(\tilde{q}_{ij})$, $\forall$i, j are calculated and given in Table IX.

Step 5: Using step 6 of the proposed model highest cost, highest time and highest quality from table IX respectively are 53.33, 43 and 0.69. For w_1= 0.5, w_2 = 0.4 and w_3 = 0.1, the values of *a*, *b*, and *c* are: *a* = 0.009, *b* = 0.0092 and *c* = 0.144.

Step 6: Using step 7 of the proposed method convert the chosen fuzzy multi-objective assignment problem into the following crisp linear programming

Minimize z = $(0.38)x_{11}$ + $(0.36)x_{12}$ + $(0.30)x_{13}$ + $(0.44)x_{14}$ + $(0.41)x_{15}$ + $(0.37)x_{16}$ + $(0.29)x_{21}$ + $(0.47)x_{22}$ + $(0.32)x_{23}$ +$(0.36)x_{24}$ + $(0.36)x_{25}$ + $(0.42)x_{26}$ + $(0.33)x_{31}$ + $(0.36)x_{32}$ + $(0.36)x_{33}$ + $(0.35)x_{34}$ + $(0.37)x_{35}$ + $(0.27)x_{36}$ + $(0.38)x_{41}$ + $(0.38)x_{42}$ + $(0.46)x_{43}$ + $(0.33)x_{44}$ + $(0.37)x_{45}$ + $(0.44)x_{46}$ + $(0.41)x_{51}$ + $(0.60)x_{52}$ + $(0.50)x_{53}$ + $(0.30)x_{54}$ + $(0.54)x_{55}$ + $(0.56)x_{56}$ + $(0.72)x_{61}$ + $(0.71)x_{62}$ + $(0.82)x_{63}$ + $(0.83)x_{64}$ + $(0.67)x_{65}$ + $(0.45)x_{66}$
subject to: $x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} = 1$; $x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} = 1$; $x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} = 1$; $x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} = 1$; $x_{51} + x_{52} + x_{53} + x_{54} + x_{55} + x_{56} = 1$; $x_{61} + x_{62} + x_{63} + x_{64} + x_{65} + x_{66} = 1$; $x_{11} + x_{21} + x_{31} + x_{41} + x_{51} + x_{61} = 1$; $x_{11} + x_{22} + x_{32} + x_{42} + x_{52} + x_{62} = 1$; $x_{13} + x_{23} + x_{33} + x_{43} + x_{53} + x_{63} = 1$; $x_{14} + x_{24} + x_{34} + x_{44} + x_{54} + x_{65} = 1$; $x_{15} + x_{25} + x_{35} + x_{45} + x_{55} + x_{65} = 1$; $x_{16} + x_{26} + x_{36} + x_{46} + x_{56} + x_{66} = 1$, $x_{ij} \geq 0$, for all i=1,2,3,4,5,6 and j=1,2,3,4,5,6.

Step 7: Solving the crisp linear programming problem, obtained in step3, the optimal solution obtained is: $x_{13} = 1$, $x_{21} = 1$, $x_{32} = 1$, $x_{45} = 1$, $x_{54} = 1$, $x_{66} = 1$.

Step 8: Using step 9 of the proposed model, the minimum fuzzy assignment cost, the fuzzy assignment time and the fuzzy assignment quality respectively are, (45, 58, 12, 12) , (37, 48, 8, 9) and (0.57, 0.72, 0.13, 0.14).

6. Conclusion

In this paper, a fuzzy multi objective assignment problem using linear programming model has developed, and it is converted into a single fuzzy objective assignment problem. The functions of L-R fuzzy numbers of fuzzy multi objective assignment problem are regard as linear and non-linear and the model is illustrated with an example using various cases. It instigates that the optimal assignment and total optimal fuzzy assignment cost, time and quality is same in all the cases, except it is unlike while both left and right functions are exponential .

Table I : Reference functions and their inverses

Function Name	**Reference Function (RF)**	**Inverse of Reference function** $\alpha \in (0,1]$
Linear	$RF_p(x) = \max\{0, 1 - \lvert x \rvert\}$	$RF_p^{-1}(x) = (1-\alpha)$
Exponential	$RF_p(x) = e^{-px},\ p \ge 1$	$RF_p^{-1}(x) = -(In\alpha)/\,p$
Power	$RF_p(x) = \max(0, 1 - x^p),\ \ p \ge 1$	$RF_p^{-1}(x) = \sqrt[p]{1-\alpha}$
Exponential power	$RF_p(x) = e^{-x^p},\ p \ge 1$	$RF_p^{-1}(x) = \sqrt[p]{-In\alpha}$
Rational	$RF_p(x) = 1/(1+x^p),\ p \ge 1$	$RF_p^{-1}(x) = \sqrt[p]{(1-\alpha)/\alpha}$

