A Bi-fuzzy Approach to a Production-Recycling-Disposal Inventory Problem with Environment Pollution Cost via Genetic Algorithm

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ABSTRACT

This paper develops a production, recycling-disposal inventory problem over a finite time horizon in fuzzy and bi-fuzzy environments. The production and recycling process are performed in a plant which is located very near to the market. The products of the plant are continuously transfer to the market. Here, the dynamic demand is satisfied by production and recycling. The use units are collected continuously from the customers and then either recycled or disposed. Recycling products can be used as new products which are sold again in the market. The rate of production, recycling and disposal are assumed to be function of time. The setup cost, idle cost and environment pollution recovery cost for production-recycling system in industry are also included. The optimum results are presented both in tabular form and graphically.

Keywords:

Production, Recycling-Disposal, Idle cost, Environment pollution cost, Bi-fuzzyifx

1. INTRODUCTION

During the last few decades, production-recycling system is an important area of inventory studies due to growing environmental concern and environmental regulations like 'Kyoto Protocol'in industry. Fig-3 represents a single plant production-recycling-disposal system in fuzzy and bi-fuzzy environments. Some units are bought back from market for a recoverable inventory after using by the customers. The serviceable stock is delivered to the customer demand. The serviceable stock can be satisfied by either production or by recycling. The non-recycling items are disposed. The non-serviceable stock is filled up by used products from the customers. The non-serviceable stock is supplied for either recycling or disposal. A number of research papers have already been published on the above type of models by (cf. Minner and Kleber (2001), Dobos and Richter (2006), Maity et al.(2009), Ilgin and Gupta(2010), Taleizadeh et. al. (2012) and others).

In the classical inventory models, normally static lot size models are formulated. But in the manufacturing environment, the static models are not adequate in analyzing the behavior of such systems and in deriving the policies for their control. Moreover it is usually observed in the market that sales of the fashionable goods, electronic gadgets, seasonable products, food grains, etc., increases with time. For these reasons, dynamic models of production inventory systems have been considered and solved by some researchers (cf. Giri and Chaudhuri (1998), Maity and Maiti (2005a) and others). In these models, demand is assumed to be continuous functions of time and till now, only a very few researchers (cf. Maity and Maiti (2005b)) have taken time dependent production function.

Here, production cost has three parts. (i) raw material cost which is constant per unit product (ii) Labour cost which is inversely proportional per unit product (iii) Environmental pollution cost is proportional to the product. The cost is expenditure due to growing environmental concern (cf. Zhang et al. (1997), Gungor and Gupta(1999), Ilgin and Gupta(2010)) and according to the rule of environmental regulations like 'Kyoto Protocol' for Industry. The 'Kyoto Protocol' was established in 1997 at Kyoto, Japan. The purpose of 'Kyoto Protocol' are (i) Clean Development Mechanism (ii) Scientific efforts aimed at understanding detail of total net carbon exchange (iii)Global-warming potential (iv) Campus carbon neutrality (v) Carbon sequestration in terrestrial ecosystems, etc. So environmental pollution cost is an important part in production system for industry. Also the recycling process can reduce the environmental pollution, save the crisis natural resources and raw materials. Therefore, the recycling process can play an important role in industry.

Prof. Zadeh(1965) first applied a new concept Fuzzy Set Theory to accommodate the uncertainty in non-stochastic sense. After that, Liu and Liu(2003) have developed a class of fuzzy ramdom optimization: expected value models. Maity et al. (2008) have developed a production recycling model with imprecise holding cost of defective units is a natural phenomenon in a production process. Bi-fuzzy sets were originally presented by Zadeh (1971) and were further elaborately by Gottwald(1979), Mendel(2002), Marusak(2009). Till now, none has considered a two plant optimal production-inventory system producing random defective units of an item with a fuzzy resource constraint and some imprecise inventory costs via optimal control theory.

Genetic Algorithms are exhaustive search algorithms based on the mechanics of natural selection and genesis (crossover, mutation etc.)(cf. Goldberg (1989), Maiti and Maiti(2007) and others). Because of its generality, it has been successfully applied to many optimization problems, for its several advantages over conventional optimization methods. Holland was inspired by Darwin's theory about evolution and constructed GAs based upon the fundamental principle of the theory: 'Survival of the fittest'. The theoretical basis for the GA is the Schema Theorem which states that individual chromosomes with short, low-order, highly fit schemata or building blocks receive an exponentially increasing number of trials in successive generations. In natural genesis, we know that chromosomes are the main carriers of hereditary information from parent to offspring and that genes, which present hereditary factors, are lined up on chromosomes. At the time of reproduction, crossover and mutation take place among the chromosomes of parents. In this way hereditary factors of parents are mixed-up and carried to their offsprings. Again according to Darwinian principle, only the fittest animals can survive in nature. So a pair of fittest parent normally reproduces a better offspring.

