Observability and Controllability of MIMO Control Systems via Difference Equations

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ABSTRACT

The paper applies the difference equations to unify the study of observability and controllability conditions of discrete-time multi-input multi-output (MIMO) nonlinear control systems. The necessary and sufficient condition for irreducibility of the set of nonlinear multi input- multi output (MIMO) is presented in terms of the greatest common left divisor of two polynomial matrices describing the behavior of the system which is suitable for constructing an observable and accessible state space realization. We show that the concepts of controllability and observability are related to systems of difference equations. It is well known that a solvable system of linear algebraic equations has a solution if and only if the rank of the system matrix is full .This method is more clear, straight-forward and therefore better suited for implementation in different computer packages such as Matlab.

Key Words

Nonlinear control system, Discrete-time system, multi input-output model, state-space realization, Polynomial matrices.

1. INTRODUCTION

To describe the behavior of the real-life processes we frequently use input-output (i/o) models. This allows representing the object of practical interest in a compact and convenient form by means of difference equations. State-space description usually becomes the basis for analysis and control of nonlinear multi input – multi output (MIMO) systems. Multiple-input multiple-output (MIMO) techniques are a key enabling technology for high-rate wireless communications. The concepts of stabilizability and detectability play very important roles in optimal control theory. All real world systems comprise multiple interacting variables. For example one tries to increase the flow of water in a shower by turning on the hot tap, but then the temperature goes up, one wants to spend more time on holiday, but then one needs to spend more time at work to earn more money. Obviously these kinds of multi tasks are complex to understand and as a result, the concepts of control system design are introduced to get an appropriate output. Of course, one could attempt to solve the problem by using several SISO (Single Input Single Output) control loops, but this might not prove satisfactory, so the researchers extended MIMO concepts in [6,7,8]. Most of the ideas presented in early parts of the book and research apply to multivariable systems. The main difficulty in the MIMO case is that we have to work with matrix, rather than scalar transfer functions.

Thus, the problem encounters and the main goal of this paper is to bridge the gap between modeling approaches and to present the algorithm allowing us to construct a minimal state-space model from an arbitrary set of nonlinear higher order i/o difference equations, whenever applicable. One of the central themes in the systems and control theory is the problem of representing a system in a form that is convenient for the particular purpose and of transforming one representation into another. Particularly, for linear systems, it is well known that an arbitrary set of higher order input – output difference equations can be always transformed into an input – output equivalent set of equations, having reduced form [4] and [9]. The main purpose of this technical note is to introduce and characterize the non linear i/o equations for non linear control systems described by the set of higher order difference equations and to transform the set of equations via nonlinear i/o equations wherever applicable. Once the set of nonlinear higher order difference equation is in the row and column reduced matrix form, it is extremely easy to transform these equations into the state space equation. So the row and column reduced forms of the set of higher order i/o equations will be instrumental to all the further developments of multi input multi output (MIMO) realization problem. The key for the success of difference equations in the nonlinear case is, its computational nature.

In MIMO control design, a key design objective is usually to achieve zero steady-state errors for certain classes of references and disturbances [8]. However, we have also seen that this requirement can produce secondary effects on the transient behavior of these errors. So the basic key elements studied are state estimation by an observer and state-estimate feedback. Using state-estimate feedback ideas, we can design a multivariable controller which stabilizes and control the systems and, at the same time it ensures zero steady-state error for constant references and disturbances.

Recently difference equations have gained popularity in the study of non linear control systems [10], both in discrete and continuous time. The difference equations utilizes the algebraic properties of polynomials with coefficients from different field of meromorphic functions of systems variable and the strong interplay between the ring of non commutative skew polynomials and the tangent linearized equations of non linear higher order input output difference equations [2]. Much kind of problems have been addressed up to now among controllability, irreducibility, system reduction, realization, transfer equivalence and model matching [6,10]. Transfer function formalism has been recently introduced into nonlinear domain and this formalism is also based on difference equations approach [9]. The coordinate-free necessary and sufficient reliability conditions were formulated in terms of polynomial matrices [1]. Moreover, it is known that if the system under consideration is not in the irreducible form, then the state space realization is not minimal, i.e. accessible. Our result can be also understood as a generalization of the polynomial realization algorithm obtained for non linear time invariant systems.

