A New Genetic Algorithm based Method for Topological Optimization of Interconnection Networks

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ABSTRACT

This paper addresses the topological optimization problem of interconnection networks where the overall cost of the network is minimized subjected to a predefined network reliability constraint. A new method based on genetic algorithm is proposed which uses more efficient encoding, cross over and mutation rate to ensure a better rate of convergence towards optimal solution. The proposed method is well equipped with some new algorithms for each of the operations stated above with suitable step wise illustration. The minimum cost of fully connected networks with up to 15 numbers of nodes and 105 numbers of links are computed with respect to different targeted values of reliability. Further the optimal layout of the interconnection networks within the above computed cost are presented that meets the network reliability constraints.

General Terms

Genetic Algorithm, Optimization , Interconnection Networks,

Keywords

Interconnection Network, Genetic Algorithm, Network Reliability, Topological Optimization.

1. INTRODUCTION

We The design of very economical interconnection network is a complex multi constraint and multi optimization problem. Because it requires the simultaneous optimization of many objectives, they may be conflicting i.e. minimizing the cost of the network while meeting some criteria such as reliability, average delay and throughput. Out of these performance criteria the reliability may be treated as the most important performance criterion as it ensures the working of such networks in the presence of different types of failures [1]. Thus the design decision is to connect all the nodes with some known probability i.e. the network reliability while minimizing the overall cost of the network [2]. The type of topological optimization problem considering the network reliability along with other objectives such as cost is an NP-hard problem [3]. This is due to exponential growth of the solution space with respect to the size of the network.

Review of literature revels mainly two approaches viz. enumerative based approach and heuristic based approach for solving this type of topological optimization problems. Some of the enumerative approaches are branch and bound [4], spanning tree [5]. The method using branch and bound technique divides the problem into several sub problems for an efficient search of the exact optimal solution. But this method is only able to find solution of the fully connected networks of maximum size of 12 nodes (60 links). Similarly, the methods based on enumeration of the spanning trees can be applied only for the small networks, as for large sized networks the computational complexity in enumerating all the spanning trees prohibits using of such techniques. So for the networks of larger size, the heuristic methods like simulated annealing [6][7][8][9], Artificial Neural Network [10], Genetic Algorithm [13][14][15][16][17] are preferred because of their strength and ability to find near optimal solutions. Pierre and et al. [7] used simulated annealing to find optimal designs for packet switched networks where delay and capacity were considered, but reliability was not. Another heuristic approach using an optimize neural network for solving the network topological optimization problem is found in [10]. Similar kind of works can be found in [11] and [12]. However mapping of the topological optimization problem onto a neural network is quite cumbersome. Further literature also records that the heuristic methods based on Artificial Neural Network may not always lead to optimal solutions. The review of literature reveals the use of genetic algorithm effectively in the past to solve many difficult engineering problems as well as the optimization problems [15][16]. Marseguerra et.al. [17] determined optimal surveillance test intervals using multi-objective Genetic Algorithm with the goal of improving reliability and availability. Elegbede and Adjallah [18] presented a multi objective combinatorial optimization method modeled with continuous and discrete variables to optimize the availability and the cost of repairable parallel series systems. Deb et al. [19] formulated a bi-objective optimization problem of minimizing total wire length and minimizing the failure rate in the printed circuit board design. They have implemented NSGA-II to solve the problem. Kumar et al. [20] presented a multi objective genetic algorithm to design telecommunication networks while simultaneously minimizing network performance and design cost under reliability constraint. The methods discussed so far though solved many optimization problems, they as such can not be able to solve the topological optimization problem of interconnection networks, where the main objective is to minimize the overall cost of the interconnection network while satisfying some reliability constraint. This motivates our study to propose a new method that uses genetic algorithm with simple and efficient chromosome encoding as well as better cross over and mutation techniques in solving such topological optimization problems in Section 2 of this paper. The rest of the paper is organized as follows: Section-3 contains the application of the proposed method on some fully connected Interconnection Networks and Section-4 concludes the paper with future scope.

