

# Graphical and Database Analysis of New Sequence of Functions Involving the Bessel Function with MatLab Implementation

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## ABSTRACT

The aim of the present paper is an attempt to introduce a new sequence of functions  $\{V_n^{(\nu, \alpha)}(x; a, k, s) / n = 0, 1, 2, \dots\}$ , which involving the Bessel function of first kind  $J_\nu(x)$ . By using operational technique, some interesting generating relations and summation formulas are obtained in sections 2 and 3. The remarkable thing of this paper is the crucial MATLAB coding of the new sequence of functions, Database and Graph established using the MATLAB (R2012a) in the section (5) and (6) for different values of parameters and  $n=1, 2, 3$  and 4. The reader can establish Database and Graph using the same program for any value of  $n$ .

## General Terms

Special function, Sequence of Function, Matlab.

## Keywords

Special Function, Generating function, Bessel function of first kind, Sequence of function, Matlab.

## 1. INTRODUCTION

In the present Era, MATLAB is a high-performance language for technical computing. It integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation. Typical uses include:

- Math and computation
- Algorithm development
- Modeling, simulation, and prototyping
- Data analysis, exploration, and visualization
- Scientific and engineering graphics
- Application development, including Graphical User Interface building

In pure mathematics, since Matlab is an integrated computer software which has three functions: symbolic computing, numerical computing and graphics drawing. Matlab is capable to carry out many functions including computing polynomials and rational polynomials, solving equations and computing many kind of mathematical expressions. One can also use Matlab to calculate the limit, derivative, integral and Taylor

series of some mathematical expressions. With Matlab, The graphs of functions with one or two variables can be easily drawn in selected domain. Therefore, functions can be studied by visualization for their main Characteristics. Matlab is also a system which can be easily expanded. Matlab provides many powerful software packages which can be easily incorporated into the clients system. Recently, there are many papers in the literature which are devoted to the application of Matlab in mathematical analysis, see the work of Stephen [22], Dunn [6], Shampine and Robert [16].

The distinct scientific communities that are working on various aspects of automatic analysis of data include Combinatorial Pattern Matching, Data Mining, Computational Statistics, Network Analysis, Text Mining, Image Processing, Syntactical Pattern Recognition, Machine Learning, Statistical Pattern Recognition, Computer Vision, and many others.

Bessel functions are a series of solutions to a second order differential equation that arise in many diverse situations. While special types of what would later be known as Bessel functions were studied by Euler, Lagrange, and the Bernoullis, the Bessel functions were first used by F. W. Bessel to describe three body motion, with the Bessel functions appearing in the series expansion on planetary perturbation [8]. The subject of Bessel Functions and applications is a very rich subject; nevertheless, due to space and time restrictions and in the interest of studying applications. Appropriate development of zeroes, modified Bessel functions, and the application of boundary conditions can be studied in the work of Jennifer Niedziela [9].

In this paper, an attempt is made to introduce a new sequence of functions  $\{V_n^{(\nu, \alpha)}(x; a, k, s) / n = 0, 1, 2, \dots\}$ , which involving the Bessel function of first kind  $J_\nu(x)$ . By using operational techniques, Some interesting generating relations and summation formulas are obtained. Program is made using the Matlab to establish the Database and Graph of the new sequence of function for different values of parameters.

The Bessel function of first kind is defined as [15, p.109]:

$$J_\nu(x) = \sum_{k=0}^{\infty} \frac{(-1)^k (x)^{\nu+2k}}{2^{\nu+2k} k! \Gamma(\nu+k+1)} \quad (1.1)$$

Operational techniques have drawn the attention of several researchers in the study of sequences of functions and polynomials.

In 1956, Chak [1] defined a class of polynomials as:

$$G_{n,k}^{(\alpha)}(x) = x^{-\alpha-kn+n} e^x (x^k D)^n [x^\alpha e^{-x}] \quad (1.2)$$

Where  $k$  is constant and  $n = 0, 1, 2, \dots, D \equiv \frac{d}{dx}$ .

Gould and Hopper [7] introduced generalized Hermite polynomials in 1962 as:

$$H_n^r(x, a, p) = (-1)^n x^{-a} \exp(px^r) D^n [x^a \exp(-px^r)] \quad (1.3)$$

Chatterjea [4] studied as a class of polynomials for generalized Laguerre polynomials in 1964:

$$T_m^{(\alpha)}(x, p) = \frac{1}{m!} x^{-\alpha-n-1} \exp(px^r) (x^2 D)^n [x^{\alpha+1} \exp(-px^r)] \quad (1.4)$$

In 1968, Singh [17] studied the generalized Triesdell polynomials defined as:

$$T_n^{(\alpha)}(x, r, p) = x^{-\alpha} \exp(px^r) (xD)^n [x^\alpha \exp(-px^r)] \quad (1.5)$$

Srivastava and Singh [19] introduced a general class of polynomials in 1971 as:

$$G_n^{(\alpha)}(x, r, p, k) = \frac{1}{n!} x^{-\alpha-kn} \exp(px^r) (x^{k+1} D)^n [x^\alpha \exp(-px^r)] \quad (1.6)$$

In 1971, the Rodrigues formula for generalized Laguerre polynomials is given by Mittal [11] as:

$$T_{kn}^{(\alpha)}(x) = \frac{1}{n!} x^{-\alpha} \exp(p_k(x)) D^n [x^{\alpha+n} \exp(-p_k(x))] \quad (1.7)$$

Where  $p_k(x)$  is a polynomial in  $x$  of degree  $k$ .

Mittal [12] also proved following relation for (1.7) as:

$$T_{kn}^{(\alpha+s-1)}(x) = \frac{1}{n!} x^{-\alpha-n} \exp(p_k(x)) T_s^n [x^\alpha \exp(-p_k(x))] \quad (1.8)$$

Where  $s$  is constant and  $T_s \equiv x(s + xD)$ .

Recently, Shukla and Prajapati [21] obtained several properties of (7).

Chandel [2] also studied a class of polynomials defined in 1973 as:

$$T_n^{\alpha,k}(x, r, p) = x^{-\alpha} \exp(px^r) (x^k D)^n [x^\alpha \exp(-px^r)] \quad (1.9)$$

In 1974, Chandel [3] established a generalization of polynomial system as:

$$G_n(h, g(x), k) = e^{-hg(x)} (x^k D)^n [e^{-hg(x)}] \quad (1.10)$$

Where  $g(x)$  is a function of  $x$ ,  $h$  and  $k$  are constants.

In the same year, Srivastava [20] discussed some operational formulas generalized function  $F_n^{(r,m)}(x, a, k, p)$  in following the form:

$$F_n^{(r,m)}(x, a, k, p) = x^{-a} \exp(px^r) (x^k D)^n [x^{a+km} \exp(-px^r)] \quad (1.11)$$

In 1975, Joshi and Parjapat [10] introduced the a class of polynomial:

$$M_{v_n}^{(\alpha)}(x, a, k) = \frac{x^{-\alpha-nk}}{n!} \exp(p_v(x)) [x^k (a + xD)]^n [x^\alpha \exp(-p_v(x))] \quad (1.12)$$

Subsequently in 1975, Patil and Thakare [14] have obtained several formulae and generating relation for:

$$P_n^{(\alpha)}(x, r, s, a, k, \lambda) = x^{-\alpha} \exp(px^r) [x^k (\lambda + xD)]^n [x^{\alpha+sn} \exp(-px^r)] \quad (1.13)$$

In 1979, Srivastava and Singh [18] studied a sequence of functions  $V_n^{(\alpha)}(x; a, k, s)$  defined as:

$$V_n^{(\alpha)}(x; a, k, s) = \frac{x^{-\alpha}}{n!} \exp\{p_k(x)\} \theta^n [x^\alpha \exp\{-p_k(x)\}] \quad (1.14)$$

By employing the operator  $\theta \equiv x^\alpha (s + xD)$ , where  $s$  is constant and  $p_k(x)$  is a polynomial in  $x$  of degree  $k$ .

A new sequence of function  $\{V_n^{(v,\alpha)}(x; a, k, s) / n = 0, 1, 2, \dots\}$  is introduced in this paper as:

$$V_n^{(v,\alpha)}(x; a, k, s) = \frac{1}{n!} x^{-\alpha} J_v(p_k(x)) (T_x^{a,s})^n [x^\alpha J_v(-p_k(x))] \quad (1.15)$$

Where  $T_x^{a,s} \equiv x^a (s + xD)$ ,  $D \equiv \frac{d}{dx}$ ,  $a$  and  $s$  are constants,  $\beta \geq 0$ ,  $k$  is finite and non-negative integer,  $p_k(x)$  is a polynomial in  $x$  of degree  $k$  and  $J_\nu(x)$  is a Bessel function of first kind defined in equation (1.1).

Some generating relations and finite summation formulae of class of polynomials or sequence of function have been obtained by using the properties of the differential operators.

$T_x^{a,s} \equiv x^a (s + xD)$ ,  $T_x^{a,1} \equiv x^a (1 + xD)$ , where  $D \equiv \frac{d}{dx}$ , is based

on the work of Mittal [13], Patil and Thakare [14], Srivastava and Singh[18].

