A Study on Intuitionistic L-Fuzzy Translation

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ABSTRACT

This paper contains some definitions and results in intuitionistic L-fuzzy translation of Intuitionistic L-fuzzy M-subgroup of a M-group, which are required in the sequel. Some properties of homomorphism and anti-homomorphism of Intuitionistic L-fuzzy translation are also established.

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L-fuzzy set, L-fuzzy M-subgroup, Homomorphism, Anti-homomorphism, Anti L-fuzzy M-subgroup, intuitionistic L-fuzzy M-subgroups, intuitionistic L-fuzzy translation.

1. INTRODUCTION:

The notion of fuzzy sets was introduced by **L.A. Zadeh** [9]. Fuzzy set theory has been developed in many directions by many researchers and has evoked great interest among mathematicians working in different fields of mathematics, such as topological spaces, functional analysis, loop, group, ring, near ring, vector spaces, automation. In 1971, **Rosenfield** [1] introduced the concept of fuzzy subgroup. Motivated by this, many mathematicians started to review various concepts and theorems of abstract algebra in the broader frame work of fuzzy settings. In [2], **Biswas** introduced the concept of anti-fuzzy subgroups of groups. **Palaniappan. N** and **Muthuraj,** [6] defined the homomorphism, anti-homomorphism of a fuzzy and an anti-fuzzy subgroups. **Pandiammal. P, Natarajan. R,** and **Palaniappan. N,** [8] defined the homomorphism, anti-homomorphism of an anti L-fuzzy M-subgroup. In this paper we define a new algebraic structure of intuitionistic L-fuzzy translation of intuitionistic L-fuzzy M-subgroup of a M-group and study some their related properties.

2. PRELIMINARIES:

2.1 Definition: Let G be a M-group. A L-fuzzy subset A of G is said to be **anti L-fuzzy M-subgroup** (ALFMSG) of G if its satisfies the following axioms:

(i) $\mu_A(\text{ mxy }) \leq \mu_A(x) \vee \mu_A(y)$,

(ii) $\mu_A(x^{-1}) \leq \mu_A(x)$, for all x and y in G.

2.2 Definition: Let (G, ·) be a M-group. An intuitionistic L-fuzzy subset A of G is said to be an **intuitionistic L-fuzzy M-subgroup (ILFMSG)** of G if the following conditions are satisfied:

(i) $\mu_A(\text{ mxy }) \geq \mu_A(x) \wedge \mu_A(y)$,

(ii) $\mu_A(x^{-1}) \geq \mu_A(x)$,

(iii) $\nu_A(\text{ mxy }) \leq \nu_A(x) \vee \nu_A(y)$,

(iv) $\nu_A(x^{-1}) \leq \nu_A(x)$, for all x and y in G.

2.3 Definition: Let A be an intuitionistic L-fuzzy subset of X and α and β in $[\,0, 1-\text{Sup}\{\,\mu_A(x) + \nu_A(x) : x \in X,\ 0 < \mu_A(x) + \nu_A(x) < 1\,\}]$. Then T = $T^{A}_{(\alpha,\beta)}$ is called an intuitionistic L-fuzzy translation of A if $\mu_T(x) = \mu^{A}_{\alpha}(x) = \mu_A(x) + \alpha$, $\nu_T(x) = \nu^{A}_{\beta}(x) = \nu_A(x) + \beta$, $\alpha+\beta \leq 1-\text{Sup}\{\mu_A(x) + \nu_A(x) : x \in X,\ 0 < \mu_A(x) + \nu_A(x) < 1\}$, for all x in X.

Remark 2.4 : When $\mu A(x) + \nu A(x) = 1$, i.e. when $\nu A(x) = 1 - \mu A(x) = \mu Ac(x)$.

Then A is called **fuzzy set.**

Example 2.5: Let X = {1, ω, ω2 }. Let A = {< 1, 0.3, 0.4 > , < ω ,0.1, 0.25 > ,

< ω2 ,0.5,0.3 >} be an IFS of X . Then [0 ,1 - Sup{ μ A(x) + ν A(x): x in X,

0 < μ A(x) + ν A(x) < 1 }] = [0, 0.2]. Take α = 0.1 and β = 0.2.Then ILFT of the

IFS A is given by T = {< 1, 0.16, 0.18>, < ω, 0.12, 0.15 >, < ω2 , 0.11, 0.16 >}

3. PROPERTIES OF INTUITIONISTIC L-FUZZY TRANSLATION:

3.1 Theorem: If T is an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy M-subgroup A of a M-group G, then $\mu_T(x^{-1}) = \mu_T(x)$ and $\nu_T(x^{-1}) = \nu_T(x)$, $\mu_T(x) \leq \mu_T(e)$ and $\nu_T(x) \geq \nu_T(e)$, for all x and e in G.