In this paper we consider a general class of nonlinear discrete-time control systems described by input-output difference equations. We begin by reviewing the basic properties of controllable and observable nonlinear dynamical systems and investigate the key relationships between their state-space and input-output representations. One of the main contributions of this paper is the derivation of a set of necessary and sufficient algebraic conditions for existence of a local observable state-space realization of an input-output equation. An immediate consequence of the necessary conditions is that a generic input-output equation does not necessarily admit an observable state-space realization. The sufficient conditions, on the other hand, can be used to construct input-output equations that are guaranteed to have a state-space realization. More importantly, the paper also provides a simple algorithm for deriving the state-space realization directly from the input-output equation whenever possible.

An uncontrollable realization may result if the input-output model itself is not minimal which contains common poles or zeros in the linear case, such a realization obviously is unsuitable for control design purposes [10]. To address this difficulty, we provide an algorithm (Polynomial Realization) for extracting a minimal realization for the external model and formulate the necessary and sufficient algebraic conditions for its existence.

This paper is organized as follows: section 2 describes the problem statement and general framework. The definitions and basic properties of controllable and observable state space are observed in section 3. Section 4 & 5 explores the main properties of input-output maps generated by a state-space system. Necessary and sufficient condition for irreducibility of nonlinear MIMO systems is given section 6. Section 7 concludes the paper.

2. CANONICAL FORMS FOR SYSTEMS WITH OUTPUT

Consider an input-output discrete-time process described by the following input-output (I/O) equation

$$y[k] = g(y[k-m], \dots, y[k-1], u[k-m], \dots, u[k \dots 1]) \quad (1)$$

Where $u[k] \epsilon \mathbb{R}$ and $y[k] \epsilon \mathbb{R}$ represent the input and output of the process respectively, and g is a smooth function. Without loss of generality, we assume that,

$$g(0, \dots, 0, 0, \dots \dots 0) = 0$$

The first issue that we address is the state-space realization of an input-output equation. State-space representation of the form

$$x[k+1] = f(X[k], u[k])$$

$$y[k] = h(X[k]) \quad (2)$$

Where $X[k] \epsilon \mathbb{R}^n$ is the state vector and $f(.,.)$ and $h(.,.)$ are smooth (C^{∞}) functions. The Jacobian of f with respect to X and $D_X f(X,u)$ is nonsingular. Now we shall seek the necessary and sufficient conditions for existence of a minimal realization and an explicit algorithm for deriving it from a non minimal observable realization whenever it exists.

For systems with one output, the C-matrix is a row vector. The essential step then is the realization that there always exists a similarity transformation of a state-space model such that the C-vector is transformed into the first unit vector. Furthermore, the A-matrix is transformed such that it contains only n free parameters. In this section we will use a third-order system with the D matrix equal to zero. The system is defined as follows:

$$A = \begin{bmatrix} -0.6129 & 0 & 0 \\ 0 & 0.7978 & -0.4494 \\ -6.4516 & 0 & 0.8867 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0323 \\ 0.1290 \\ 0.2247 \end{bmatrix}$$

$$C = [9.667 \quad 0.1124 \quad 1.629] \text{ and } D = 0$$

Now the system is converted into observer canonical form. The observer canonical form of a third-order system is given by

$$x(k+1) = \begin{bmatrix} 0 & 0 & -a_0 \\ 1 & 0 & -a_1 \\ 0 & 1 & -a_2 \end{bmatrix} x(k) + \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix}$$

$$y(k) = [0 \quad 0 \quad 1]\, x(k)$$

Such that

$$y(k) = \frac{c_{n-1}q^{n-1} + \dots + c_1 q + c_0}{q^n + a_{n-1}q^{n-1} + \dots + a_1 q + a_0} u(k)$$

In order to build this state-space model, we first need the transfer function polynomials corresponding to the matrices A, B, C and D. These polynomials can be used to build the state-space model.

