A Comparative Study of Tsalli's and Kapur's Entropy in Communication Systems

Vijay Kumar, PhD.
Associate Professor,
JRE School of Engineering,
Greater Noida.

Y.K Sharma, PhD.
Assistant Professor(Sr. Scale),
Ansal University,
Gurgaon.

Prince Goyel
Research Scholar,
Maharishi Dayanand
University, Rohtak.

ABSTRACT
The total channel capacity of the system of channels composed of two independent subsystems of channel. On the basis of noise index Δ and composability factor C, we compared Kapur's entropy and Tsalli's entropy and found some promising results, which helps in the further generalization of communication systems and makes them more stable.

KEYWORDS
Generalized entropies, Tsalli's entropy, Kapur's entropy.

1. INTRODUCTION
In the process of transmission of signals, a communication system has to do with the amount of information added to the signals, having appearance of the errors in transmission of signals may be due to *noise* or *perturbation.* The communication system considered here is of statistical nature and is thus a bundle of different components. If one element of the system bundle is inappropriate to the setting, the communication system can under perform. Communication is a continuous process and a channel is the 'pipe' along which a information is conveyed. Shannon[9] realized that the noise added to one digital pulse would generally make the overall amplitude different from that of another pulse. Further, the noise amplitudes for two different pulses are independent in the channel as amplitude variations. From a thermodynamic viewpoint, the heat in atomic motions disturb the whole system. On the contrary, in communication systems, noise added in a single pulse of the channel disturbs the signal. Composability is an important issue in communication system development in which different sets of components from the system of channels may be composed into different sub-systems of channels. Kapur[6,7, 8] proposed a generalization of the Boltzmann-Gibbs (BG) entropic measure to deal with the problems of non-extensivity in statistical mechanics. Tsalli's[12] non-extensive statistical formalism is a useful framework for the analysis of many interesting properties of nonlinear systems. Suyari [11] characterized these entropies by generalizing Shannon-Khinchin's axiom. Abe [1,2,3] described the pseudo-additivity of a thermodynamic system containing two sub-systems by taking the composition of Tsalli's entropy. Callen [4] described the condition of equilibrium by the maximum of total entropy of the system. Gupta and Kumar[5] discuss a general composition law and show that the generalized Kapur's entropy is compatible with the composition law, defined consistently with the condition of the existence of equilibrium and satisfies pseudo-additivity. In this paper, comparison of Tsalli's entropy and Kapur's entropy for the purpose of noise index and composability factor will discuss in section (2).

Channels exhibit complex systems and attracting great attention. A common feature of these systems is that they stay in non equilibrium stationary states for significantly long periods. In these systems, channels are generally inhomogeneous. Let $X(A,B)$ be the total channel capacity of the channel and is divided into two independent subsystems of channels, *A* and *B*.

Let $X(A,B)$ satisfy additivity as:

$$X(A,B) = X(A) + X(B)$$

According to Boltzmann-Gibbs-Shannon entropy [9]

$$H(p) = -\sum_{i=1}^{N} p_i \ln(p_i) \tag{1.1}$$

where, N is the total no. of states at a given scale.

For independent variables A and B,

$$H(A,B) = H(A) + H(B) \tag{1.2}$$

According to Landsberg and Vedral [10], for any non-extensive entropy

$$H_\alpha(A,B) = H_\alpha(A) + H_\alpha(B) + \tau(\alpha)\, H_\alpha(A).H_\alpha(B) \tag{1.3}$$

where, $\tau(\alpha)$ is a function of the entropic index.

Abe [2], proved pseudo-additivity as:

$$H_\alpha^{(T)}(A,B) = H_\alpha^{(T)}(A) + H_\alpha^{(T)}(B) + (1-\alpha)\, H_\alpha^{(T)}(A)\, H_\alpha^{(T)}(B) \tag{1.4}$$

for Tsallis entropy $H_\alpha^{(T)}(p)$ is defined as:

$$H_\alpha^{(T)}(p) = \frac{1}{1-\alpha}\left[\sum_{i=1}^{N} (p_i)^\alpha - 1\right] \tag{1.5}$$

where, $\alpha > 0$ and is called as non-extensive index.

when $\alpha \to 1$, the measure (1.5) reduces to (1.1).