Table II: Fuzzy assignment with fuzzy cost, fuzzy time, fuzzy quality

Works

Persons	1	2	…	j	…	n
1	$\tilde{c}_{11};\tilde{t}_{11};\tilde{q}_{11}$	$\tilde{c}_{12};\tilde{t}_{12};\tilde{q}_{12}$	…	$\tilde{c}_{1j};\tilde{t}_{1j};\tilde{q}_{1j}$	…	$\tilde{c}_{1n};\tilde{t}_{1n};\tilde{q}_{1n}$
2	$\tilde{c}_{21};\tilde{t}_{21};\tilde{q}_{21}$	$\tilde{c}_{22};\tilde{t}_{22};\tilde{q}_{22}$	…	$\tilde{c}_{2j};t_{2j};\tilde{q}_{2j}$	…	$\tilde{c}_{2n};t_{2n};\tilde{q}_{2n}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
i	$\tilde{c}_{i1};t_{i1};\tilde{q}_{i1}$	$\tilde{c}_{i2};t_{i2};\tilde{q}_{i2}$	…	$\tilde{c}_{ij};t_{ij};\tilde{q}_{ij}$	…	$\tilde{c}_{in};t_{in};\tilde{q}_{in}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
n	$\tilde{c}_{n1};t_{n1};\tilde{q}_{n1}$	$\tilde{c}_{n2};t_{n2};\tilde{q}_{n2}$	…	$\tilde{c}_{nj};t_{nj};\tilde{q}_{nj}$	…	$\tilde{c}_{nn};t_{nn};\tilde{q}_{nn}$

TABLE III: Fuzzy assignment problem with fuzzy cost, fuzzy time and fuzzy quality

	I	**II**	**III**	**IV**	**V**	**VI**
A	(6,7,2,2) (9,11,2,2) (0.16,0.19,0.01,0.02)	(5,7,2,3) (9,10,3,2) (0.11,0.13,0.01,0.01)	(8,10,3,2) (4,5,2,2) (0.04,0.06,0.03,0.03)	(5,8,1,2) (12,13,4,2) (0.18,0.20,0.02,0.1)	(7,10,1,2) (10,11,1,1) (0.16,0.18,0.02,0.02)	(4,6,1,3) (11,13,3,2) (0.07,0.09,0.02,0.02)
B	(3,5,1,2) (7,10,1,2) (0.12,0.15,0.03,0.03)	(7,8,2,3) (12,14,3,3) (0.16,0.18,0.02,0.02)	(8,7,2,4) (5,8,2,1) (0.08,0.09,0.03,0.01)	(5,7,2,2) (9,12,1,1) (0.09,0.15,0.03,0.03)	(6,7,1,3) (8,10,1,1) (0.21,0.23,0.01,0.02)	(7,9,3,2) (8,12,2,1) (0.18,0.22,0.03,00.03)
C	(8,10,2,2) (4,5,1,2) (0.20,0.22,0.02,0.02)	(7,12,2,2) (5,7,1,2) (0.15,0.17,0.02,0.02)	(8,9,1,2) (7,9,1,2) (0.11,0.14,0.01,0.02)	(6,8,2,2) (6,8,1,4) (0.22,0.23,0.02,0.02)	(7,9,1,1) (7,8,1,3) (0.22,0.24,0.02,0.03)	(5,7,1,2) (4,6,1,1) (0.16,0.18,0.01,0.02)
D	(7,10,4,2) (6,8,2,2) (0.18,0.20,0.03,0.02)	(7,10,1,2) (7,8,2,2) (0.21,0.23,0.02,0.02)	(11,13,3,2) (7,10,1,2) (0.17,0.20,0.02,0.02)	(9,10,3,2) (4,6,2,2) (0.10,0.11,0.01,0.04)	(10,11,3,2) (5,7,1,1) (0.13,0.14,0.01,0.01)	(7,10,2,4) (9,11,4,4) (0.14,0.16,0.04,0.02)
E	(6,8,2,2) (11,13,3,2) (0.09,0.11,0.01,0.01)	(24,26,4,2) (2,4,2,2) (0.14,0.18,0.02,0.02)	(12,14,2,2) (12,13,1,2) (0.03,0.04,0.01,0.02)	(9,10,1,2) (5,6,1,1) (0.03,0.05,0.02,0.02)	(16,17,2,3) (8,10,2,2) (0.14,0.15,0.01,0.01)	(8,11,2,2) (16,18,2,2) (0.12,0.14,0.02,0.01)
F	(16,19,2,1) (18,20,2,2) (0.07,0.08,0.01,0.03)	(24,26,2,2) (6,8,2,2) (0.32,0.35,0.02,0.02)	(21,23,1,1) (20,21,2,2) (0.06,0.08,0.02,0.02)	(20,22,2,2) (18,20,2,2) (0.22,0.24,0.01,0.02)	(22,24,2,2) (10,11,2,2) (0.06,0.12,0.02,0.03)	(8,10,2,2) (11,13,2,1) (0.10,0.15,0.02,0.03)