3. CONTROLLABILITY AND OBSERVABILITY OF STATE-SPACE REALIZATIONS

Controllability and observability represent two major concepts of modern control system theory. These concepts were introduced by R. Kalman in 1960. A system is observable if and only if the system state $x(t)$can be found by observing the input u and output y over a period of time from $x(t)$ to $x(t+h)$. In order to be able to do whatever we want with a dynamic system with a control input, the system must be controllable [2, 6, 9]. The system controllability is roughly defined as an ability to do whatever we want with our system, or in more technical terms, the ability to transfer our system from any initial state H to any desired final state I J in a finite time. In order to see what is going on inside the system under observation, the system must be observable. It is well known that a solvable system of non linear equations has a solution if and only if the rank of the system matrix is full. Observability and controllability tests will be connected to the rank rests of certain matrices [6]. Throughout the paper, without loss of generality, we assume that the origin is an equilibrium state of the system, i.e., $f(0,0) = 0$, and restrict the state vector to an open neighborhood of the origin. The linearization of the system in (2) about $X = 0$ is denoted by

$$X[k+1] = A_0 X[k] + B_0 u[k] \quad (3)$$

$$y[k] = C_0 X[k] \quad (4)$$

Where $A_0 = D_X f(0,0), B_0 = D_u f(0,0), and\ C_0 = D_X h(0)$. The controllability and observability matrices of (3) are denoted by $Q_0 := [A_0^{n-1}B, A_0^{n-2}B, \dots., A_0 B, B]$ and $P_0 = (C, CA_0, \dots, CA_0^{n-1})$ respectively. Initially we define Controllability and Observability of (2).

Definition 1 The state-space representation of

$$x[k+1] = f(X[k], u[k])$$
$$y[k] = h(X[k])$$

is said to be controllable, if its linearization is also controllable of rank $(Q_0) = n$, and strongly observable, its linearization about the origin is also observable of rank $(P_0) = n$. It is said to be strongly minimal if it is both locally controllable and observable.

The block of inputs and outputs are

$$u[k] = (u[k], .., u[k+m-1]) \text{ and}$$
$$Y[k] = (y[k], ..., y[k+m-1])$$

respectively, for some fixed block size $m \geq 1$we define the state transition map of the system resulting from an initial state X and input sequence $u_1, u_{2,...,} u_i$ by

$$f^i(X, u) := f_{u_i} \circ f_{u_{i-1}} \circ ... \circ f_{u_2} \circ f_{u_1}(X)$$

Applying the state equation (2), sequentially to evaluate $X[k+1], ... , X[k+m]$ yields

$$X[k+m] = f^m(X[k], u[k]) \qquad (5)$$
$$y[k] = h^m(X[k], u[k])$$

Where

$$h^m(X, u) = (h(X), h \circ f_u^1(X), ..., h \circ f_u^{m-1}(X)).$$

f^m and h^m is mapped with Jacobian with respect to X and u.

$$-\frac{\partial J^0(x(t), t)}{\partial t} = v(x(t), u, t) + \left[\frac{\partial J^0(x(t), t)}{\partial x}\right] f(x(t), u, t)$$

The solution for this equation must satisfy the boundary condition $J^0(x(t_f), t_f) = g(x(t_f))$

Theorem 1

The rank of the controllability sub matrix

$C^{(j)}(A, B) = [B, AB, ... , A^{j-1}B]$ is $\sigma_j = \sum_{i=1}^{j} \rho_i$ and that of the full matrix is σ_k.

Proposition 1

The partial derivatives of functions f^m and h^m in (5) are given by

$$D_X f^m(X, u) = Ad^* f_{u_m} I_{n \times n}(X),$$
$$D_u f^m = D_x f^m [B_1 .... B_m]$$
$$y[k] = g(y[k-m], ..., y[k-1], u[k-m], ..., u[k-1])$$
$$D_X h^m = \begin{bmatrix} C_0 \\ C_1 \\ \vdots \\ C_{m-1} \end{bmatrix}$$

$$D_u h^m = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ C_1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & 0 \\ C_{m-1}B_1 & \cdots & C_{m-1}B_{m-1} \cdots & 0 \end{bmatrix} \qquad (6)$$

Where

$$B_i(X, u) = Ad_{f_{u1}} \ldots Ad_{f_{ui}-1} B_{ui}(X)$$
$$C_i(X, u) = Addd *_{f_{u1}} \ldots Ad *_{f_{ui}} C(X)$$
$$Ad_\varphi v(x) := [D\varphi(x)]^{-1} v \circ \varphi(x)$$
$$Ad_\varphi^* w(x) := [W \circ \varphi(x)] D\varphi(x)$$

for a smooth function $\varphi(x) \in \mathbb{R}^n$, vector field $v(x) \in \mathbb{R}^n$ and co vector field $W(x) \in \mathbb{R}^{1Xn}$.