In this paper, the production and disposal rates are function of time and unknown taken as control variables. Moreover, the recycling rate is unknown constant and control variable. Here, production cost is greater than the recycling cost. Also non-serviceable holding cost is less than serviceable holding cost. The total cost is minimized and solved by genetic algorithm technique. None has considered a plants production, recycling-disposal inventory problem in fuzzy and bi-fuzzy environments. The optimum production, recycling-disposal and stock levels are determined for known dynamic demand function. The model is illustrated through numerical examples and results are also presented graphically.

2. NECESSARY KNOWLEDGE ABOUT FUZZY AND BI-FUZZY SETS

A fuzzy set is a class of objects in which there is no sharp boundary between those objects that belong to the class and those that do not. Let X be a collection of objects and x be an element of X, then a fuzzy set $\widetilde{A}$ in X is a set of ordered pairs $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x))/x \in X\}$
where $\mu_{\widetilde{A}}(x)$ is called the membership function or grade of membership of x in $\tilde{A}$ which maps X to the membership space M which is considered as the closed interval [0,1].

Fuzzy Number: A fuzzy number $\widetilde{A}$ is a convex normalized fuzzy set on real line $\Re$ such that
(i) it exists exactly one $x_0 \in \Re$ with $\mu_{\widetilde{A}}(x_0) = 1$ (x_0 is called the mean value of $\widetilde{M}$),
(ii) $\mu_{\widetilde{A}}(x)$ is piecewise continuous.

Example 1: In particular if $\widetilde{A} = (a_1, a_2, a_3)$ be a Triangular Fuzzy Number (TFN)(cf. Fig. 1) then $\mu_{\widetilde{A}}(x)$ is defined as follows

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & \text{for } a_1 \le x < a_2 \\ \frac{a_3-x}{a_3-a_2} & \text{for } a_2 < x \le a_3 \\ 0 & \text{otherwise} \end{cases}$$

where a_1, a_2 and a_3 are real numbers.

Lemma-1: Let $\tilde{a} = (a_1, a_2, a_3))$ be a triangular fuzzy number and r is a crisp number. The expected value of $\tilde{a}$ is

$$\begin{aligned} E[\tilde{a}] &= \frac{1}{2}\left[(1-\rho)a_1 + a_2 + \rho a_3\right], \ 0 < \rho < 1. \\ &= \frac{a_1 + 2a_2 + a_3}{4}, \qquad \rho = 0.5. \end{aligned} \tag{1}$$

Proof: The Proof of the lemma-1 is in reference in Liu and Liu(2002).

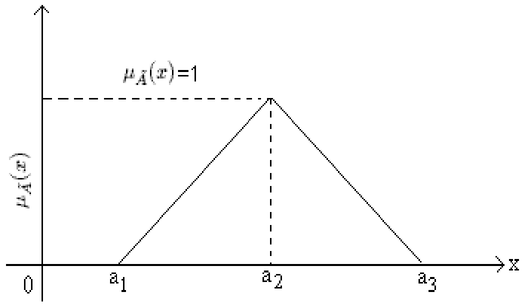

Fig. 1. Triangular Fuzzy Number(TFN)

2.1 Bi-fuzzy set

Generally speaking, a level-2 fuzzy set is a fuzzy set in which the elements are also fuzzy sets, and the bi-fuzzy variable is a fuzzy variable with fuzzy parameters. Level-2 fuzzy sets were originally presented by Zadeh(1971). Such sets are fuzzy sets whose elements themselves are ordinary fuzzy sets. They are very useful in circumstances where it is difficult to determine some elements for a fuzzy set.