Some useful Operational Techniques are given below:

$$\begin{aligned} & \exp(tT_x^{a,s})(x^\beta f(x)) \\ &= x^\beta (1 - ax^a t)^{-\left(\frac{\beta+s}{a}\right)} f\left(x(1 - ax^a t)^{-1/a}\right) \end{aligned} \quad (1.16)$$

$$\begin{aligned} & \exp(tT_x^{a,s})(x^{\alpha-am} f(x)) \\ &= x^\alpha (1 + at)^{-1+\left(\frac{\alpha+s}{a}\right)} f\left(x(1 + at)^{1/a}\right) \end{aligned} \quad (1.17)$$

$$\begin{aligned} & (T_x^{a,s})^n(xuv) \\ &= x \sum_{m=0}^{\infty} \binom{n}{m} (T_x^{a,s})^{n-m} (v) (T_x^{a,1})^m (u) \end{aligned} \quad (1.18)$$

$$\begin{aligned} & (1 + xD)(1 + a + xD)(1 + 2a + xD) \\ & (1 + 3a + xD) \dots (1 + (m-1)a + xD) x^{\beta-1} \\ &= a^m \left(\frac{\beta}{a}\right)_m x^{\beta-1} \end{aligned} \quad (1.19)$$

$$(1 - at)^{-\frac{\alpha}{a}} = (1 - at)^{-\frac{\beta}{a}} \sum_{m=0}^{\infty} \binom{\alpha - \beta}{a}_m \frac{(at)^m}{m!} \quad (1.20)$$

## 2. GENERATING RELATIONS

$$\begin{aligned} & \sum_{n=0}^{\infty} V_n^{(v,\alpha)}(x; a, k, s) x^{-an} t^n \\ &= (1 - at)^{-\left(\frac{\alpha+s}{a}\right)} J_\nu(p_k(x)) \\ & J_\nu\left(-p_k\left(x(1 - at)^{-1/a}\right)\right) \end{aligned} \quad (2.1)$$

$$\begin{aligned} & \sum_{n=0}^{\infty} V_n^{(v,\alpha-am)}(x; a, k, s) x^{-an} t^n \\ &= (1 + at)^{-1+\left(\frac{\alpha+s}{a}\right)} J_\nu(p_k(x)) \\ & J_\nu\left(-p_k\left(x(1 + at)^{1/a}\right)\right) \end{aligned} \quad (2.2)$$

$$\begin{aligned} & \sum_{n=0}^{\infty} \binom{m+n}{m} V_n^{(v,\alpha)}(x; a, k, s) t^n \\ &= (1 - at)^{-\left(\frac{\alpha+s}{a}\right)} \frac{J_\nu(p_k(x))}{J_\nu\left(p_k\left(x(1 - at)^{-1/a}\right)\right)} \\ & V_n^{(v,\alpha)}\left(x(1 - at)^{-1/a}; a, k, s\right) \end{aligned} \quad (2.3)$$

**Proof of (2.1):**

From (1.15), we consider

$$\begin{aligned} & \sum_{n=0}^{\infty} V_n^{(v,\alpha)}(x; a, k, s) t^n = \\ & x^{-\alpha} J_\nu(p_k(x)) \exp(tT_x^{a,s}) \left[ x^\alpha J_\nu(-p_k(x)) \right] \end{aligned} \quad (2.4)$$

Using operational technique (1.16), above equation (2.4) reduces to:

$$\begin{aligned} & \sum_{n=0}^{\infty} V_n^{(v,\alpha)}(x; a, k, s) t^n = \\ & x^{-\alpha} J_\nu(p_k(x)) x^\alpha (1 - ax^a t)^{-\left(\frac{\alpha+s}{a}\right)} J_\nu\left(-p_k\left(x(1 - ax^a t)^{-1/a}\right)\right) \\ &= (1 - ax^a t)^{-\left(\frac{\alpha+s}{a}\right)} J_\nu(p_k(x)) J_\nu\left(-p_k\left(x(1 - ax^a t)^{-1/a}\right)\right) \end{aligned} \quad (2.5)$$

And replacing  $t$  by  $tx^{-a}$ , this gives (2.1).

**Proof of (2.3):**

Again from (1.15), we have

$$\begin{aligned} & \sum_{n=0}^{\infty} x^{-an} V_n^{(v,\alpha-am)}(x; a, k, s) t^n \\ &= x^{-\alpha} J_\nu(p_k(x)) \exp(tT_x^{a,s}) \left[ x^{\alpha-am} J_\nu(-p_k(x)) \right] \end{aligned} \quad (2.6)$$

applying the operational technique (1.17), we get

$$\begin{aligned} & \sum_{n=0}^{\infty} x^{-an} V_n^{(v,\alpha-am)}(x; a, k, s) t^n \\ &= x^{-\alpha} J_\nu(p_k(x)) x^\alpha (1 + at)^{\frac{\alpha+s}{a}-1} J_\nu\left[-p_k\left(x(1 + at)^{1/a}\right)\right] \\ &= (1 + at)^{\frac{\alpha+s}{a}-1} J_\nu(p_k(x)) J_\nu\left[-p_k\left(x(1 + at)^{1/a}\right)\right] \end{aligned} \quad (2.7)$$

This proves (2.2).

**Proof of (2.3):**

We can write (1.15) as,

$$\begin{aligned} & (T_x^{a,s})^n \left[ x^\alpha J_\nu(-p_k(x)) \right] \\ &= n! x^\alpha \frac{1}{J(p_k(x))} V_n^{(v,\alpha)}(x; a, k, s) \end{aligned} \quad (2.8)$$

Or

$$\begin{aligned} & \exp\left(t(T_x^{a,s})\right)\left\{\left(T_x^{a,s}\right)^n\left[x^\alpha J_\nu(-p_k(x))\right]\right\} \\ &= n! \exp\left(t T_x^{a,\alpha}\right)\left[x^\alpha \frac{1}{J(p_k(x))} V_n^{(v,\alpha)}(x; a, k, s)\right] \\ & \sum_{m=0}^{\infty} \frac{t^m}{m!} \left(T_x^{a,s}\right)^{m+n} \left[x^\alpha J_\nu(-p_k(x))\right] \\ &= n! \exp\left(T_x^{a,s}\right)\left[x^\alpha \frac{1}{J(p_k(x))} V_n^{(v,\alpha)}(x; a, k, s)\right] \quad (2.9) \end{aligned}$$

Using the operational technique (1.16), above equation can be written as

$$\begin{aligned} & \sum_{m=0}^{\infty} \frac{t^m}{m!} \left(T_x^{a,s}\right)^{m+n} \left[x^\alpha J_\nu(-p_k(x))\right] \\ &= n! x^\alpha (1-ax^a t)^{-\left(\frac{\alpha+s}{a}\right)} \frac{1}{J_\nu\left(p_k\left(x(1-ax^a t)^{-1/a}\right)\right)} \\ & V_n^{(v,\alpha)}\left(x(1-ax^a t)^{-1/a}; a, k, s\right) \quad (2.10) \end{aligned}$$

use of (2.8) gives

$$\begin{aligned} & \sum_{m=0}^{\infty} \frac{t^m (m+n)!}{m! n!} x^\alpha \frac{1}{J_\nu(p_k(x))} V_{m+n}^{(v,\alpha)}(x; a, k, s) \\ &= x^\alpha (1-ax^a t)^{-\left(\frac{\alpha+s}{a}\right)} \frac{V_n^{(v,\alpha)}\left(x(1-ax^a t)^{-1/a}; a, k, s\right)}{J_\nu\left[p_k\left(x(1-ax^a t)^{-1/a}\right)\right]} \quad (2.11) \end{aligned}$$

Therefore

$$\begin{aligned} & \sum_{m=0}^{\infty} \binom{m+n}{n} V_{m+n}^{(v,\alpha)}(x; a, k, s) t^m \\ &= (1-ax^a t)^{-\left(\frac{\alpha+s}{a}\right)} \frac{J_\nu(p_k(x))}{J_\nu\left(p_k\left(x(1-ax^a t)^{-1/a}\right)\right)} \\ & V_n^{(v,\alpha)}\left(x(1-ax^a t)^{-1/a}; a, k, s\right) \quad (2.12) \end{aligned}$$

And replacing  $t$  by  $tx^{-a}$ , this gives the result (2.3).