The representations of $D_u f^m$ *and* $D_X h^m$ in proposition 1 extend the controllability and observability matrices to non equilibrium states and inputs. In particular, $D_u f^m(0,0) = Q_0$ and $D_x h^m(0,0) = P_0$ when $m = n$, these matrices at $X = 0$ *and* $u = 0$ coincide with the controllability and observability matrices of the linearized system given by (3).

Proposition 2

i) If the system given by (2) is locally controllable, then there exits an open neighborhood $X \subset \mathbb{R}^n$ of the origin and a smooth function $\psi : \mathcal{X} \times \mathcal{X} \to \mathbb{R}^{\boldsymbol{m}}, m \geq n,$ Such that if $u = \psi(X_m, X_0)$ then

$$X_{m,} = f^m(x_0, u), \; \forall x_{0}, x_m \in \mathcal{X}.$$

ii) If the system given by (2) is locally observable, then there exist an open neighborhoods $X \subset \mathbb{R}^n$, $\mathcal{U} \subset \mathbb{R}, \mathcal{Y} \subset \mathbb{R}$ of the origin and a smooth function $\Omega : y^m \times \mathcal{U}^m \to \mathcal{X}$, $m \geq n$, such that if $y = h^m(x, u)$ then $x = \Omega(y, u), \forall x \in \mathcal{X}$ and $u \in \mathcal{U}^m$.

The proposition implies that if the system is locally controllable then any two states in a neighborhood of the origin can be transferred to one another by means of a finite control sequence. Local observability implies that any state in a neighborhood of the origin can be uniquely determined from a finite sequence of the input and output vectors.

Proof of the Proposition:

Consider the map $z = f^m(x, u)$. The Jacobian of f^m with respect to u $Q_0 = D_u f^m(0,0)$ has rank n, since the system is locally controllable. There exits an $m \times m$ permutation martrix $\Pi = [\,\Pi_1 \;\; \Pi_2\,]$, $\Pi_1 \in \mathbb{R}^{m \times n}$, such that $Q_0 \Pi_1$ is invertible.

Partitioning $\Pi^{-1} = \begin{bmatrix} \Pi_1^{-1} \\ \Pi_2^{-2} \end{bmatrix}$,

Let $u_1 = \Pi_1^{-1}$u and define the map $F : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ by $z = F(x, u_1) = f^m(x, \Pi_1, u_1)$. Since $D_u F(0,0) = Q_0 \Pi_1$ is invertible by the implicit function theorem. There are open neighborhoods $X_1 \subset \mathbb{R}^n$, $X_2 \subset \mathbb{R}^n, \mathcal{U}_1 \subset \mathbb{R}^m$, and function $F^{-1} : \mathcal{X}_1 \times \mathcal{X}_2$ such that $u_1 = F^{-1}(z, x) \Leftrightarrow z = F(x, u_1)$ and $\mathcal{X} = \mathcal{X}_1 \cap \mathcal{X}_2$ proves the preposition. Also it is noted that if $n = m$, then $\Pi_1 = \Pi = I_{n \times n}$ and $\psi = F^{-1}$ is unique.