TABLE IV: Ranks of Fuzzy Cost, Fuzzy Time and Fuzzy Quality for case (i)

	I	II	III	IV	V	VI
A	6.5 10 0.1775	6.2 9.25 0.12	8.75 4.5 0.05	6.75 12 0.1875	8.75 10.75 0.17	5.5 11.75 0.08
B	4.25 8.75 0.135	7.75 13 0.17	8 6.25 0.08	6 10.5 0.12	7 9 0.2225	7.75 9.75 0.2
C	9 4.75 0.21	9.5 6.25 0.16	8.75 8.25 0.1275	7 7.75 0.225	8 8 0.2325	6.25 5 0.1725
D	8 7 0.1875	8.75 7.5 0.22	11.75 8.75 0.185	9.25 5 0.1125	10.25 6 0.135	9 10 0.145
E	7 11.75 0.1	24.5 3 0.16	13 12.75 0.0375	9.75 5.5 0.04	16.75 9 0.145	9.5 17 0.1275
F	17.25 19 0.08	25 7 0.335	22 20.5 0.075	21 19 0.2325	23 10.5 0.0925	9 11.75 0.1275

TABLE V: Ranks of Fuzzy Cost, Fuzzy Time and Fuzzy Quality for case (ii)

	I	II	III	IV	V	VI
A	6.5 10 0.1775	6.2 9.25 0.12	8.75 4.5 0.05	6.75 12 0.1875	8.75 10.75 0.17	5.5 11.75 0.08
B	4.25 8.75 0.135	7.75 13 0.17	8 6.25 0.08	6 10.5 0.12	7 9 0.2225	7.75 9.75 0.2
C	9 4.75 0.21	9.5 6.25 0.16	8.75 8.25 0.1275	7 7.75 0.225	8 8 0.2325	6.25 5 0.1725
D	8 7 0.1875	8.75 7.5 0.22	11.75 8.75 0.185	9.25 5 0.1125	10.25 6 0.135	9 10 0.145
E	7 11.75 0.1	24.5 3 0.16	13 12.75 0.0375	9.75 5.5 0.04	16.75 9 0.145	9.5 17 0.1275
F	17.25 19 0.08	25 7 0.335	22 20.5 0.075	21 19 0.2325	23 10.5 0.0925	9 11.75 0.1275

TABLE VI: Ranks of Fuzzy Cost, Fuzzy Time and Fuzzy Quality for case (iii)

	I	II	III	IV	V	VI
A	7 10.5 0.1825	7 9.75 0.1225	9.25 5 0.0575	7.25 12.5 0.19	9.25 10.75 0.175	6.25 12.25 0.085
B	4.75 9.25 0.1425	8.5 13.75 0.175	9 6.5 0.0825	6.5 10.75 0.1275	7.75 9.25 0.2275	8.25 10 0.2075
C	9.5 5.25 0.215	10 6.75 0.165	9.25 8 0.225	7.5 8.75 0.23	8.25 8.75 0.24	6.75 5.25 0.1775
D	8.5 7.5 0.1925	9.25 8 0.225	12.25 9.25 0.19	9.75 5.5 0.1225	10.75 6.25 0.1375	10 11 0.15
E	7.5 12.25 0.1025	25 3.5 0.165	13.5 13.25 0.0425	10.25 5.75 0.045	17.5 9.5 0.1475	10 17.5 0.13
F	17.5 19.5 0.0875	25.5 7.5 0.34	22.25 21 0.075	21.5 19.5 0.2375	23.5 11 0.1	9.5 12 0.135

TABLE VII: Ranks of Fuzzy Cost, Fuzzy Time and Fuzzy Quality for case (iv)

	I	II	III	IV	V	VI
A	8 11.5 0.185	7.75 11.5 0.1275	11 6 0.0725	7.5 15 0.2025	9.5 11.25 0.185	6.25 14 0.095
B	5 9.5 0.1575	9.25 15.25 0.185	9.5 7.75 0.1025	7.5 11.25 0.1425	7.75 9.75 0.23	10 11.25 0.2225
C	10.5 5.5 0.225	11 7 0.175	9.5 9 0.135	8.5 8.5 0.24	8.75 8.75 0.2475	7 5.75 0.18
D	11 8.5 0.21	9.5 9 0.235	14 9.5 0.2	11.5 6.5 0.12	12.5 6.75 0.1425	10.5 13 0.175
E	8.5 14 0.1075	27.5 4.5 0.175	14.5 13.5 0.045	10.5 6.25 0.055	18.25 10.5 0.1525	11 18.5 0.1425
F	18.75 20.5 0.0875	26.5 8.5 0.35	22.75 22 0.085	22.5 20.5 0.24	24.5 12 0.1075	10.5 13.25 0.1425