### 3. FINITE SUMMATION FORMULAE

$$\begin{aligned} & V_n^{(v,\alpha)}(x; a, k, s) \\ &= \sum_{m=0}^{\infty} \frac{1}{m!} (ax^a)^m \left(\frac{\alpha}{a}\right)_m V_{n-m}^{(v,0)}(x; a, k, s) \quad (3.1) \end{aligned}$$

$$\begin{aligned} & V_n^{(v,\alpha)}(x; a, k, s) \\ &= \sum_{m=0}^{\infty} \frac{1}{m!} (ax^a)^m \left(\frac{\alpha-\beta}{a}\right)_m V_{n-m}^{(v,\beta)}(x; a, k, s) \quad (3.2) \end{aligned}$$

**Proof of the (3.1) equation:**

From equation (1.15), We have:

$$\begin{aligned} & V_n^{(v,\alpha)}(x; a, k, s) \\ &= \frac{1}{n!} x^{-\alpha} J_\nu(p_k(x)) \left(T_x^{(a,s)}\right)^n \left[xx^{\alpha-1} J_\nu(-p_k(x))\right] \quad (3.3) \end{aligned}$$

Using the operational technique (1.18), we have:

$$\begin{aligned} & V_n^{(v,\alpha)}(x; a, k, s) \\ &= \frac{1}{n!} x^{-\alpha} J_\nu(p_k(x)) x \sum_{m=0}^n \binom{n}{m} \left(T_x^{(a,s)}\right)^{n-m} \left[J_\nu(-p_k(x))\right] \left(T_x^{a,1}\right)^m \left(x^{\alpha-1}\right) \\ &= \frac{1}{n!} x^{-\alpha} J_\nu(p_k(x)) x \sum_{m=0}^n \frac{n!}{m!(n-m)!} x^{\alpha(n-m)} \times \\ & \left[(s+xD)(s+a+xD)(s+2a+xD)\dots\right. \\ & \left.(s+(n-m-1)a+xD)\right] \left[J_\nu(-p_k(x))\right] x^{am} \times \\ & \left[(1+xD)(1+a+xD)(1+2a+xD)\dots\right. \\ & \left.(1+(m-1)a+xD)\right] \left(x^{\alpha-1}\right) \\ &= J_\nu(p_k(x)) \sum_{m=0}^n \frac{1}{m!(n-m)!} x^{am} \\ & \prod_{i=0}^{n-m-1} (s+ia+xD) \left[J_\nu(-p_k(x))\right] a^m \left(\frac{\alpha}{a}\right)_m \quad (3.4) \end{aligned}$$

Put  $\alpha = 0$  and replacing  $n$  by  $n-m$  in (3.3), we get:

$$\begin{aligned} & V_{n-m}^{(v,0)}(x; a, k, s) \\ &= \frac{1}{(n-m)!} J_\nu(p_k(x)) \left(T_x^{a,s}\right)^{n-m} \left[J_\nu(-p_k(x))\right] \\ & \Rightarrow \frac{1}{(n-m)!} \left(T_x^{a,s}\right)^{n-m} \left[J_\nu(-p_k(x))\right] \\ &= \frac{1}{J_\nu(p_k(x))} V_{n-m}^{(v,0)}(x; a, k, s) \quad (3.5) \end{aligned}$$

This gives

$$\begin{aligned} & \frac{1}{(n-m)!} \prod_{i=0}^{n-m-1} (s+ia+xD) \left[J_\nu(-p_k(x))\right] \\ &= x^{\alpha(n-m)} \frac{1}{J_\nu(p_k(x))} V_{n-m}^{(v,0)}(x; a, k, s) \quad (3.6) \end{aligned}$$

From equation (3.4) and (3.6) we have the main the result

**Proof the equation (3.2):**

Equation (1.15) can be written as:

$$\begin{aligned} & \sum_{n=0}^{\infty} V_n^{(v,\alpha)}(x; a, k, s) t^n \\ &= x^{-\alpha} J_\nu(p_k(x)) \exp\left(T_x^{(a,s)}\right) \left[x^\alpha J_\nu(-p_k(x))\right] \quad (3.7) \end{aligned}$$

Applying the (1.16) in equation (3.1), we have:

$$\begin{aligned} & \sum_{n=0}^{\infty} V_n^{(v,\alpha)}(x; a, k, s) t^n \\ &= x^{-\alpha} J_\nu(p_k(x)) x^\alpha (1-ax^a t)^{-\left(\frac{\alpha+s}{a}\right)} J_\nu\left(-p_k\left(x(1-ax^a t)^{-1/a}\right)\right) \\ &= (1-ax^a t)^{-\left(\frac{\alpha+s}{a}\right)} J_\nu(p_k(x)) J_\nu\left(-p_k\left(x(1-ax^a t)^{-1/a}\right)\right) \quad (3.8) \end{aligned}$$

Applying the result from equation (1.20); equation (3.8) reduces to:

$$\begin{aligned}
 &= (1-ax^at)^{-\left(\frac{\beta+s}{a}\right)} \sum_{m=0}^{\infty} \left(\frac{\alpha-\beta}{a}\right)_m \frac{(ax^at)^m}{m!} \\
 &J_\nu(p_k(x)) J_\nu\left(-p_k\left(x(1-ax^at)^{-1/a}\right)\right) \\
 &= \sum_{m=0}^{\infty} \left(\frac{\alpha-\beta}{a}\right)_m \frac{(ax^at)^m}{m!} x^{-\beta} J_\nu(p_k(x)) \\
 &\exp\left(T_x^{a,s}\right)\left[x^\beta J_\nu(-p_k(x))\right] \\
 &= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left(\frac{\alpha-\beta}{a}\right)_m \frac{(ax^at)^m t^{n+m}}{m!n!} x^{-\beta} \\
 &J_\nu(p_k(x))\left(T_x^{a,s}\right)^n\left[x^\beta J_\nu(-p_k(x))\right] \\
 &= \sum_{n=0}^{\infty} \sum_{m=0}^n \left(\frac{\alpha-\beta}{a}\right)_m \frac{(ax^at)^m t^n}{m!(n-m)!} x^{-\beta} \\
 &J_\nu(p_k(x))\left(T_x^{a,s}\right)^{n-m}\left[x^\beta J_\nu(-p_k(x))\right] \tag{3.9}
 \end{aligned}$$

Now equating the coefficient of  $t^n$ , we get

$$\begin{aligned}
 &V_n^{(v,\alpha)}(x;a,k,s) \\
 &= \sum_{m=0}^n \left(\frac{\alpha-\beta}{a}\right)_m \frac{(ax^at)^m}{m!(n-m)!} x^{-\beta} \\
 &J_\nu(p_k(x))\left(T_x^{a,s}\right)^{n-m}\left[x^\beta J_\nu(-p_k(x))\right] \tag{3.10}
 \end{aligned}$$

Using the equation (1.15) in (3.10) we have the result (3.2)

#### 4. SPECIAL CASES

The interesting special cases/relationships between (1.15) and class of polynomials (1.2)-(1.14) can also be obtained for appropriate values of  $\alpha, \nu, a, k$  and  $s$ .

The, Matlab is one of the important aspect mainly in the field of sciences and engineering, Therefore, the imperative Matlab coding established for each parameter of equation (1.15) and some interesting database and graphs also established in the section 5. Using this coding reader may obtain large number of graphs of equation (1.15), which gives the eccentric characteristics in the area of sequence of functions or class of polynomials

#### (6.1) First Database and Graph:

**Table 1.** Database of sequence of function  $V_n(1,1,1,1,x)$  for  $n=1:4$   
 $V_n(\alpha,\nu,a,k,s,x); \alpha=1;\nu=1; a=1; k=1; s=1;n=1:4$

| x    | V1(1,1,1,1,x) | V2(1,1,1,1,x) | V3(1,1,1,1,x) | V4(1,1,1,1,x) |
|------|---------------|---------------|---------------|---------------|
| -2.0 | 1.1817        | -1.2520       | -7.1588       | 51.7684       |
| -1.5 | 1.1095        | -2.3219       | 3.0743        | -0.5750       |

### 5. PROGRAMMING OF THE NEW SEQUENCE OF FUNCTIONS IN MATLAB

Code of New sequence of functions:

```

function [Vn] = gnsbj2(alpha,nu,a,k,s,x)
%Graph of Vn(alpha,nu,a,k,s,x)
%P=Pn(alpha,nu,a,k,s,x)=(1/n!)*x^alpha*besselj(nu,x^k)*Tn^alpha(a,s)
%(x^a*(s+x*D)(x^alpha)*besselj(nu,-x^k)), where n=1,2,3,...
syms x
%n=input('please enter n:');
n=2;
E1= besselj(nu,-x.^k);
y=(x.^alpha).*E1;
for i=1:n
y=(x.^a).*(s.*y+x.*diff(y));
end
E2=besselj(nu,x.^k);
v=(1./factorial(n)).*(1./(x.^alpha)).*E2.*y;
%Vn=subs(v,x,-.5:1:.5);
Vn=subs(v,x);
end
Plot The Graph:
Command window:
hold on
V1= ezplot(gnsbj1(1,1,1,1,x),[-2:.5:2]);
V2= ezplot(gnsbj2(1,1,1,1,x),[-2:.5:2]);
V3= ezplot(gnsbj3(1,1,1,1,x),[-2:.5:2]);
V4= ezplot(gnsbj4(1,1,1,1,x),[-2:.5:2]);
title('Vn(alpha,nu,a,k,s,x);n=1,2,3,4')
hold off
set(V1,'color','r')
set(V2,'color','b')
set(V3,'color','g')
set(V4,'color','m')
legend('V1(1,1,1,1,x)','V2(1,1,1,1,x)','V3(1,1,1,1,x)','V4(1,1,1,1,x)')