The following Notations and assumptions are used throughout this paper.

Notations

ICN	Completely connected interconnection network.
$G(N,L)$	Probabilistic Graph G of the given Interconnection Network having N number of nodes and L number of links.
P	Population
P_s	Population size
C_p	Crossover probability
Z	Objective function
$R(X)$	Reliability of network X
x_{ij}	A decision variable.(1 if a link exist between node i and node j, 0 otherwise)
R_0	Minimum reliability constraint.
C_{ij}	Cost of the link connecting node i and j
R_{ij}	Reliability of link connecting node i and j
$f(X)$	Fitness of the network x
$Max\_Cost(P)$	Maximum network cost of a population P
$Cost(X)$	Cost of every network x
P1 and P2	Selected two networks for chromosome level cross over
C1 and C2	Generated Child networks after chromosome level cross over.
Pt1; Pt2	Two nodes chosen from network
$Cost_{max}$	Maximum permissible Cost/Budget Constraint
K	The set of nodes having degree less than 2.
$|K|$	The number of nodes in K set.
NR	Network Reliability

Assumptions

1. Nodes are perfectly reliable.

2. Links are either operational or failed.

3. Failures of links are s-independent.

1.1 Problem Statement

An interconnection network can be modeled as a graph G (N, L), where N is the set of nodes (Stations, terminals or computer sites) and L is the communication links among them. As in practically, the nodes are far away from each other. The distance between them may vary. If the nodes of the network are fixed and the associated links are predefined, the main objective is to find out best layout of links within a minimum cost while satisfying a given reliability constraint, or mathematically the objective function is expressed as:

Minimize
$$Z = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} C_{ij}\, x_{ij}$$
---------(Eq1)

Subject to
$$R(X) \geq R_0$$

And degree of each node of $G \geq 2$

1.2 Fitness Function:

A network will be included in the population if it satisfies two conditions:

1. Each node of the network must have at least degree 2.

2. The computed reliability of the network must be greater than the given value of the reliability constraint.

For the networks that satisfy the above conditions, the fitness function can be calculated as:

$$F(X) = Max\_Cost(P) - Cost(X)$$
----------(Eq2)

2. PROPOSED METHOD

2.1 Definitions

Definition 1. Spanning Tree is a maximal set of edges of G that contains no cycle, or as a minimal set of edges that connect all vertices.

Definition 2. A steady state is defined as the state of a system that does not change over time or in which any one change is continually balanced by another.

2.2 Encoding

The proposed method uses the following variable length integer encoding string representation:

For a network with N number of nodes and L number of links, the length of the chromosomes is the maximum number of links of the network, which requires much fewer number of integers to represent the chromosome than other encoding techniques [23]. Every possible link is numbered sequentially starting from 1.

(a)

(b)
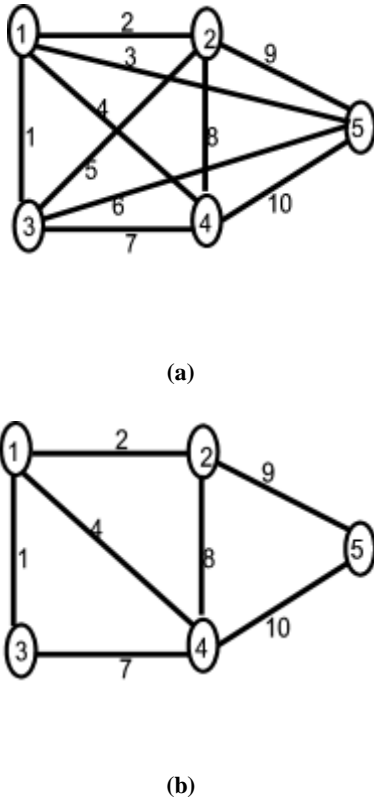
Figure 1: (a) A fully connected network with 10 links that are labeled from 1 to 10. (b) A partially connected network with 7 links using the same labeling as (a)

Example :

Chromosome Encoding of the networks in Figure 1 are:

(a) { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and

(b) {1, 2, 4, 7, 8, 9, 10}

2.3   Initial Population

The proposed method only considers the candidate networks in the initial population that must meet the reliability constraint while minimizing the overall cost and satisfying the 2-connectivity test. Thus by enhancing the efficiency of the search for the optimal solutions.