Also, Abe [2], proved pseudo-additivity as:

$$H_\alpha^{(NT)}(A,B) = H_\alpha^{(NT)}(A) + H_\alpha^{(NT)}(B) + (\alpha-1)\, H_\alpha^{(NT)}(A)\, H_\alpha^{(NT)}(B) \tag{1.6}$$

for the normalized Tsalli's Entropy $H_\alpha^{(NT)}(p)$ and is defined as:

$$H_\alpha^{(NT)}(p) = \frac{H_\alpha^{(T)}(p)}{\sum_{i=1}^{N}(p_i)^\alpha} = \frac{1}{1-\alpha}\left[1 - \frac{1}{\sum_{i=1}^{N}(p_i)^\alpha}\right] \qquad (1.7)$$

Gupta and Kumar[5], proved pseudo-adddditivity as:

$$H_\alpha^K(A,B) = H_\alpha^K(A) + H_\alpha^K(B) + \tau(\alpha)\, H_\alpha^K(A)\cdot H_\alpha^K(B) \qquad (1.8)$$

for the generalized Kapur entropy $H_\alpha^K(p)$ of order α and is defined as:

$$H_\alpha^K(p) = \frac{1-\left(\sum_{k=1}^{n} p_k^{\;1/\alpha}\right)^\alpha}{1-\alpha}, \quad \alpha > 0 \qquad (1.9)$$

when $\alpha \to 1$, $H_\alpha^K(p)$ reduces to $H(p)$

For Tsallis, Normalized Tsallis and Kapur's Entropy, the entropic index $\tau(\alpha)$ is $(1-\alpha)$, $(\alpha-1)$ and $(1-\alpha)$ respectively and is known as the frequency of the noise. Also, it is seen that τ has different values for different entropy measures. Since, the entropic index $\tau(\alpha)$ of Tsallis entropy and Kapur's entropy is same. Therefore, the noise in sub-system depends on the noise index $\Delta = (1-\alpha).H(A)\cdot H(B)$ and composability factor

$$C = \frac{\Delta}{H(A)+H(B)}$$

Taking, $\Delta^T = (1-\alpha).H_\alpha^T(A)\cdot H_\alpha^T(B)$, in case of Tsalli's entropy and $\Delta^K = (1-\alpha).H_\alpha^K(A)\cdot H_\alpha^K(B)$, in case of Kapur's entropy.

The composability factor C for individual entropies are as follows:

$C^T = \dfrac{\Delta^T}{H_\alpha^T(A)+H_\alpha^T(B)}$, in case of Tsallis entropy and

$C^K = \dfrac{\Delta^K}{H_\alpha^K(A)+H_\alpha^K(B)}$, in case of Kapur's entropy

2. COMPARISON OF TSALLI'S AND KAPUR'S ENTROPY AS PER THE NOISE INDEX Δ AND COMPOSABILITY FACTOR C

In this section, we will study the comparison between the Noise index Δ and Composability factor C for different values of parameter α .

Consider an example of the binomial distribution of two sets A and B for the given values of X are as follows:

X	**0**	**1**	**2**	**3**	**4**
$P_A(x)$	0.2138	0.4	0.2866	0.0866	0.013
$P_B(x)$	0.5311	0.3542	0.098	0.015	0.0017

The value of the noise index Δ and composability factor C is shown in the following two cases as:

2.1 Case I: For $0 < \alpha < 1$

2.1.1 Noise Index Δ :

Table. 2.1.1. Value of the noise index Δ^T and Δ^K for $0 < \alpha < 1$

H_α	$H_\alpha^{(T)}$	Δ^T	$H_\alpha^{(K)}$	Δ^K
$H_{0.1}(A)$	3.425	9.6519	0.6650	0.3113
$H_{0.1}(B)$	3.1312		0.5201	
$H_{0.2}(A)$	2.9653	6.0284	0.7286	0.3318
$H_{0.2}(B)$	2.5412		0.5694	
$H_{0.5}(A)$	2.08	1.6648	0.9127	0.3231
$H_{0.5}(B)$	1.6008		0.7081	
$H_{0.9}(A)$	1.4307	0.1539	1.216	0.1129
$H_{0.9}(B)$	1.076		0.9292	

2.1.2 Composability factor C :

Table. 2.1.2. Value of the composability factor C^T an C^K for $0 < \alpha < 1$

H_α	$H_\alpha^{(T)}$	C^T	$H_\alpha^{(K)}$	C^K
$H_{0.1}(A)$	3.425	1.4721	0.6650	0.2626
$H_{0.1}(B)$	3.1312		0.5201	
$H_{0.2}(A)$	2.9653	1.0947	0.7286	0.2936
$H_{0.2}(B)$	2.5412		0.5694	
$H_{0.5}(A)$	2.08	0.4522	0.9127	0.1993
$H_{0.5}(B)$	1.6008		0.7081	

$H_{0.9}(A)$	1.4307	0.0613	1.216	0.0526
$H_{0.9}(B)$	1.076		0.9292	

From the tables (2.1.1) and (2.1.2), we plot graphs for different values of parameter α and are as shown:

Fig. 2.1.3. Comparison of Tsalli's and Kapur's Entropy for $0 < \alpha < 1$ as per Δ