```

### 6. DIFFERENT DATABASES AND GRAPHS USING MATLAB

The new sequence  $\{V_n^{(v,\alpha)}(x;a,k,s)/n=0,1,2,\dots\}$  introduced in equation (1.15), takes place in the form of  $V_n(\alpha,\nu,a,k,s,x)$  to establish Database and Graph for different values of parameters  $\alpha, \nu, a, k, s$  and  $(n = 0, 1, 2, 3, \dots)$ . Four different Database for different values of parameters for  $n=1,2,3$  in the interval  $-2 \leq x \leq 2$  with difference .5 and  $-0.5 \leq x \leq 0.5$  with difference 0.1 are established, as shown in Data base in table 1, 2, 3, 4 & 5 and their corresponding Graphs are plotted.

|      |         |         |         |         |
|------|---------|---------|---------|---------|
| -1.0 | 0.5304  | -0.9386 | 1.3300  | -1.6043 |
| -0.5 | 0.0862  | -0.0839 | 0.0676  | -0.0488 |
| 0    | 0       | 0       | 0       | 0       |
| 0.5  | -0.0862 | -0.0839 | -0.0676 | -0.0488 |
| 1.0  | -0.5304 | -0.9386 | -1.3300 | -1.6043 |
| 1.5  | -1.1095 | -2.3219 | -3.0743 | -0.5750 |
| 2.0  | -1.1817 | -1.2520 | 7.1588  | 51.7684 |

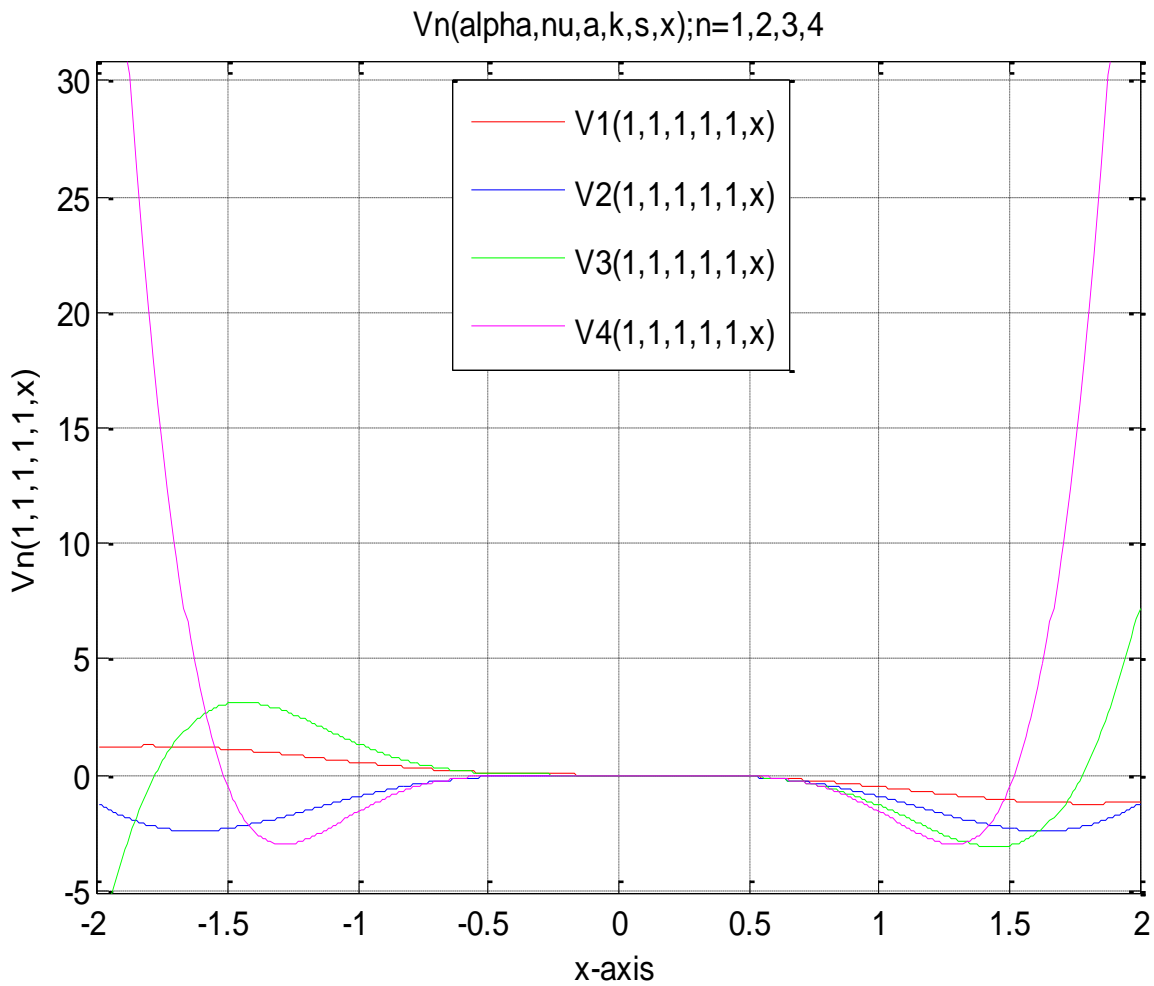


Fig. 1 Based on table (1)

**(6.2) Second Database and Graph:**

**Table 2.** Database of sequence of function  $V_n(-1,2,1,2,1)$  for  $n=1:4$

| $V_n(\alpha, \nu, a, k, s, x); \alpha=-1; \nu=2; a=1; k=2; s=1; n=1:4$ |                    |                    |                    |                    |
|--|--------------------|--------------------|--------------------|--------------------|
| x  | $V1(-1,2,1,2,1,x)$ | $V2(-1,2,1,2,1,x)$ | $V3(-1,2,1,2,1,x)$ | $V4(-1,2,1,2,1,x)$ |
| -2.0   | 1.4455             | -14.1741           | 26.3861            | 313.2720           |
| -1.5   | -0.5153            | -0.3966            | 9.0717             | -42.1274           |
| -1.0   | -0.0483            | 0.1034             | -0.1567            | 0.1564             |
| -0.5   | -0.0001            | 0.0001             | -0.0001            | 0.0001             |
| 0  | 0                  | 0                  | 0                  | 0                  |

|     |         |          |          |          |
|-----|---------|----------|----------|----------|
| 0.5 | 0.0001  | 0.0001   | 0.0001   | 0.0001   |
| 1.0 | 0.0483  | 0.1034   | 0.1567   | 0.1564   |
| 1.5 | 0.5153  | -0.3966  | -9.0717  | -42.1274 |
| 2.0 | -1.4455 | -14.1741 | -26.3861 | 313.2720 |