Definition 1 In Mendel (2002), a type-2 fuzzy set, denoted $\tilde{A}$, is characterized by a type-2 membership function $\mu_{\tilde{A}}(x, u)$, where $x \in X$ and $u \in J)x \subseteq [0, 1]$, i.e.,

$$\tilde{V} = \{\tilde{V}, \mu_{\tilde{V}}(\tilde{V})) \mid \forall x \in \tilde{\Gamma}(U) : \mu_{\tilde{V}} > 0\} \tag{2}$$

where each ordinary fuzzy set $\tilde{V}$ is defined by

$$\tilde{V} = \{(x, \mu_{\tilde{V}}(x)) \mid \forall x \in U : \mu_{\tilde{V}} > 0\} \tag{3}$$

For convenience, the membership grades $\mu_{\tilde{V}}(\tilde{V})$ of the fuzzy sets $.\tilde{V} \in \tilde{\Gamma}(U)$ are called 'outer-layer' membership grades, whereas the membership grades $\mu_{\tilde{V}}(\tilde{x})$ of the elements $x \in U$ are called inner-layermembership grades. Since level-2 fuzzy sets are still fuzzy sets, their mathematical behavior is defined by the fuzzy set operators. Type-2 fuzzy sets were introduced by Zadeh(1975) as another extension of the concept of an ordinary fuzzy set, and it was elaborated by Mendel(2002). Such sets are fuzzy sets whose membership grades them as ordinary fuzzy sets. They are very useful in circumstances where it is difficult to determine an exact membership function for a fuzzy set.Normally speaking, a Fu-Fu variable is a fuzzy variable under fuzzy environment.

Example 2: $\tilde{\tilde{\xi}} = (s_L, \tilde{\xi}, s_R)$ with $\rho = (\rho_L, \rho_M, \rho_R)$ is called Fu-Fu variable,(cf. Fig. 2), if the outer-layer and inner-layer membership functions are as follows

$$\mu_{\tilde{\tilde{\xi}}}(x) = \begin{cases} \left(\dfrac{x - s_L}{\tilde{\rho} - s_L}\right) & \text{if } s_L \le x \le \tilde{\rho} \\ 0 & \text{otherwise} \\ \left(\dfrac{s_R - x}{s_R - \tilde{\rho}}\right) & \text{if } \tilde{\rho} \le x \le s_R \end{cases}$$

and

$$\mu_{\tilde{\rho}}(x) = \begin{cases} \left(\dfrac{x' - \rho_L}{\rho_M - \rho_L}\right) & \text{if } \rho_L \le x' \le \rho_M \\ 0 & \text{otherwise} \\ \left(\dfrac{\rho_R - x'}{\rho_R - \rho_M}\right) & \text{if } \rho_M \le x' \le \rho_R \end{cases}$$

where $\tilde{\rho}$ is the center of $\tilde{\xi}$, which is a triangular fuzzy variable, $s_L \in R$ and $s_R \in R$ are the smallest possible value and the largest possible value of $\tilde{\tilde{\xi}}$, $s_L \in R$, $s_M \in R$ and $s_R \in R$ are the the smallest possible value, the most promising value and the largest possible value of $\tilde{\rho}$, respectively.

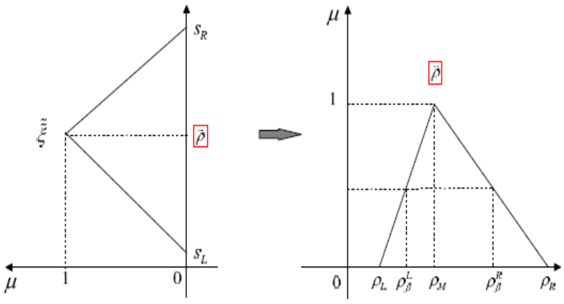

Fig. 2. Triangular Bi-fuzzy variable

Lemma-2: The expected value for the bi-fuzzy variable $\tilde{\tilde{c}} = (\tilde{c} - l_1, \tilde{c}, \tilde{c} + r_1)$ with $\tilde{c} = (c - l_2, c, c + r_2)$ we obtain that

$$E[\tilde{\tilde{c}}] = c + \frac{(r_1 + r_2) - (l_1 + l_2)}{4} \tag{4}$$

Proof: Let $\tilde{\tilde{c}} = (\tilde{c} - l_1, \tilde{c}, \tilde{c} + r_1)$, where $\tilde{c} = (c - l_2, c, c + r_2)$. Therefore

$$\begin{aligned} E(\tilde{\tilde{c}}) &= \frac{E(\tilde{c} - l_1) + 2E(\tilde{c}) + E(\tilde{c} + r_1)}{4} \text{ (UsingLamma} - 1) \\ &= \frac{E(\tilde{c}) - l_1 + 2E(\tilde{c}) + E(\tilde{c}) + r_1}{4} \\ &= \frac{4E(\tilde{c}) - l_1 + r_1}{4} \\ &= E(\tilde{c}) + \frac{r_1 - l_1}{4} \\ &= c + \frac{r_2 - l_2}{4} + \frac{r_1 - l_1}{4} \\ &= c + \frac{(r_1 + r_2) - (l_1 + l_2)}{4} \end{aligned}$$