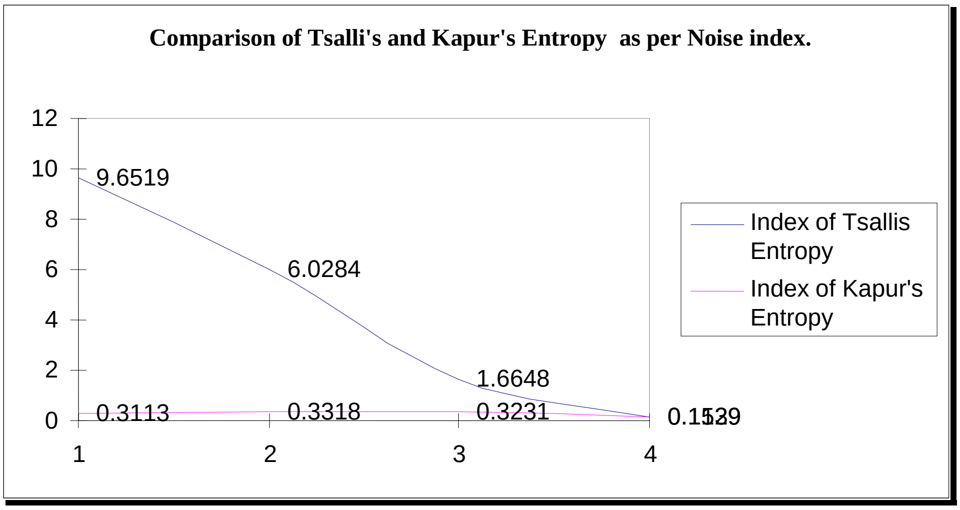
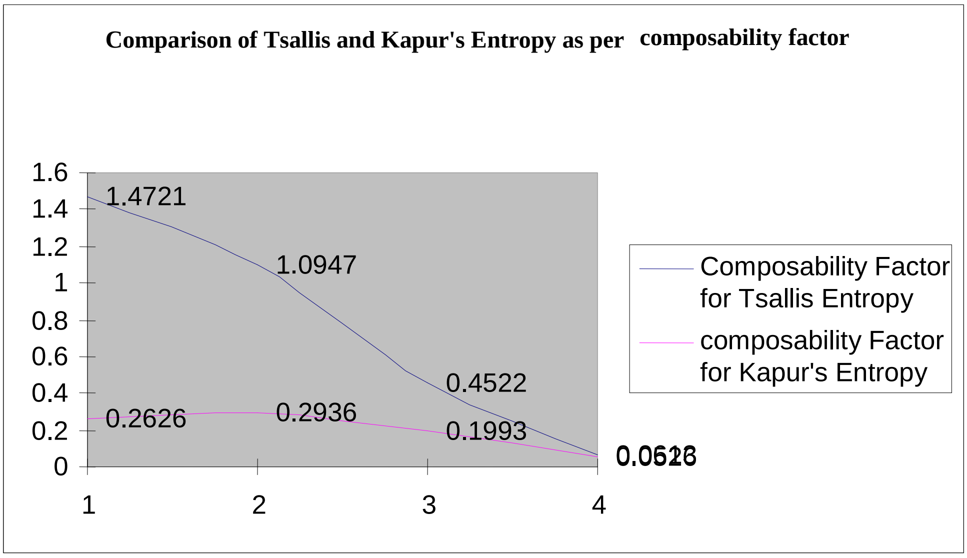


Fig. 2.1.4. Comparison of Tsalli's and Kapur's Entropy for $0 < \alpha < 1$ as per C



From the fig. (2.1.3) and (2.1.4), it is seen that noise index Δ and composability factor C in case of Kapur's entropy is very less as compared to Tsalli's entropy, when $\alpha \in (0,1)$. Also, when the value of α comes closer to one, noise index Δ and composability factor C for both the entropies is almost same.

2.2 Case II: For $\alpha > 1$

2.2.1 Noise Index Δ:

Table. 2.2.1 Value of the noise index Δ^T and Δ^K for $\alpha > 1$.

H_α	$H_\alpha^{(T)}$	Δ^T	$H_\alpha^{(K)}$	Δ^K
$H_{1.1}(A)$	1.228	0.1161	1.43	0.1542
$H_{1.1}(B)$	0.946		1.079	
$H_{1.5}(A)$	0.9386	0.3475	2.052	1.5518
$H_{1.5}(B)$	0.7406		1.5125	
$H_{2.1}(A)$	0.6709	1.6648	3.5633	9.8664
$H_{2.1}(B)$	0.5587		2.5172	

2.2.2 Composability factor C :

Table. 2.2.2 Composability factor C^T and C^K for $\alpha > 1$

H_α	$H_\alpha^{(T)}$	C^T	$H_\alpha^{(K)}$	C^K
$H_{1.1}(A)$	1.228	0.05340	1.43	0.0614
$H_{1.1}(B)$	0.946		1.079	
$H_{1.5}(A)$	0.9386		2.052	
$H_{1.5}(B)$	0.7406	0.2069	1.5125	.4353
$H_{2.1}(A)$	0.6709	0.3353	3.5633	1.6226
$H_{2.1}(B)$	0.5587		2.5172	

From the tables (2.2.1) and (2.2.2) , noise index Δ and composability factor C , we plot graphs for the parameter $\alpha > 1$.