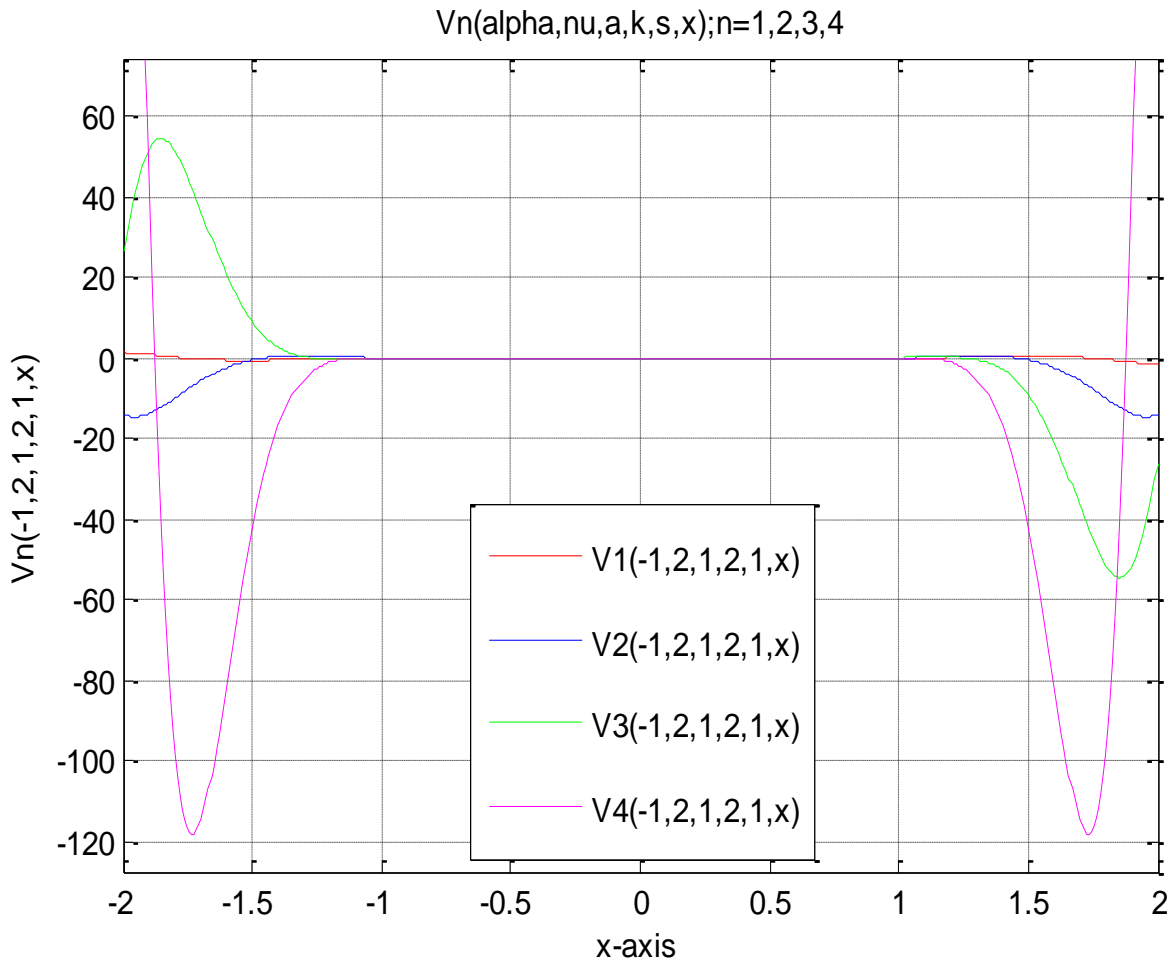


Fig. 2 Based on table (2)

**(6.3) Third Database and Graph:**

Table 3. Database of sequence of function V1 for different values of parameters

| $V_n(\alpha, \nu, a, k, s, x); \alpha=2,4,6; \nu=2; a=3; k=2; s=1; n=1$ |                    |                    |                    |                    |
|---|--------------------|--------------------|--------------------|--------------------|
| x   | $V_1(2,2,3,2,1,x)$ | $V_1(4,2,3,2,1,x)$ | $V_1(6,2,3,2,1,x)$ | $V_1(8,2,3,2,1,x)$ |
| -2.0  | 2.5998             | 0.4784             | -1.6431            | -3.7645            |
| -1.5  | -2.8178            | -3.9233            | -5.0288            | -6.1343            |
| -1.0  | -0.0879            | -0.1143            | -0.1407            | -0.1671            |
| -0.5  | -0.0001            | -0.0001            | -0.0001            | -0.0001            |
| 0   | 0                  | 0                  | 0                  | 0                  |
| 0.5   | 0.0001             | 0.0001             | 0.0001             | 0.0001             |
| 1.0   | 0.0879             | 0.1143             | 0.1407             | 0.1671             |
| 1.5   | 2.8178             | 3.9233             | 5.0288             | 6.1343             |
| 2.0   | -2.5998            | -0.4784            | 1.6431             | 3.7645             |

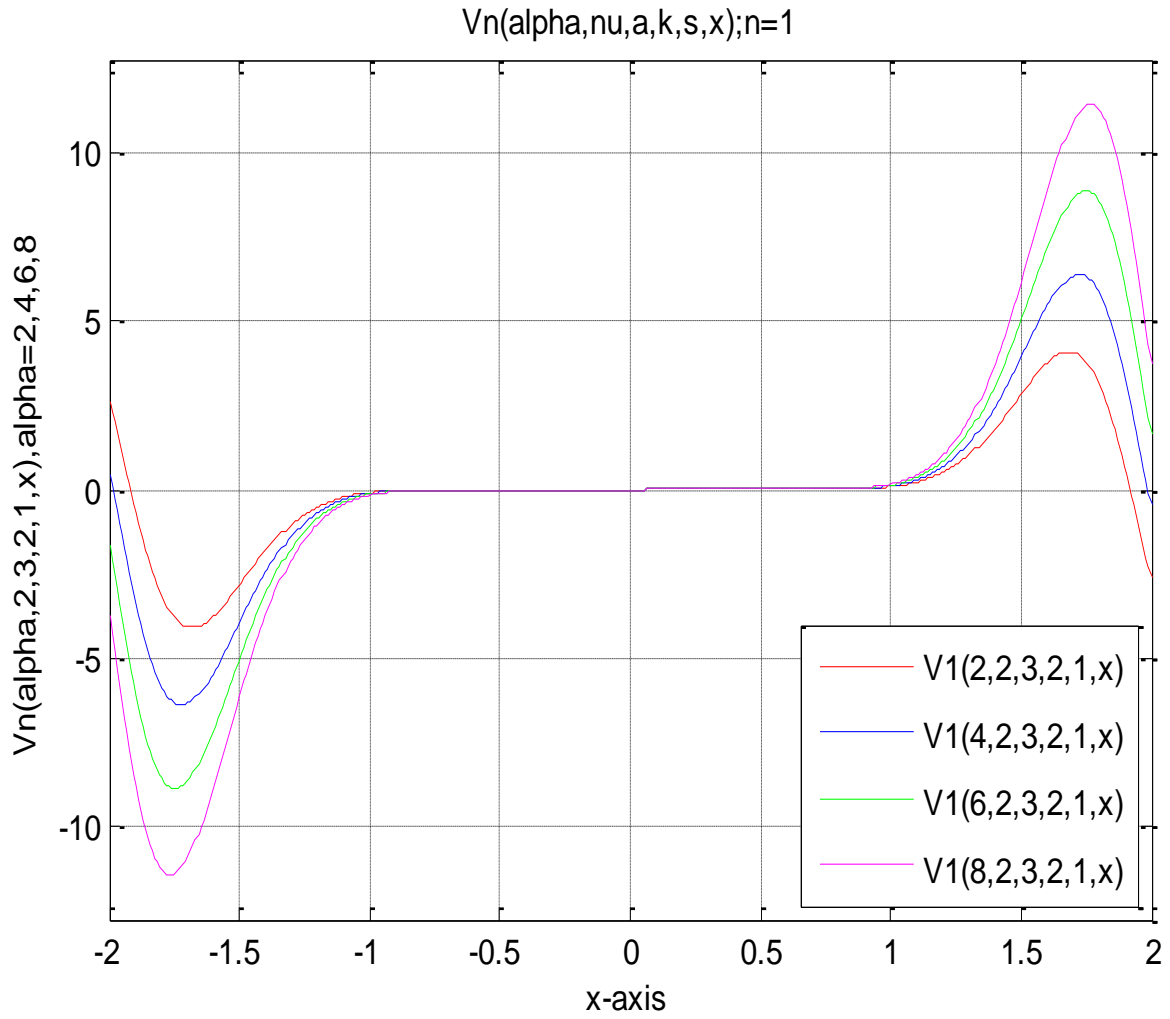


Fig. 3 Based on table (3)

**(6.4) Fourth Database and Graph:**

**Table 4.** Database of sequence of function V2 for different values of parameters  
 $V_n(\alpha, \nu, a, k, s, x); \alpha=2,4,6,8; \nu=2; a=3; k=2; s=1; n=2$

| x    | V2(2,2,3,2,1,x) | V2(4,2,3,2,1,x) | V2(6,2,3,2,1,x) | V2(8,2,3,2,1,x) |
|------|-----------------|-----------------|-----------------|-----------------|
| -2.0 | -335.4357       | -334.6040       | -299.8294       | -231.1119       |
| -1.5 | 30.4355         | 58.7832         | 94.5931         | 137.8653        |
| -1.0 | 0.4155          | 0.6573          | 0.9520          | 1.2995          |
| -0.5 | 0.0000          | 0.0001          | 0.0001          | 0.0001          |
| 0    | 0               | 0               | 0               | 0               |
| 0.5  | 0.0000          | 0.0001          | 0.0001          | 0.0001          |
| 1.0  | 0.4155          | 0.6573          | 0.9520          | 1.2995          |
| 1.5  | 30.4355         | 58.7832         | 94.5931         | 137.8653        |
| 2.0  | -335.4357       | -334.6040       | -299.8294       | -231.1119       |