Initial Population

Input: ICN

while (Population size is not reached) do

   a) *Randomly generate two spanning trees [22] from the given fully connected network*

   b) *Make it a two connected network using repair algorithm*

   c) *Apply crossover to generate children*

   d) *Estimate fitness of both parents and children*

   e) *Networks that satisfy the fitness criteria will included in initial population*

End while

2.4   Chromosome level Cross Over

Crossover is the genetic operator that mixes two chromosomes together to form new offspring. The intuition behind crossover is exploration of new solution by exploitation of existing solutions. Chromosomes with high fitness got opportunity to be selected to take part in the crossover. The steps involved in crossover operator are present in the following algorithm:

Cross Over

Input: *Networks P1 and P2*

Step1 :   *Randomly select a common link (i ,j) from P1 and P2 and select two points Pt1 = i and Pt2 = j for cross over.*

Step2 :   *The links adjacent to Pt1 and Pt2 in P2 are assigned to one child C1 and the links adjacent to Pt1 and Pt2 in P1 are assigned to child C2*
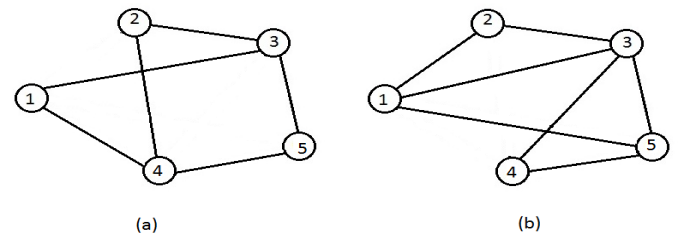
Step3 :   *Determine added links*

            P'1 = P1- C2 and P'2 = P2- C1

Step4 :   *Assign C1 = C1 U P'1 and C2 = C2 U P'2*

Example :

Let two networks P1 and P2 are selected after using some selection procedure for crossover. The two networks are shown in Figure 2.

(a)   (b)

Figure 2: Two selected parent (P1 and P2) networks for crossover

Step:1   Let it chooses a common link (2,3) so the two cross over points are Pt1= 2 and Pt2=3.

Step:2   The links adjacent to Pt1 and Pt2 in P1 are

         P1(Pt1) = {(2,1), (2,3)} and P1(Pt2) = {(2,3), (1,3), (3,4), (3,5)}.

         Similarly the links adjacent to Pt1 and Pt2 in P2 are P2(Pt1) ={(2,4), (2,3)} and P2(Pt2) = {(2,3), (1,3), (3,5)}.

Step:3   The Child C1 will be assigned with the adjacent links to the points of P2.
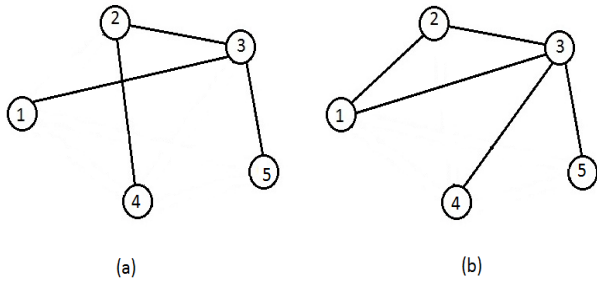
         The links adjacent to the two points Pt1 and Pt2 in network P2 will be assigned to C1.

         C1 = {(2,3), (2,4), (1,3), (3,5)}

         Similarly the links adjacent to the two points Pt1 and Pt2 in P1 will be assigned to C2.