Proof: Let x and e be elements of G.

Now,

$$\begin{aligned}\mu_T(x) &= \mu_A(x) + \alpha \\ &= \mu_A((x^{-1})^{-1})+\alpha \\ &\geq \mu_A(x^{-1}) + \alpha = \mu_T(x).\end{aligned}$$

Therefore, $\mu_T(x) = \mu_T(x^{-1})$, for x in G.

And,

$$\begin{aligned}\nu_T(x) &= \nu_A(x)+\beta \\ &= \nu_A((x^{-1})^{-1} \\ &= \nu_T(x).\end{aligned}$$

Therefore, $\nu_T(x) = \nu_T(x^{-1})$, for x in G.

Now,

$$\begin{aligned}\mu_T(e) &= \mu_A(e) + \alpha \\ &= \mu_A(xx^{-1}) + \alpha \\ &\geq \{\,\mu_A(x) \wedge \mu_A(x^{-1})\,\}+ \alpha\end{aligned}$$

$= \mu_A(x) + \alpha = \mu_T(x)$. Therefore, $\mu_T(e) \geq \mu_T(x)$,for x in G.

And $\nu_T(e) = \nu_A(e) + \beta$

$= \nu_A(xx^{-1}) + \beta$

$\leq \{ \nu_A(x) \vee \nu_A(x^{-1}) \} + \beta$

$= \nu_A(x) + \beta = \nu_T(x)$.

Therefore, $\nu_T(e) \leq \nu_T(x)$, for x in G.

3.2 Theorem: If T is an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy M-subgroup A of a M-group G, then

(i)$\mu_T(xy^{-1}) = \mu_T(e)$ implies $\mu_T(x) = \mu_T(y)$,

(ii)$\nu_T(xy^{-1}) = \nu_T(e)$ implies $\nu_T(x) = \nu_T(y)$, for all x, y and e in G.

Proof: Let x, y and e be elements of G.

Now, $\mu_T(x) = \mu_A(x) + \alpha$

$= \mu_A(xy^{-1}y) + \alpha$

$\geq (\mu_A(xy^{-1}) + \alpha) \wedge (\mu_A(y) + \alpha)$

$=\mu_T(xy^{-1}) \wedge \mu_T(y)$

$= \mu_T(e) \wedge \mu_T(y) = \mu_T(y)$

$= \mu_A(y)+\alpha= \mu_A(yx^{-1}x) + \alpha$

$\geq \{ \mu_A(yx^{-1}) \wedge \mu_A(x) \} + \alpha$

$= (\mu_A(yx^{-1}) +\alpha \wedge (\mu_A(x) + \alpha)$

$= \mu_T(yx^{-1}) \wedge \mu_T(x)$

$= \mu_T(e) \wedge \mu_T(x) = \mu_T(x)$.

Therefore, $\mu_T(x) = \mu_T(y)$, for all x & y in G.

Similarly, $\nu_T(x) = \nu_T(y)$, for all x & y in G.

3.3 Theorem: If T is an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy M-subgroup A of a M-group G, then T is an intuitionistic L-fuzzy M-subgroup of a M-group G, for all x and y in G.

Proof: Assume that T is an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy M-subgroup A of a M-group G. Let x and y in G.

We have, $\mu_T(mxy^{-1}) = \mu_A(mxy^{-1}) + \alpha$

$\geq\{ \mu_A(x) \wedge \mu_A(y^{-1})\} + \alpha = \{\mu_A(x) \wedge \mu_A(y)\} + \alpha = \mu_T(x) \wedge \mu_T(y)$.

Therefore, $\mu_T(mxy^{-1}) \geq \mu_T(x) \wedge \mu_T(y)$, for all x and y in G .

And,$\nu_T(mxy^{-1}) = \nu_A(mxy^{-1}) + \beta$

$\leq \{ \nu_A(x) \vee \nu_A(y^{-1}) \} + \beta$

$= \{ \nu_A(x) \vee \nu_A(y) \} + \beta = \nu_T(x) \vee \nu_T(y)$.

Therefore, $\nu_T(mxy^{-1}) \leq \nu_T(x) \vee \nu_T(y)$, for all x and y in G.