4. PROPERTIES OF DYNAMICAL INPUT-OUTPUT MAPS

The key properties of an input-output map, generated by the dynamical systems which possess a state equation in the form of (2) by [7]. These properties will serve as the necessary conditions for the existence of a minimal state-space realization [13,14]. We begin by formulating the block representation of the input-output map. Evaluating

$$y[k], y[k+1], ..., y[k-m+1]$$

recursively in terms of $y[k-m], ..., y[k-1]$ and $u[k-m], ..., u[k+m-2]$ using (1), we obtain

$$y[k] = G(y[k-m], u[k-m], u[k])$$

where the i^{th} row of G, denoted by $g_i, i = 1,\ldots,m$ is given by

$$g_i(y,u,v) = g(y_i,\ldots,y_m, g_1(y,u,v),\ldots g_{i-1}(y,u,v), u_i,\ldots,u_m, v_1,\ldots,v_{i-1} \quad (7)$$

for $i \geq 2$ and of course

$$g_1(y,u,v) = g(y_1\ldots,y_m,u_1\ldots,u_m).$$

we also define

$$g_{m+1}(y,u,v) = g(g_1(y,u,v),\ldots,g_m(y,u,v),v_1,\ldots,v_m)$$

We now derive an input-output map corresponding to an observable state-space realization given in (2). By proposition (2), $x[k]$ can be solved for locally in terms of $y[k]$ and $u[k]$.

That is, $x[k] = \Omega(y[k],u[k])$ for some local function Ω .

Using this expression for $x[k]$ in (5) yields

$$x[k+m] = \Theta(y[k],u[k]) := f^m(\Omega(y[k],u[k],u[k])$$

Thus the input-output map $g = h \circ \Theta$ that is

$$y[k] = h(x[k]) = h(\Theta(y[k-m],u[k-m])) \quad (8)$$

The corresponding block input-output equation becomes,

$$y[k] = G(y[k-m],u[k-m],u[k])$$
$$:= h^m(\Theta(y[k-m],u[k-m]),u[k])$$

It is worth mentioning that this input-output equation is locally unique if and only if $m = n$. In all other cases for which $m < n$, each different function $\Omega(.,.)$ yields a different input-output map.

Definition 2 The input-output

$$y[k] = g(y[k-1],\ldots,y[k-m],u[k-1],\ldots,u[k-m])$$

is said to be a Dynamical Input Output Map if it coincides locally with the unique input-output map corresponding to an observable state-space realization of order m.

An observable state-space realization of order n can always generate dynamical input output map of higher order $m \geq n$. For example let

$$x[k+1] = f(x[k],u[k], \; y[k] = h(x[k])$$

be an n^{th} order locally observable system. The constant matrices $C_z \in \mathbb{R}^{p\times n-m}$ and $A_Z \in \mathbb{R}^{m-n\times m-n}$ such that (A_Z, C_Z) is observable, and that A_z and $A_0 = D_x f(0,0)$ have no common eigen values. It can be verified that the m^{th} order system

$$x[k+1] = f(X[k],u[k])$$
$$z[k+1] = A_z z[k]$$
$$y[k] = h(x[k]) + C_z z[k] \quad (9)$$

is locally observable. Moreover its output matches that of the original system provided $z_0 = 0$.

Theorem 2 Let $G(.,.,.)$ be the block input output map of a dynamical system given in (11). Then $D_y G(y,u,v)$ is nonsingular and $(D_y G(y,u,v))^{-1}[D_u G(y,u,v)]$ is independent of the third variable v on a neighborhood of the origin.

Proof of the Theorem

Let the m^{th} order system

$$x[k+1] = f(x[k],u[k], \; y[k] = h(x[k])$$

be an observable realization of the dynamical input output map. Then the block input output map of the dynamical input output map is given by

$$G(y,u,v) = h^m(D_u\Theta(y,u),v)$$

Differentiating this with respect to y and u using chain rule, we get

$$D_y G(y,u,v) = D_x h^m(x,v) D_y \Theta(y,u)$$
$$D_u G(y,n,v) = D_x h^m(x,v) D_n \Theta(y,u) \quad (10)$$

Differentiating $y = h^m(\Omega(y,u),u$ and

$\Theta(y,u) = f^m(\Omega(y,u),u$ with respect y yields

$$D_y\Theta(y,u) = D_x f^m(x,u) D_y \Omega(y,u)$$
$$I_{mXm} = D_x h^m(x,u) D_y \Omega(y,u)$$

This implies that $D_y\Theta(y,u)$ is invertible. Thus

$$G(y,u,v))^{-1} D_u G(y,u,v) = D_y\Theta(y,u))^{-1} D_u\Theta(y,u) \quad (11)$$

is independent of v .