         C2 = {(2,1), (2,3), (1,3), (3,4), (3,5)}

The children after exchanging the links adjacent to the chosen point between P1 and P2 are shown in Figure 3.

(a)   (b)

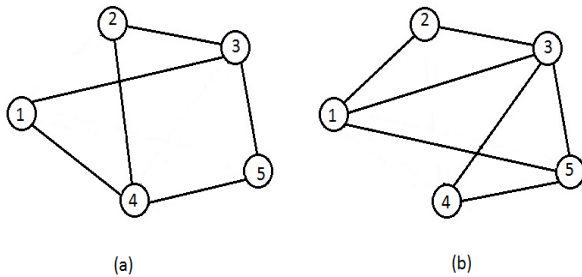**Fig 3. The intermediate generated after applying the second step of Cross Over Algorithm on P1 and P2.**

Step:4   The links that are not adjacent to the Pt1 and Pt2 in P1 will be added to C1.

Added links to C1are {(1,4), (4,5)}

The links that are not adjacent to the Pt1 and Pt2 in P2 will be added to C2.

Added links to C2 are {(1,5), (4,5)}

The new generated off springs are shown in Figure 4.

(a)   (b)

Fig 4. Two children networks C1 and C2 after applying the step 3 and step 4 of Cross Over Algorithm on P1 and P2.

2.5 Repair Algorithm

A repair algorithm is used here as mutation operator to add some additional links to the generated networks so as to satisfy the minimum 2-connectivity criteria. The updating is applied in the following steps:

Repair Algorithm

Input: G(N,L)

1.   If $|K|$=1 then join that node with a node having least cost.

2.   If $|K|$=2 say N1 and N2 then

    2.1   Compute C1=The minimum cost between N1 and any   other node Nk of the network.

        If Nk=N2 then join N1 and N2.

        else

2.2   Compute C2=The minimum cost between N2 and any other node NL of the network.

    if NL=N1 the join N1 with N2

    else

2.3   Compute C=Cost between N1 and N2

if C< (C1+C2) then join N1 and N2 otherwise join N1 with Nk and N2 with NL.

3.   If $|K| \geq 2$ then

Assign a link from ICN to the existing network by joining any two nodes of K-set with minimum cost.

Example:

The children networks C1 and C2 satisfy the 2-connectivity criteria. But the cases may arise where the child networks may not be a 2-connected as in Figure 5.
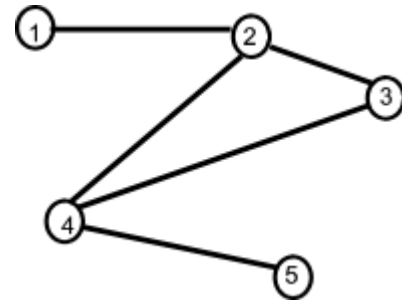
Figure 5: A network not satisfying the 2-connectivity criteria

The cost matrix of the above network is given below.

$$\begin{vmatrix} - & 25 & 52 & 36 & 47 \\ & - & 65 & 23 & 40 \\ & & - & 43 & 29 \\ & & & - & 71 \end{vmatrix}$$

The network C2 does not satisfy the 2-connectivity criteria. So the Repair Algorithm is applied on C2.

In network C2, node 1 and 5 has degree less than 2, so K= {1, 5} satisfying step 2 of Repair Algorithm.

Here N1=1 and N2=5.

Step 2.1   C1=25, Nk is here N2   and Nk ≠ N5,

Step 2.2   C5=29, Nk is here N3   and Nk ≠ N1,

Step 2.3   C=47, and C1+C2=54

As C< C1+C2, Node N1 and N5 will be joined.