Hence T is an intuitionistic L-fuzzy M-subgroup of a M-group G.

3.4 Theorem: If T is an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy M-subgroup A of a M-group G, then H = { $x\in G : \mu_T(x) = \mu_T(e)$ and $\nu_T(x) = \nu_T(e)$ } is a M-subgroup of G.

Proof: Let x, y and e be elements of G.

Given H = { $x\in G : \mu_T(x) = \mu_T(e)$ and $\nu_T(x) = \nu_T(e)$ }.

Now, $\mu_T(x^{-1}) = \mu_T(x) = \mu_T(e)$ and $\nu_T(x^{-1}) = \nu_T(x) = \nu_T(e)$.

Therefore, $\mu_T(x^{-1}) = \mu_T(e)$ and $\nu_T(x^{-1}) = \nu_T(e)$. Therefore, $x^{-1}\in$ H.

Now, $\mu_T(xy^{-1}) \geq \mu_T(x) \wedge \mu_T(y)$

$= \mu_T(e) \wedge \mu_T(e)$

$= \mu_T(e)$,

and $\mu_T(e) = \mu_T((xy^{-1})(xy^{-1})^{-1})$

$\geq \mu_T(xy^{-1}) \wedge \mu_T(xy^{-1})$

$= \mu_T(xy^{-1})$.

Therefore, $\mu_T(e) = \mu_T(xy^{-1})$, for all x and y in G.

Similarly,$\nu_A(e)=\nu_A(xy^{-1})$, for all x & y in G.

Therefore, xy^{-1} in H. Hence H is a M-subgroup of G.

3.5 Theorem: Let T be an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy M-subgroup A of a M-group G. If $\mu_T(xy^{-1}) = 1$, then $\mu_T(x) = \mu_T(y)$ and if $\nu_T(xy^{-1}) = 0$, then $\nu_T(x) = \nu_T(y)$.

Proof: Let x and y be elements of G.

Now, $\mu_T(x) = \mu_T(xy^{-1}y)$

$\geq \mu_T(xy^{-1}) \wedge \mu_T(y)$

$= 1 \wedge \mu_T(y)$

$= \mu_T(y) = \mu_T(y^{-1})$

$= \mu_T(x^{-1}xy^{-1})$

$\geq \mu_T(x^{-1}) \wedge \mu_T(xy^{-1})$

$= \mu_T(x) \wedge \mu_T(xy^{-1})= \mu_T(x)$.

Therefore, $\mu_T(x) = \mu_T(y)$, for all x & y in G.

Now, $\nu_T(x) = \nu_T(xy^{-1}y) \leq \nu_T(y)$

$= \nu_T(y^{-1})$

$= \nu_T(x^{-1}xy^{-1})$

$\leq\nu_T(x^{-1})\vee\nu_T(xy^{-1})$

$=\nu_T(x)\vee\nu_T(xy^{-1})$

$=\nu_T(x)\vee 0= \nu_T(x)$

Therefore,$\nu_T(x)=\nu_T(y)$,for all x & y in G.

3.6 Theorem: Let G be a M-group. If T is an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy M-subgroup A of G, then $\mu_T(xy) = \mu_T(x) \wedge \mu_T(y)$ and $\nu_T(xy) = \nu_T(x) \vee \nu_T(y)$, for each x and y in G with $\mu_T(x) \neq \mu_T(y)$ and $\nu_T(x) \neq \nu_T(y)$.

Proof: Let x and y be elements of G.

Assume that $\mu_T(x) > \mu_T(y)$ and $\nu_T(x) < \nu_T(y)$.

Then, $\mu_T(y) = \mu_T(x^{-1}xy)$

$\geq \mu_T(x^{-1}) \wedge \mu_T(xy) = \mu_T(x) \wedge \mu_T(xy)$

$= \mu_T(xy)$

$\geq \mu_T(x) \wedge \mu_T(y) = \mu_T(y)$.

Therefore, $\mu_T(xy) = \mu_T(y) = \mu_T(x) \wedge \mu_T(y)$, for all x and y in G.

Then, $\nu_T(y) = \nu_T(x^{-1}xy)$

$\leq \nu_T(x^{-1}) \vee \nu_T(xy)$

$= \nu_T(xy)$

$\leq \nu_T(x) \vee \nu_T(y) = \nu_T(y)$.

Therefore, $\nu_T(xy) = \nu_T(y) = \nu_T(x) \vee \nu_T(y)$, for all x and y in G.