TABLE VIII: Ranks of Fuzzy Cost, Fuzzy Time and Fuzzy Quality for case (v)

	I	II	III	IV	V	VI
A	6.75 10.25 0.18	6.625 9.5 0.12125	9 4.75 0.05375	7 12.25 0.18875	9 10.625 0.1725	5.875 12 0.0825
B	4.5 9 0.13875	8.125 13.375 0.1725	8.5 6.375 0.08125	6.25 10.625 0.12375	7.375 9.125 0.225	8 9.875 0.20375
C	9.25 5 0.2125	9.75 6.5 0.1625	9 8.5 0.13	7.25 8.25 0.2275	8.125 8.375 0.23625	6.5 5.125 0.175
D	8.25 7.25 0.19	9 7.75 0.2225	12 9 0.1875	9.5 5.25 0.1175	10.5 6.125 0.13625	9.5 10.5 0.1475
E	7.25 12 0.10125	24.75 3.25 0.1625	13.25 13 0.04	10 5.625 0.0425	17.125 9.25 0.14625	9.75 17.25 0.12875
F	17.375 19.25 0.08375	25.25 7.25 0.3375	22.125 20.75 0.0725	21.25 19.25 0.235	23.25 10.75 0.09625	9.25 11.875 0.13125

TABLE IX: Ranks of Fuzzy Cost, Fuzzy Time and Fuzzy Quality for case (vi)

	I	II	III	IV	V	VI
A	15 22 0.3633	14.3333 21.66667 0.25	20.66 11 0.13	14.3333 28.33 0.3966	18.3333 22 0.36	11.6666 26.6666 0.18
B	9.3333 18.3333 0.3	17.3333 29 0.36	17.66 14.66 0.1933	14 22 0.27	14.6666 19 0.4533	18.6666 21.6666 0.43
C	20 10.3333 0.44	21 13.3333 0.34	18.33 17.33 0.2633	16 16 0.47	17 16.6666 0.4833	13.3333 11 0.3533
D	20.3333 16 0.4066	18.3333 17 0.46	26.66 18.33 0.39	21.66 12 0.23	23.6666 13 0.28	19.6666 24 0.3333
E	16 26.66 0.21	53.33 8 0.34	28 26.33 0.0833	20.33 12 0.1	35.33 20 0.3	21 36 0.2766
F	36.66 40 0.1666	52 16 0.69	45 43 0.16	44 40 0.4733	48 23 0.2033	20 25.6666 0.2733

TABLE X: Total Optimal Fuzzy Assignment Cost, Time and Quality

Linear and non-linear functions	Optimal Assignment	Total Optimal Fuzzy Assignment Cost, Time and Quality
L (x) = R (x) = max $\{0,1-\lvert x\rvert\}$	A-III, B-I, C-II, D-V, E-IV, F-VI	(45, 58, 12, 12) (37, 48, 8, 9) (0.57, 0.72, 0.13, 0.14)
L (x) = R (x) = e^{-x}	A-III, B-I, C-II, D-VI, E-IV,F-V	(56, 71, 11, 14) (40, 50, 11, 13) (0.54, 0.71, 0.16, 0.15)
L (x) = max $\{0,1-\lvert x\rvert\}$ **R (x) =** e^{-x}	A-III, B-I, C-II, D-V, E-IV,F-VI	(45, 58, 12, 12) (37, 48, 8, 9) (0.57, 0.72, 0.13, 0.14)
L (x) = e^{-x} **R (x) = max** $\{0,1-\lvert x\rvert\}$	A-III, B-I, C-II, D-V, E-IV, F-VI	(45, 58, 12, 12) (37, 48, 8, 9) (0.57, 0.72, 0.13, 0.14)
L (x) = e^{-px} **R (x) = max** $\{0,1-\lvert x^p\rvert\}$	A-III, B-I, C-II, D-V, E-IV, F-VI	(45, 58, 12, 12) (37, 48, 8, 9) (0.57, 0.72, 0.13, 0.14)
L (x) = max $\{0,1-\lvert x^p\rvert\}$ **R (x) =** e^{-px}	A-III, B-I, C-II, D-V, E-IV, F-VI	(45, 58, 12, 12) (37, 48, 8, 9) (0.57, 0.72, 0.13, 0.14)

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