Now the repaired network satisfying the minimum 2-connectivity criteria is given in Fig. 6.
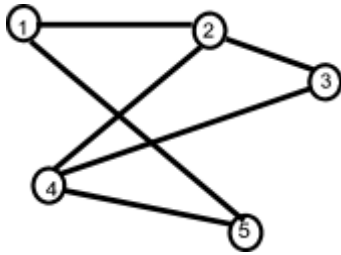
Figure 6: The example given in Figure 5 after applying Repair Algorithm

2.6 Selection

In selection only the fittest species can survive, breed and pass their genes to the next generation. The proposed method uses the Roulette Wheel selection algorithm for selecting individuals with a probability proportional to the fitness. The network reliability is computed using the method [21]. So the initial population taken for the example network (Figure 1) is given in Table I:

Table-I.   Candidates of Initial Population

Network	NR	Cost
P1	0.957906	280
P2	0.918540	200
C1	0.957906	243
C2	0.905418	260

Applying the fitness function:

$$F(X) = Max\_Cost(P) - Cost(X), \text{------(Eq 3)}$$

The Table-II shows the selection of candidate networks for the next generation by applying its fitness function (Eq 3).

Table-II. Selection of Candidate networks based on fitness function

Network	NR	Cost	$F(X)$
P1	0.957906	280	0
P2	0.918540	200	80
C1	0.957906	243	37
C2	0.905418	260	20

The candidate chromosomes selected for the next generation can be computed on its probability value of the fitness.

$$\Pr_i = \frac{F_i}{\sum_{j=1}^{N} F_j}$$

Here the networks P2 and C1 are found to be fittest for the next generation.

2.7 Proposed algorithm

The proposed algorithm to solve optimization problem as formulated in Section 1 is presented below:

Optimize Network

Input: (ICN, G , Cost_mat, Rel_mat, $Cost_{max}$)

1. Generate initial population

2. While (steady state is not reached) do

    2.1   Apply selection process with crossover probability to select chromosomes into mating pool.

    2.2   Apply crossover to get new solution network

    2.3   Apply repair algorithm to ensure two connected network

    2.4   Find fitness using fitness function $f(X)$

    2.5   Select the fittest chromosomes for next generation.

3. End while

4. Declare the chromosome having minimum cost.

3. RESULT AND DISCUSSION

For the purpose of simulation, the following parameters are set:

Population size (Ps)=10, Crossover probability (Cp) =0.8, the mutation rates from 0.002 to 0.044 and the reliability of each link = 0.9. The costs of the links are set as per [16].

The considered mutation rate is in average smaller than its maximum value, which ensures the preservation of the original network.

The cost of few fully connected interconnection networks having up to 15 numbers of nodes are computed under different reliability constraints (Table III).

Further, the optimized networks satisfying the different reliability constraints thus generated for different input networks are presented in the figure 7.

Table-III. Application of proposed algorithm on some sample Networks

Sl. No	N	R_0	Computed Optimum Cost Of the Network	NR of the Optimum Network	CPU Time in sec [16]	CPU Time in sec using proposed method	Optimum Network obtained using proposed method
1	5	0.9	167	0.9774075	*	11.3599	Fig. 7(a)
2	6	0.9	188	0.9672261	52.39	19.2113	Fig. 7(b)
3	7	0.9	260	0.9556194	135.98	23.9944	Fig. 7(c)
4	8	0.9	275	0.9637000	260.86	59.1566	Fig. 7(d)
5	9	0.85	295	0.9287880	378.91	102.838	Fig. 7(e)
6	11	0.85	429	0.8771490	1532.3	367.799	Fig. 7(f)
7	12	0.85	459	0.9115021	*	1098.12	Fig. 7(g)
8	15	0.9	644	0.986545	*	1205.87	Fig. 7(h)