3.7 Theorem: Let (G,•) and (G^{l}, •) be any two M-groups. If f : G→ G^{l} is a homomorphism, then the homomorphic image (pre-image) of an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy M-subgroup A of a M-group G is an intuitionistic L-fuzzy M-subgroup of a M-group G^{l}.

Proof: Let (G,•) and (G^{l}, •) be any two M-groups and f : G→ G^{l} be a homomorphism. That is f(x y) = f(x) f(y), f(mx y) = mf(x) f(y), for all x and y in G and m in M.

Let V = f($T^{A}_{(\alpha,\beta)}$), where $T^{A}_{(\alpha,\beta)}$ is an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy M-subgroup A of a M-group G.

We have to prove that V is an intuitionistic L-fuzzy M-subgroup of a M-group G^{l}.

Now, for f(x) and f(y) in G^{l}, we have

$\mu_V[mf(x)(f(y)^{-1})] = \mu_V[mf(x) f(y^{-1})]$

$= \mu_V [f(mx y^{-1})]$

$\geq \mu^{A}_{\alpha} (mxy^{-1}) = \mu_A (mx y^{-1})+ \alpha$

$\geq \{\mu_A(x) \wedge \mu_A(y^{-1}) \} +\alpha \geq \{ \mu_A(x) \wedge \mu_A(y) \} + \alpha = (\mu_A(x) +\alpha) \wedge (\mu_A(y)+\alpha) = \mu^{A}_{\alpha} (x) \wedge \mu^{A}_{\alpha} (y)$.

which implies that $\mu_V[mf(x) (f(y)^{-1})] \geq \mu_V(f(x)) \wedge \mu_V(f(y))$, for all f(x) and f(y) in G^{l}.

Similarly, V is an intuitionistic L-fuzzy M-subgroup of a M-group G^{l}.

Hence the homomorphic image of an intuitionistic L-fuzzy translation of A of G is an intuitionistic L-fuzzy M-subgroup of a M-group G^{l}.

3.8 Theorem: Let (G,•) and (G^{l}, •) be any two M-groups. If f : G→ G^{l} is an anti-homomorphism, then the anti-homomorphic image (pre-image) of an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy normal M-subgroup A of a M-group G is an intuitionistic L-fuzzy normal M-subgroup of a M-group G^{l}.

Proof: Let (G,•) and (G^{l}, •) be any two M-groups and f : G→ G^{l} be an anti-homomorphism.

That is f(xy) = f(x)f(y), f(mxy) = mf(x) f(y), for all x and y in G and m in M.

Let V = f($T^{A}_{(\alpha,\beta)}$), where $T^{A}_{(\alpha,\beta)}$ is an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy normal M-subgroup A of a M-group G.

We have to prove that V is an intuitionistic L-fuzzy normal M-subgroup of a M-group G^{l}.

Now, for f(x) and f(y) in G^{l},clearly V is an intuitionistic L-fuzzy M-subgroup of a M-group G^{l}. We have, $\mu_V(mf(x)f(y)) = \mu_V(f(myx))$

$\geq \mu_T(myx)$

$= \mu_A(myx) + \alpha$

$= \mu_A(mxy) + \alpha$

$= \mu_T(mxy)$

$\leq \mu_V(f(mxy))$

$= \mu_V(mf(y) f(x))$

which implies that $\mu_V(mf(x)f(y)) = \mu_V(mf(y) f(x))$, for f(x) and f(y) in G^{l}.

And, $\nu_V(mf(x) f(y)) = \nu_V(f(myx))$

$\leq \nu_T(myx)$

$= \nu_A(myx)+\alpha$

$= \nu_A(mxy) + \alpha$

$= \nu_T(mxy)$

$\geq \nu_V(f(mxy))$

$= \nu_V(mf(y) f(x))$

which implies that $\nu_V(mf(x)f(y)) = \nu_V(mf(y) f(x))$, for f(x) and f(y) in G^{l}.

Therefore, V is an intuitionistic L-fuzzy normal M-subgroup of a M-group G^{l}.

Hence the anti-homomorphic image of an intuitionistic L-fuzzy translation of A of a M-group G is an intuitionistic L-fuzzy normal M-subgroup of a M-group G^{l}.

4. CONCLUSION

In this paper, we define a new algebraic structure of Intuitionistic L-fuzzy translation and Homomorphism and anti-homomorphism of Intuitionistic L-fuzzy Translation, we wish to define Level subset of Intuitionistic L-fuzzy Translatin and other some L-fuzzy Translation are in progress.

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