5. OBSERVABLE STATE SPACE REALIZATION

We construct a suitable state vector corresponding to an input output map and use it to formulate and prove the necessary and sufficient conditions in order to map the dynamical input output map. An input map of (1) with the block of output map of (5) is defined along with the state vector be, $x[k] = G(y[k-m],u[k-m],0)$. In order for $x[k]$ to qualify as a state vector, we need to show that $x[k+1]$ is a function of $x[k]$ and $u[k]$, and does not depend on $u[j], \forall j < k$. Incrementing k ,

$$X[k+1] = G(y[k-m+1],u[k-m+1],0)$$
$$=: G_1(y([k-m],u[k-m],v[k])$$

Where

$$V[k] = (u[k],0,\ldots,0) \quad \text{and}$$
$$G1 = (g_2, g_3 \ldots\ldots, g_m, g_{m+1}) \quad (12)$$

Since the Jacobian of $G(y,u,0)$ with respect to y is nonsingular, by the implicit function theorem $y[k-m]$ can be solved in terms of $x[k]$, $u[k-m]$, and $v[k]$. Then there exists a smooth function G_y^{-1} such that

$$G(G_y^{-1}(x,u,v),u,v) = x \quad (13)$$

locally. Thus

$$X[k+1] = G_1(G_y^{-1}(x[k],u[k],0),u[k],0),u[k],v[k]) \quad (14)$$

The following lemma shows that (14) is an observable realization of the input output map provided the necessary conditions stated earlier in theorem 3.

Lemma 1 (Necessary) If the input output map (5) is such that $D_y\, G(y,u,v)$ is a nonsingular and $[D_yG(y,u,v]^{-1}[D_uG(y,u,v)]$ is independent of the third variable v on a neighbourhood of the origin. Defining the state vector $x[k] = G(y[k], u[k],0)$, then (14) is an observable state-space realization for the i/o map.

Proof of the lemma: Consider the state equation , it is enough to show that its right hand side does not depend of $u[k]$ .That is

$$D_uG_1(G_y^{-1}(x,u,0),u,v) = D_yG_1(y,u,v)D_uG_y^{-1}(x,u,0) + D_uG_1(y,u,v) = 0 \quad (15)$$

From the definition of G_1,

$$G_1(G_y^{-1}(x,u,v),u,v) = (x_2, x_3, \ldots, x_m, g_m(x,v))$$

Thus $G_1(G_y^{-1}(x,u,v),u,v)$ does not depend on u, this implies

$$D_uG_1\left(G_y^{-1}(x,u,v),u,v\right) = D_yG_1(y,u,v)D_uG_y^{-1}(x,u,v) + D_uG_1(y,u,v) = 0 \quad (16)$$

Taking partial derivatives of (13) with respect to u,

$$D_uG_y^{-1}(x,u,v) = -(D_yG(y,u,v))^{-1}D_uG(y,u,v) = 0$$

does not depend on $\boldsymbol{v}$ by the hypothesis. Replacing $D_uG_y^{-1}(x,u,v)$ in (16) by $D_uG_y^{-1}(x,u,0)$ proves (15) and its necessary. Since $y[k] = g_1(y[k],u[k]) = x[k]$ then $y[k]$ is an output of (14). Finally (14) with $y[k]$ as output is an observable realization, since its observability matrix is identity.

6. IRREDUCIBILITY OF THE I/O EQUATIONS & REALIZATION

Definition 3 : A Control system of any set of i/o equations is said to be irreducible if there does not exist any non-zero autonomous variable in differential field K. Otherwise system the i/o equations are called reducible.

Theorem 3 (Polynomial Realization Algorithm) The nonlinear control system of any set of i/o is reducible in the sense of Definition 3, if and only if the polynomial matrices $P(\partial)$ and $Q(\partial)$ associated to system of i/o equations, are not relatively left prime.