* Not computed in [16].
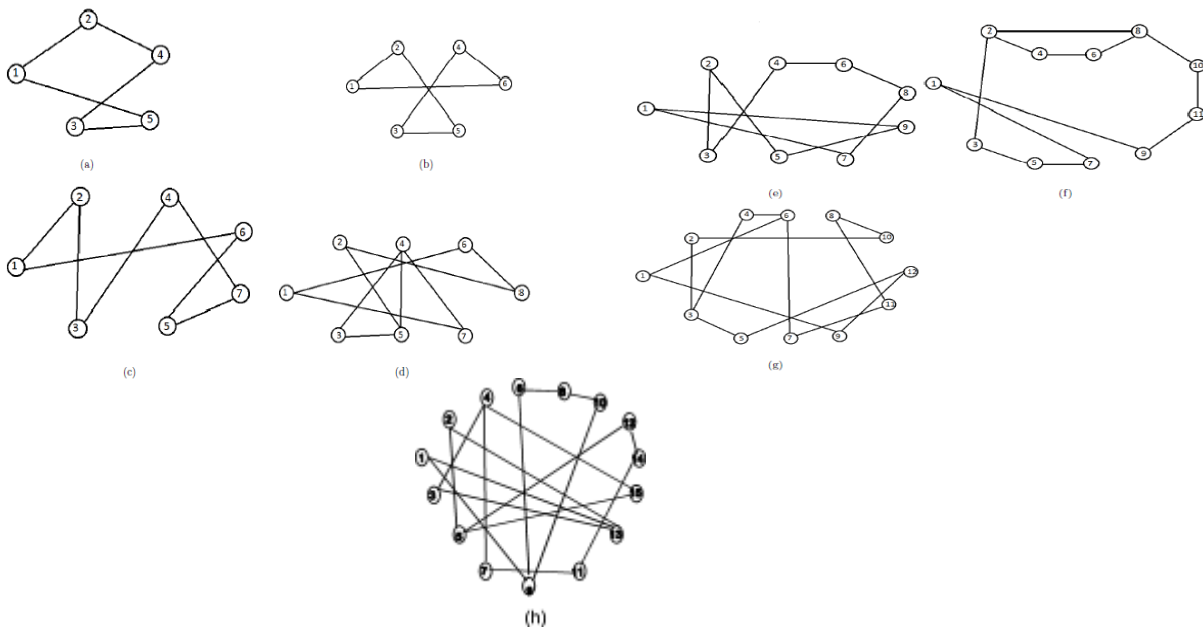
Figure 7: The optimal networks obtained after applying the proposed algorithm.

The proposed method is compared against method [16] in terms of Cpu time taken to get optimal solutions for different input networks of same interest. From this comparison it is quite clear that the proposed method requires much less Cpu time in minimizing the overall cost of the network under different reliability constraints.

From, Table III and Figure. 7, the following observations can be made:

For the optimized network having 7 numbers of nodes, the cost of the network is computed to be 260 within 23.99440 CPU seconds while the value of the reliability is 0.9556194. The optimized network with 11 numbers of nodes is generated

within 367 Cpu secs whose reliability and the minimal cost is computed to be 0.8771490 and 429 respectively. However, for a comparatively larger sized network with 15 numbers of nodes and 105 numbers of links, the optimized network is

generated in a cpu time of 1205 secs with a cost of 644 and reliability of 0.986545.

From the different observations as discussed above, it can be ensured that the proposed method generates an optimized network with a minimal computed cost while meeting the reliability constraint.

4. CONCLUSION

This paper presents a genetic algorithm based method to address a topological optimization problem of interconnection network to minimize the overall cost of the interconnection network while satisfying some reliability constraint. The proposed method uses an efficient variable length chromosome encoding technique with chromosome level cross over to ensure a better convergence towards optimal solution i.e. generation of a network having the least cost. Further, a Repair algorithm is proposed for the situation when the generated networks are not 2-connected. The simulated results reveals the proposed method to be very efficient and less time consuming in optimizing the cost of different interconnection networks with in different reliability constraints. The works carried out in this paper can be further extended to solve similar topological optimization problems with additional complexity like network delay, throughput etc. as objective and/or constraints.

5. ACKNOWLEDGMENTS

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