Particular case: When $l_2 = 0 = r_2 \Rightarrow \tilde{\tilde{c}} = \tilde{c} \Rightarrow E(\tilde{\tilde{c}}) = c + \frac{r_1 - l_1}{4}$

Lemma-3: Assume that ξ and η are fuzzy/ bi-fuzzy variables with finite expected values. Then for any real numbers a and b, we have

$$E[a\xi + b\eta] = aE[\xi] + bE[\eta] \tag{5}$$

Proof: The proof of the Lemma-3 is in reference Xu and Zhou (2009).

3. ASSUMPTIONS AND NOTATIONS

The model under consideration is developed with the following assumptions and notations.

3.1 Assumptions:

For the product recycling, disposal model, it is assumed that,

(i) demand rate is known and time dependent,
(ii) shortages are not allowed,
(iii) production rate is time dependent and unknown taken as decision variable ,
(iv) this is a single item inventory model with finite time length,
(v) recycling item is same to a new product, it's rate is constant and unknown taken as decision variable.
(vi) disposal item is rejected unit, it's rate is constant and unknown taken as decision variable .
(vii) lead time is zero,
(viii) all return units have the same level of quality,
(ix) holding cost of non-serviceable item is less than that for serviceable product.
(x) holding cost of non-serviceable item is less than that for serviceable product.
(xi) Unit production cost is less than the unit recycling cost.
(xii) environment pollution cost for production in industry is also included.
(xiii) The holding costs, setup costs and idle costs are taken to be fuzzy in nature.
(xiv) The unit production cost is taken to be bi-fuzzy in nature.

3.2 Notations:

$x_{Si}(t)$ serviceable stock at time t for i^{th} production cycle.
$x_{Sj}(t)$ serviceable stock units at time t for j^{th} production and recycling cycle.
$x_R(t)$ stock of non serviceable units at time t for production cycles.
$x_{Rj}(t)$ stock of non serviceable at time t for j^{th} production and recycling cycle.
$u(t)$ $u_0 + u_1 t$, production rate(decision variable) for each production up to m cycles.
$p(t)$ p_0, constant recycling rate taken as a decision variable.
$u'(t)$ $u'_0 + u'_1 t$, production rate (decision variable) for each production from m+1 cycle to m+n cycle.
$d(t)$ $d_0 - d_1 e^{-\beta t}$, demand function, where d_{0p}, d_{1p} are known and $\beta > 0$. $(\alpha_0 + \alpha_1 t)d(t)$ return function.
z(t) $(z_0 + z_1 t)$, disposal rate(decision variable).
C_p recycling cost per unit.
$\tilde{\tilde{C}}_u$ $\tilde{\tilde{C}}_{u0} + \frac{\tilde{\tilde{C}}_{u1}}{u(t)} + \tilde{\tilde{C}}_{u2}(u(t))^{\gamma_1} + \tilde{\tilde{C}}_{u3}(u(t))^{\gamma_2}$, bi-fuzzy production cost per unit.
Where $\tilde{\tilde{C}}_{u0}$ is bi-fuzzy raw material cost,
$\tilde{\tilde{C}}_{u1}$ is bi-fuzzy labour cost,
$\tilde{\tilde{C}}_{u2}$ is bi-fuzzy wear and tear cost,
$\tilde{\tilde{C}}_{u3}$ is bi-fuzzy environmental pollution cost and $\gamma_i, i = 1, 2$ are positive constants.
C_r purchasing cost per recovered item.
C_z disposal cost per unit item.
$\tilde{h}_R$ fuzzy holding cost of non-serviceable product per unit per unit time.

$\tilde{h}_S$ fuzzy holding cost of serviceable product per unit per unit time.
$\tilde{s}_u$ fuzzy setup cost for the first i^{th} production cycle.
$\tilde{s}_p$ fuzzy setup cost for the first j^{th} recycling cycle.
m number of only production cycles, positive integer.
n number of production and recycling cycles, positive integer.
t_u time interval of each production cycle.
t_p time interval of each production and recycling cycle.
$t^{'}_{ui}$ time duration of production for i^{th} production cycle.
$t^{'}_{pj}$ time duration of production for j^{th} production and recycling cycle.
$\tilde{Id}_u$ fuzzy idle cost for each production cycle.
$\tilde{Id}_p$ fuzzy idle cost for each recycle cycle.