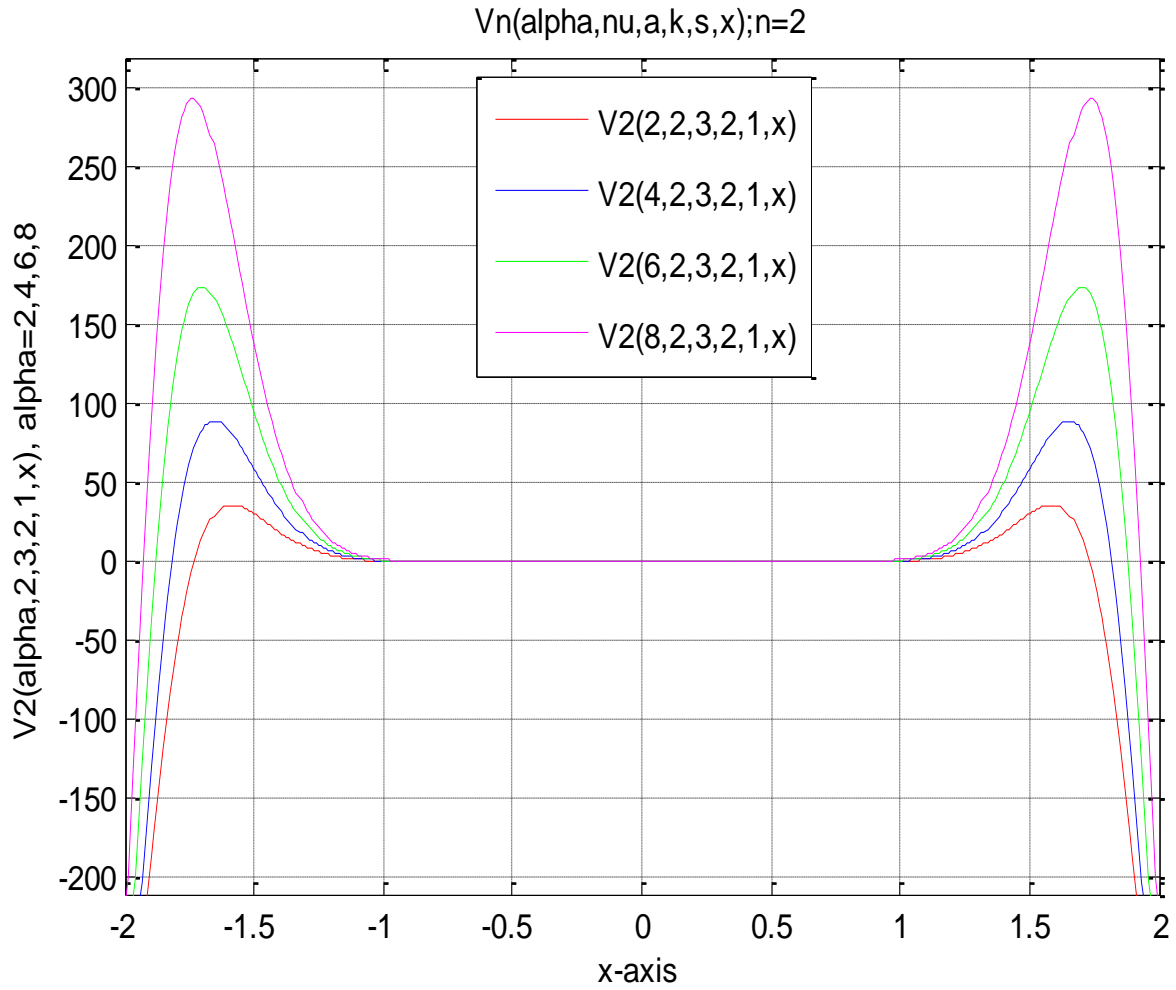


Fig. 4 Based on table (4)

**(6.5) Fifth Database and Graph:**

**Table 5.** Database of sequence of function V2 for different values of parameters  
 $V_n(\alpha, \nu, a, k, s, x); \alpha=\nu=1; a=2:5; k=3:6; s=4:7; n=2$

| x    | $V_2(1,1,2,3,4,x)$ | $V_2(1,1,3,4,5,x)$       | $V_2(1,1,4,5,6,x)$       | $V_2(1,1,5,6,7,x)$       |
|------|--------------------|--------------------------|--------------------------|--------------------------|
| -0.5 | -0.0097            | $-0.9899 \times 10^{-3}$ | $-0.9151 \times 10^{-4}$ | $-0.7926 \times 10^{-5}$ |
| -0.4 | -0.0010            | $-0.0436 \times 10^{-3}$ | $-0.0165 \times 10^{-4}$ | $-0.0058 \times 10^{-5}$ |
| -0.3 | -0.0001            | $-0.0008 \times 10^{-3}$ | $-0.0001 \times 10^{-4}$ | $-0.0000 \times 10^{-5}$ |
| -0.2 | -0.0000            | $-0.0000 \times 10^{-3}$ | $-0.0000 \times 10^{-4}$ | $-0.0000 \times 10^{-5}$ |
| -0.1 | -0.0000            | $-0.0000 \times 10^{-3}$ | $-0.0000 \times 10^{-4}$ | $-0.0000 \times 10^{-5}$ |
| 0    | 0                  | 0                        | 0                        | 0                        |
| 0.1  | -0.0000            | $-0.0000 \times 10^{-3}$ | $-0.0000 \times 10^{-4}$ | $-0.0000 \times 10^{-5}$ |
| 0.2  | -0.0000            | $-0.0000 \times 10^{-3}$ | $-0.0000 \times 10^{-4}$ | $-0.0000 \times 10^{-5}$ |
| 0.3  | -0.0001            | $-0.0008 \times 10^{-3}$ | $-0.0001 \times 10^{-4}$ | $-0.0000 \times 10^{-5}$ |
| 0.4  | -0.0010            | $-0.0436 \times 10^{-3}$ | $-0.0165 \times 10^{-4}$ | $-0.0058 \times 10^{-5}$ |
| 0.5  | -0.0097            | $-0.9899 \times 10^{-3}$ | $-0.9151 \times 10^{-4}$ | $-0.7926 \times 10^{-5}$ |

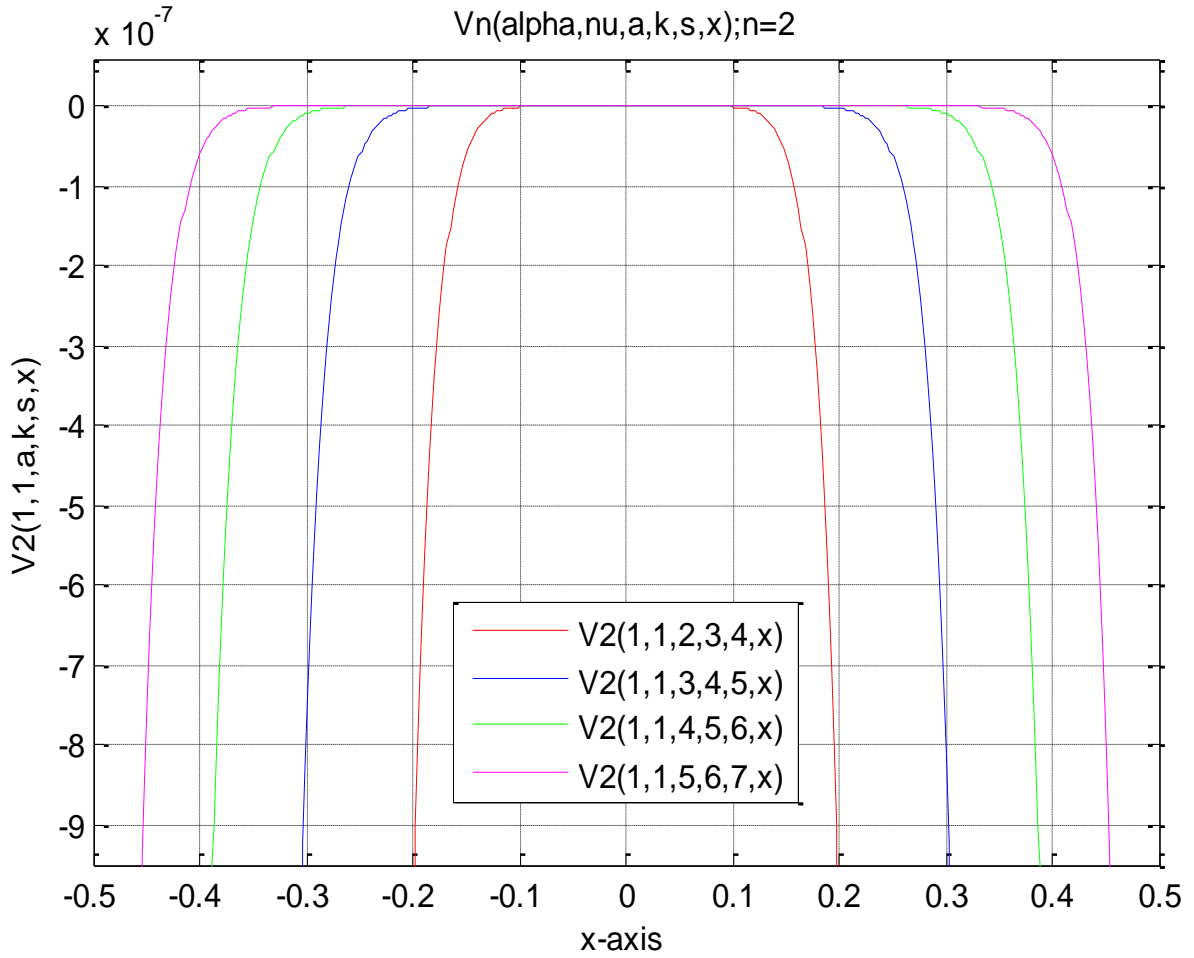


Fig. 5 Based on table (5)