Consider a class of matrices $P(\partial)$ whose elements are polynomials $P(\partial) \in K[\partial,\sigma,\delta]$ of finite, but unbounded degree. A σ differential field is a triple (K,σ,δ) where K is a field, σ is an automorphism of K and δ is a σ-derivation. The σ-differential field K will be the starting point for constructions used in characterizing theoretic properties of different nonlinear control systems. Any automorphism σ and σ-derivation *induce* a (left) non-commutative skew polynomial ring. The left skew polynomial ring given by σ and δ is the ring $K[\partial,\sigma,\delta]$ of polynomials in ∂ over K with the usual addition, and the (non-commutative) multiplication given by the commutation rule

$\partial \cdot a = \sigma(a) \cdot \partial + \delta(a)$ for any $a \in K$.

Proof (Sufficiency). Assume that the polynomial matrices $P(\partial)$ and $Q(\partial)$ are not relatively prime. This means that $P(\partial)$ and $Q(\partial)$ has a common left divisor $GL(\partial)$, which is not uni modular,

$$P(\partial)dy - Q(\partial)du = GL(\partial)[\,\tilde{P}(\partial)dy - \tilde{Q}(\partial)du] = 0$$

Because of non- uni modularity of $GL(\partial)$, will imply the existence at least one non zero analytic function, hence the system is reducible.

7. CONCLUSION

In this paper we investigated the realization problem of a general class of nonlinear discrete time systems described by multi input and multi output difference equations and derived the necessary and sufficient conditions for existence of their observable state space realizations. Minimal (accessible and observable) realization problem of nonlinear MIMO systems described by the set of i/o equations are addressed based on the polynomial representation of the system.

REFERENCES

[1] Antonis – Ioannis G Vardulakis, Nicholas P. Karampetakis, Efstathios N. Antoniou, Evangelia Tictopoulou, *On the realization theory of polynomial matrices and the algebraic structure of pure generalized state space systems,* Int. J. Appl. Math. Comput. Sci., Vol. 19, No. 1, 77–88, 2009.

[2] J. Grizzle and Moraal, *Non linear discrete time control. IEEE Transactions on Automatic Control*, 857 – 868, 2007

[3] M. Halas and C.H.Moog, *Transfer function approach to the model matching problem of nonlinear systems* in Proc. 17th IFAC World Congress , Seoul, Korea, 175-182, 2008.

[4] J. Hauser, S. Sastry, and P. Kokotovi´c, *Nonlinear control via approximate input output linearization: The ball and beam example,* IEEE Transactions on Automatic Control, vol. 37, no. 3, pp. 392–398, 1992.

[5] Michel Verhaegen, Vincent Verdult, and Niek Bergboer, *Filtering and System Identification: An Introduction to using Matlab Software*, 2007.

[6] B. Jakubczyk and E. Sontag, *Controllability of nonlinear discrete time systems: An Algebraic approach,* SIAM Journal of Control and Optimization, 1-33, 1999.

[7] Jerzy Klamka, *Controllability of non linear discrete systems,* Int. J. Appl. Math. Comput. Sci., Vol.12, No.2, 173–180, 2002.

[8] U. Kotta, *Towards a solution of the state space realization problem of a set of multi input multi output nonlinear difference equations*, Proceedings of the European Control conference, Germany 2001.

[9] U. Kotta , A. S.I .Zinober and P.Liu, *Transfer equivalence and realization of nonlinear higher order input output difference equations,* Applicable Algebra in Engineering, Communications and Computing, Vol 4 , 2002

[10] U. Kotta, Z. Bartosiewicz, E. Pawłuszewicz, and M. Wyrwas, *Irreducibility, reduction and transfer equivalence of nonlinear input-output equations on homogeneous time scales,* Systems and Control, vol. 58, no. 9, pp. 646–651, 2009.

[11] X. Lu and D. Bell, *Realization theory for differential algebraic input output systems,* IMA Journal of Mathematical Control & Information, vol. 10, no. 1, pp. 33–47, 1993.

[12] Paulraj.A.J, Gore, Nabar and Bolcskei, .*An Overview of MIMO Communications – A key to Gigabit wireless, IEEE Trans Comm, 449-461, 2003.*

[13] N. Sadegh. *Realization of nonlinear systems described by input output difference equations*, Proceedings of the American Control Conference, 1998

[14] N. Sadegh, *Minimal realization of nonlinear systems described by input-output difference equations*, IEEE Transactions on Automatic Control, vol. 46, no. 5, pp. 698–710, 2001.