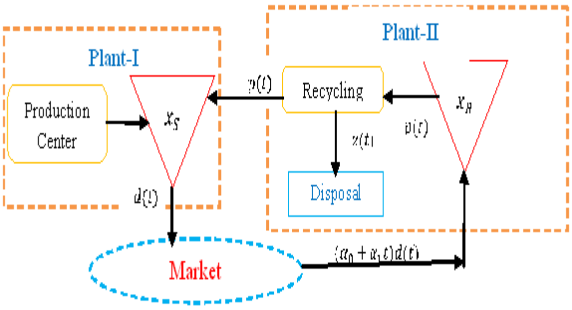

Fig. 3. Block diagram for production, recycling and disposal model

4. RECYCLING MODEL FORMULATION IN FUZZY AND BI-FUZZY ENVIRONMENTS

This paper develops a single plant production, recycling-disposal system over a finite time horizon in fuzzy and bi-fuzzy environments. The holding costs, setup costs, idle costs are fuzzy in nature. But the production cost is bi-fuzzy in nature as the purchasing of raw materials faces is how to make purchasing decisions, in order to obtain required raw materials at a lower price and at the same time meet production demand in terms of item, quality, quantity, due date, and so on.

The production and the recycling process are performed in a plant which is located very near to the market and the products of plants are continuously transfer to the market. Here, the dynamic demand is satisfied by production and recycling. The used units are bought back and then either recycled or disposed in the said plant. The use units are collected continuously from the customers. Recycling products can be used as new products which are sold again. The rate of production, recycling and disposal are assumed to be function of time. The setup cost, idle cost and environment pollution cost for production in industry are also included. The production cost has three parts. (i) raw material cost which is constant per unit product (ii) Labour cost which is inversely proportional per unit product (iii) Environmental pollution cost is proportional to the product. The cost is expenditure due to growing environmental concern and according to the rule of environmental regulations like 'Kyoto Protocol' for Industry. At the beginning, production satisfies the demand. After sometime, production and recycling fill up the demand. The first m cycles are presented for production and next n cycles exist both for production and recycling. The period of each of first m cycle and last n cycle are t_u and t_p respectively. The time interval of each of first m cycles is equal. Similarly the time interval of each of last n cycles is equal. Production takes $t^{'}_{ui}$ duration in $i^{th}, i = 1, 2..., m$ production cycle. Also production and recycling takes $t^{'}_{pj}$ duration in $j^{th}, j = 1, 2, ..., n$ production and recycling cycle. We collect reused product at the rate of $(\alpha_0 + \alpha_1 t)d(t)$ continuously from the market. At the time of collection we also consider the disposal at the rate of $(z_0 + z_1 t)$. The total time horizon $T = mt_u + nt_p$. The optimization problem is to maximize total profit over the finite planning horizon, T and it is given in fig-3.

In plant-I, the bi-fuzzy cost function $\tilde{\tilde{J}}_1$ is given below:

$$\begin{aligned} Min\ \tilde{\tilde{J}}_1 &= \text{bi-fuzzy production costs + fuzzy holding costs for} \\ &+ \text{serviceable stocks fuzzy set up costs + fuzzy idle costs} \\ &= \sum_{i=1}^{m} \int_{(i-1)t_u}^{(i-1)t_u+t^{'}_{ui}} \tilde{\tilde{C}}_u u(t)dt + \sum_{i=1}^{m} \int_{(i-1)t_u}^{it_u} \tilde{h}_S x_{Si}(t)dt \\ &+ m\tilde{S}_u + \sum_{i=1}^{m}(t_u - t^{'}_{ui})\tilde{Id}_u \\ &+ \sum_{j=1}^{n} \int_{mt_u+(j-1)t_p}^{mt_u+(j-1)t_p+t^{'}_{pj}} \tilde{\tilde{C}}_u u^{'}(t) \\ &+ \sum_{j=1}^{n} \int_{mt_u+(j-1)t_p}^{mt_u+jt_p} \tilde{h}_S x_{Sj}(t)dt \\ &+ n\tilde{S}_p + \sum_{j=1}^{n}(t_p - t^{'}_{pj})\tilde{Id}_p \end{aligned} \quad (6)$$