**(6.6) Different Graphs Of The Sequence Of Functions For Different Range**

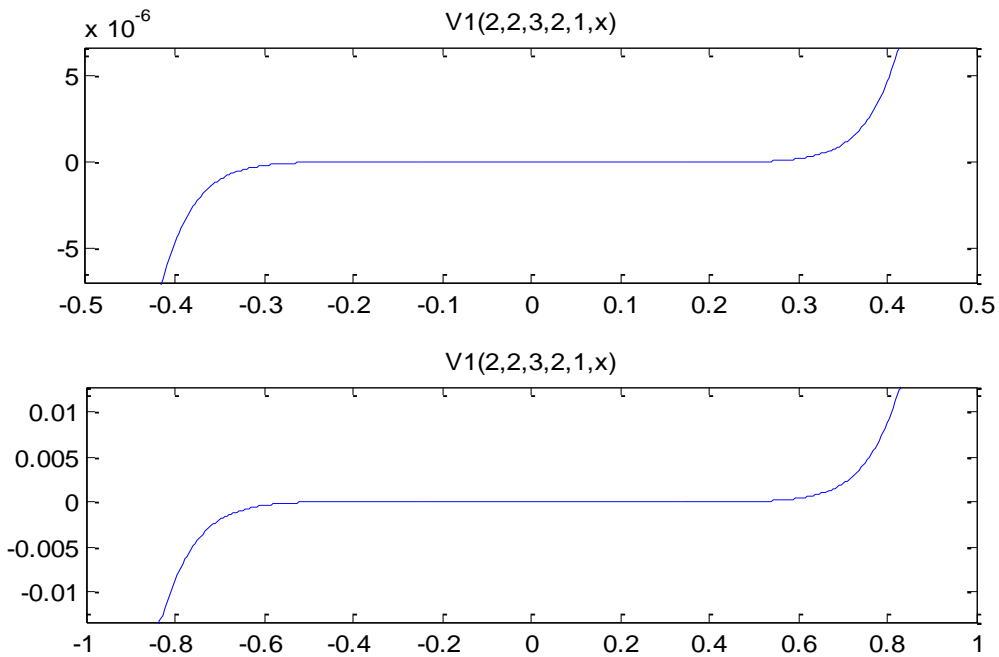


Fig. 6 Graphs of the sequence for the intervals  $[-0.5, 0.5]$  and  $[-1, 1]$  for  $n=1$

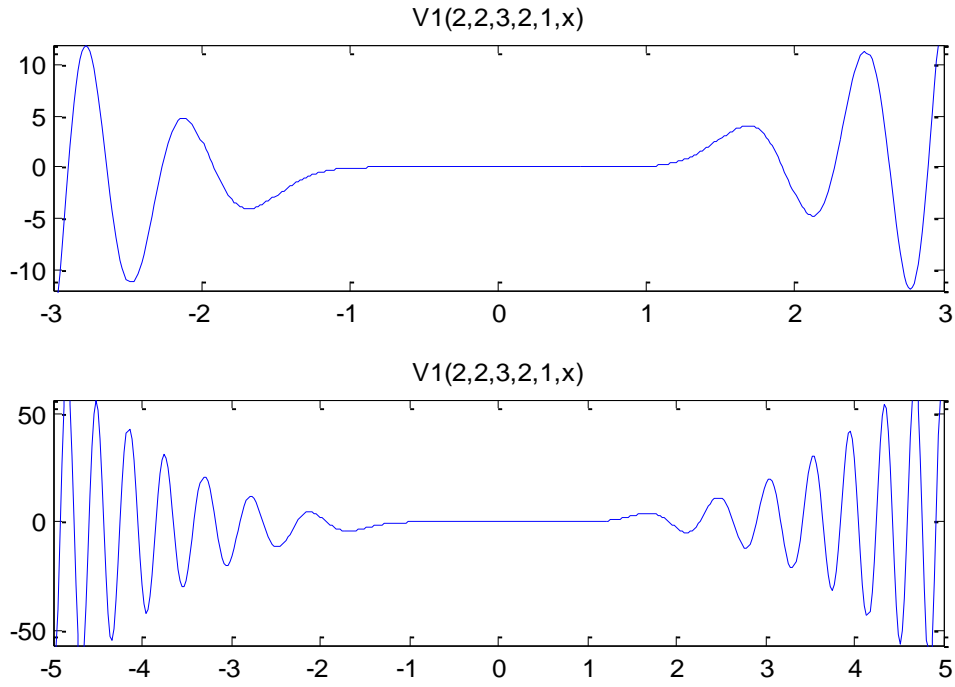


Fig. 7 Graphs of the sequence for the intervals  $[-3, 3]$  and  $[-5, 5]$  for  $n=1$

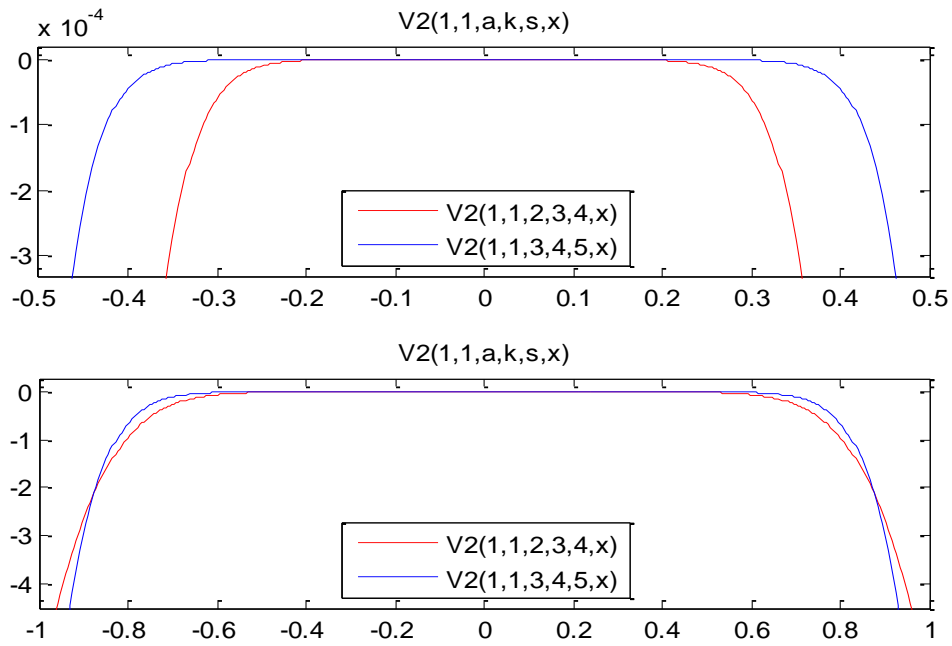
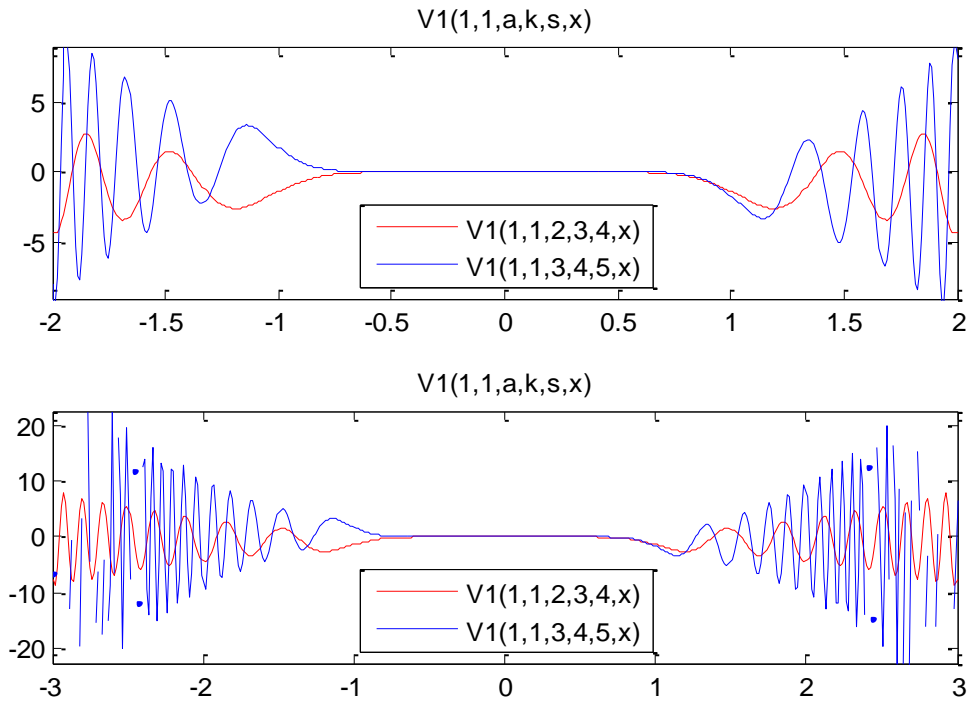
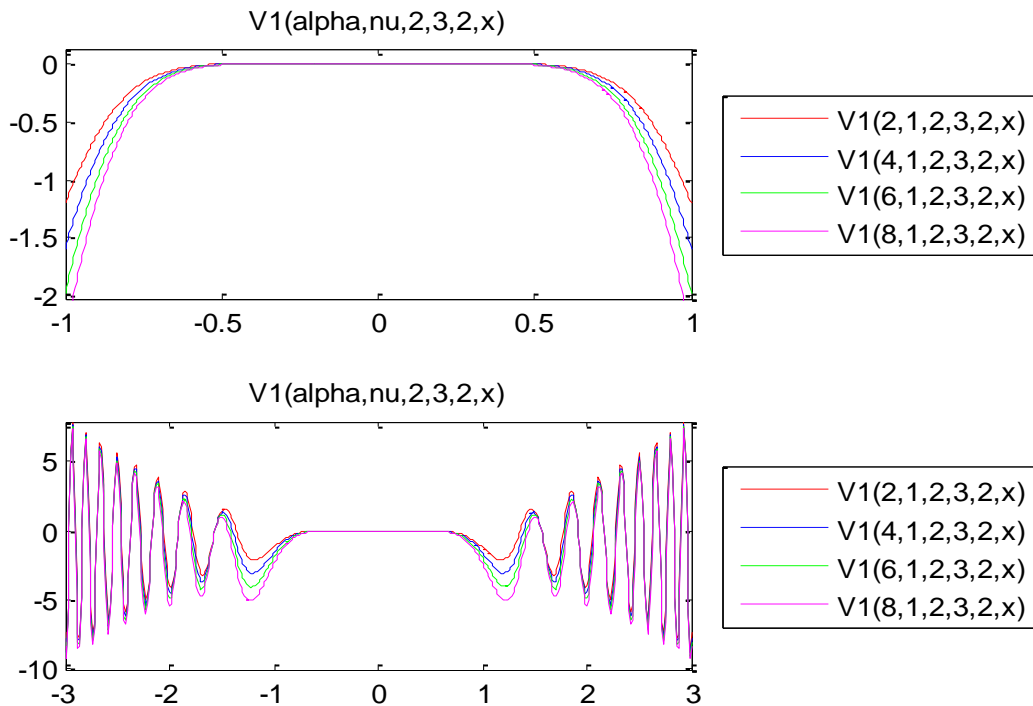