Fig. 2.2.3. Comparison of Tsalli's and Kapur's Entropy for $\alpha > 1$ as per Δ

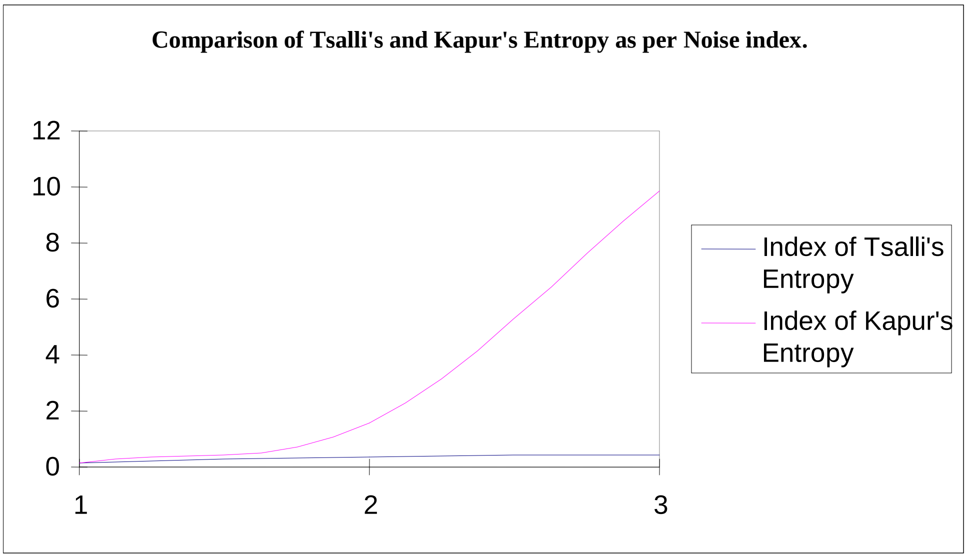
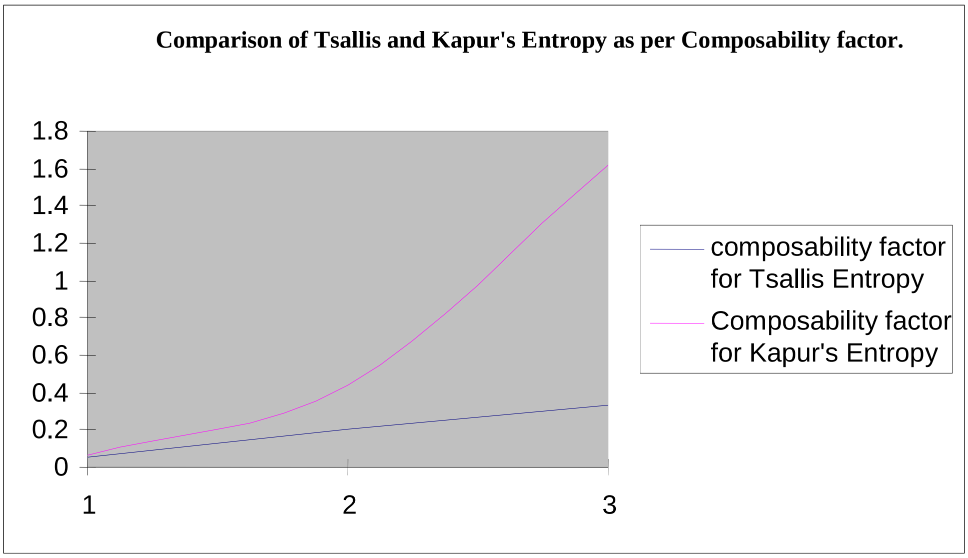


Fig. 2.2.4. Comparison of Tsalli's and Kapur's Entropy for $\alpha > 1$ as per C



From the graphs (2.2.3) and (2.2.4), it conclude that, for $\alpha > 1$, noise index Δ and composability factor C in case of Tsallis entropy is very less as compared to Kapur's entropy

3. CONCLUSION

The concept of composability puts a stringent constraint on possible forms of entropies. Also, on the basis of noise index Δ and composability factor C , we compared Kapur's entropy and Tsalli's entropy and found that, for $0 < \alpha < 1$, Kapur's entropy gives more promising results than Tsalli's entropy and hence used for further generalization in communication systems. For $\alpha > 1$, Tsallis entropy finds suitable than Kapur's entropy for the purpose of noise.

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