Fig. 8 Graphs of the sequence for the intervals  $[-0.5, 0.5]$  and  $[-1, 1]$  for  $n=2$



**Fig. 9** Graphs of the sequence for the intervals  $[-2, 2]$  and  $[-3, 3]$  for  $n=1$



**Fig. 10** Graphs of the sequence for the intervals  $[-1, 1]$  and  $[-3, 3]$  for  $n=1$

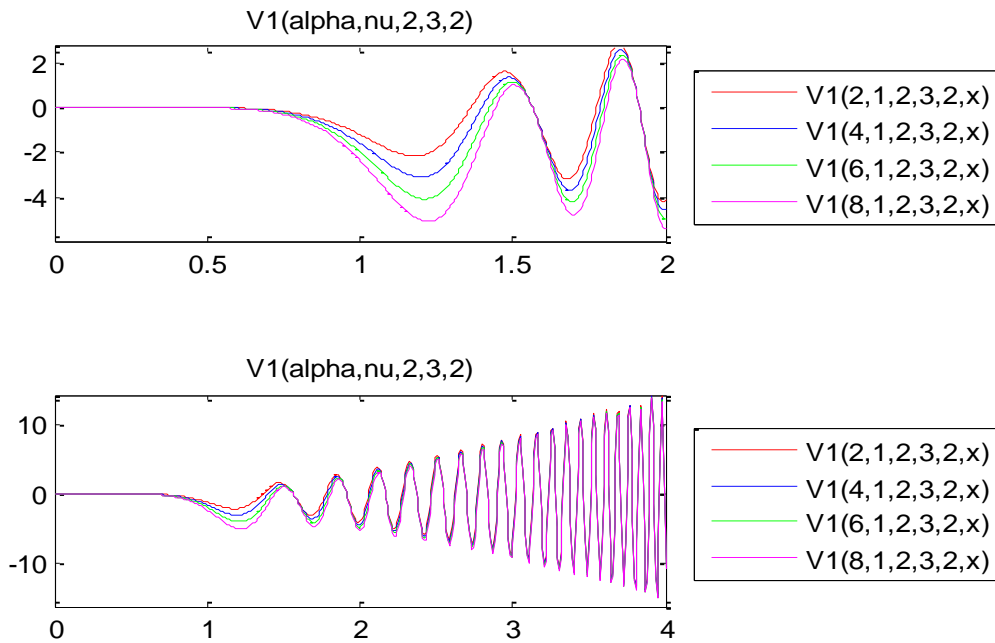


Fig. 11 Graphs of the sequence for the intervals  $[0, 2]$  and  $[0, 4]$  for  $n=1$

## 7. INTERPRETATION OF THE ABOVE DATABASE AND GRAPHS

The different values of the parameters, as assigned for equation (1.15) are taken to establish different databases and their graphs. In this section interpretation of the above database and graphs are made for different parameters of the new sequence of functions.

- (I) The first database and graph of the sequence of functions  $V_n(1,1,1,1,x)$  for  $n=1, 2, 3$  and  $4$  are established for the range  $-2 \leq x \leq 2$ . From the database (1) and Figure (1), it is easily observed that  $V_1(1,1,1,1,x)$  and  $V_3(1,1,1,1,x)$  are anti-symmetric in the interval about the line  $x=0$  whereas  $V_2(1,1,1,1,x)$  and  $V_4(1,1,1,1,x)$  are symmetric about the line  $x=0$  for the given interval.
- (II) The second database and graph of the sequence of functions  $V_n(-1,2,1,2,1,x)$  for  $n=1, 2, 3$  and  $4$  are established for the range  $-2 \leq x \leq 2$ . From the database (2) and Figure (2), it is easily observed that  $V_1(-1,2,1,2,1,x)$  and  $V_3(-1,2,1,2,1,x)$  are anti-symmetric in the interval about the line  $x=0$  whereas  $V_2(-1,2,1,2,1,x)$  and  $V_4(-1,2,1,2,1,x)$  are symmetric about the line  $x=0$  for the given interval.
- (III) The third database and graph of the sequence of functions  $V_1(\alpha,2,3,2,1,x)$  for  $\alpha=2, 4, 6, 8$  are established for the range  $-2 \leq x \leq 2$ . From the database (3) and Figure (3), it is easily observed that  $V_1(2,2,3,2,1,x)$ ,  $V_1(4,2,3,2,1,x)$ ,  $V_1(6, 2,3,2,1,x)$  and

$V_1(8,2,3,2,1,x)$  are anti-symmetric about the line  $x=0$  for the given interval.

- (IV) The fourth database and graph of the sequence of functions  $V_2(\alpha,2,3,2,1,x)$  for  $\alpha=2, 4, 6, 8$  are established for the range  $-2 \leq x \leq 2$ . From the database (4) and Figure (4), it is easily observed that  $V_2(2,2,3,2,1,x)$ ,  $V_2(4,2,3,2,1,x)$ ,  $V_2(6, 2,3,2,1,x)$  and  $V_2(8,2,3,2,1,x)$  are symmetric about the line  $x=0$  for the given interval.
- (V) The fifth database and graph of the sequence of functions  $V_2(1,1,a,k,s,x)$  for different values of the parameters are established for the range  $-0.5 \leq x \leq 0.5$ . From the database (5) and Figure (5), it is easily observed that  $V_2(1,1,2,3,4,x)$ ,  $V_2(1,1,3,4,5,x)$ ,  $V_2(1,1,4,5,6,x)$  and  $V_2(1,1,5,6,7,x)$  are symmetric about the line  $x=0$  for the given interval. The graph of the sequence for all the parameters lies below the line  $y=0$  in the interval  $-0.5 \leq x \leq 0.5$ .
- (VI) In the figure 6 and 7 for the sequence  $V_1(2,2,3,2,1,x)$ , where  $x$  lies in the intervals  $[-0.5, 0.5]$ ,  $[-1,1]$ ,  $[-3,3]$  and  $[-5,5]$ ; it is observed that graphs for the said range of the sequence of function are anti-symmetric. The figures 8 and 9 of the sequences  $V_1(1,1,2,3,4,x)$  and  $V_1(1,1,3,4,5,x)$  for the intervals  $[-0.5, 0.5]$ ,  $[-1,1]$ ,  $[-2,2]$  and  $[-3,3]$  are also anti-symmetric. Figure 10 and 11 of the sequences  $V_1(\alpha,1,2,3,2,x)$  where  $\alpha=2,4,6,8$ ; for the interval  $[-1,1]$ ,  $[-3,3]$ ,  $[0,2]$  and  $[0,4]$  also can be easily interpreted.

## 8. CONCLUSION

This paper presents several cases of MATLAB applications in mathematical analysis including generating the graph and database of sequence of function. Program made for sequence of function is easy to establish and implementation for Graphs and Data-Bases. Authors come to the conclusion on the basis of above interpretation that the sequence of functions is either symmetric or anti-symmetric for different values of the parameters for even and odd values of  $n$  in particular. The present paper has enabled to find the values of sequence of functions for different values of parameters and have paved the way for comparison of trends.

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