In plant-II, the bi-fuzzy cost function $\tilde{\tilde{J}}_2$ is given below:

$$\begin{aligned} Min\ \tilde{\tilde{J}}_2 &= \text{fuzzy holding costs for NS stocks} \\ &+ \text{fuzzy recycling stock + collect cost + disposal cost} \\ &= \sum_{i=1}^{m} \int_{(i-1)t_u}^{it_u} \tilde{h}_R x_{Ri}(t)\}dt \\ &+ \sum_{j=1}^{n} \int_{mt_u+(j-1)t_p}^{mt_u+jt_p} \tilde{h}_R x_{Rj}(t)dt \\ &+ \sum_{j=1}^{n} \int_{mt_u+(j-1)t_p}^{mt_u+(j-1)t_p+t^{'}_{pj}} C_p p(t)dt \\ &+ \int_0^T \Big[C_r(\alpha_0 + \alpha_1 t)d(t) + C_z z(t)\Big]dt \end{aligned} \quad (7)$$

subject to

$$\frac{dx_{Si}(t)}{dt} = \begin{cases} u(t) - d(t) & \text{if}(i-1)t_u \le t \le (i-1)t_u + t^{'}_{ui} \\ -d(t) & \text{if}(i-1)t_u + t^{'}_{ui} \le t \le it_u \end{cases} \quad (8)$$

$$\frac{dx_{Sj}(t)}{dt} = \begin{cases} u^{'}(t) + p(t) - d(t) & \text{if} mt_u + (j-1)t_p \le t \le mt_u + (j-1)t_p + t^{'}_{pj} \\ -d(t) & \text{if} mt_u + (j-1)t_p + t^{'}_{pj} \le t \le mt_u + jt_p \end{cases} \quad (9)$$

$$\frac{dx_R(t)}{dt} = \Big\{ (\alpha_0+\alpha_1 t)d(t) - z(t)\,\text{if} 0 \le t \le mt_u \quad (10)$$

$$\frac{dx_{Rj}(t)}{dt} = \begin{cases} (\alpha_0+\alpha_1 t)d(t) - p(t) - z(t) \\ \text{if} mt_u + (j-1)t_p \le t \le mt_u \\ +(j-1)t_p + t'_{pj} \\ (\alpha_0+\alpha_1 t)d(t) - z(t) \\ \text{if} mt_u + (j-1)t_p + t'_{pj} \le t \le mt_u + jt_p \end{cases} \quad (11)$$

where

$$\begin{aligned} d(t) &= d_0 - d_1 e^{-\beta t}\, x_S(0) = 0 = x_S(it_u), x_S(jt_p) = 0 \\ x_R(0) &= 0, u(t) = u_0 + u_1 t,\, p(t) = p_0, z(t) = z_0 + z_1 t \\ u'(t) &= u'_0 + u'_1 t\, i = 1,2,...,m,\, j = 1,2,...,n. \end{aligned}$$

Where the bi-fuzzy variables are given by $\tilde{\tilde{C}}_{ui} = (\tilde{C}_{ui} - C_{ui1}, \tilde{C}_{ui}, \tilde{C}_{ui} + C_{ui3}), with\ \tilde{C}_{ui} = (C_{ui} - C_{ui2}, C_{ui}, C_{ui} + C_{ui4}), i = 1,2,3.$

And the TFNs are given by

$$\begin{aligned} \tilde{h}_i &= (h_{i1}, h_{i2}, h_{i3}), i = R, S, \\ \tilde{I}d_j &= (Id_{j1}, Id_{j2}, Id_{j3}), j = u, p. \end{aligned}$$

5. EQUIVALENT CRISP MODEL

In plant-I, the expected cost function $E[\tilde{\tilde{J}}_1]$ is given below:

$$\begin{aligned} Min\, E[\tilde{\tilde{J}}_1] &= \sum_{i=1}^{m} \int_{(i-1)t_u}^{(i-1)t_u + t'_{ui}} E[\tilde{\tilde{C}}_u] u(t) dt \\ &+ \sum_{i=1}^{m} \int_{(i-1)t_u}^{it_u} E[\tilde{h}_S]\, x_{Si}(t) dt + \sum_{i=1}^{m} (t_u - t'_{ui}) E[\tilde{I}d_u] \\ &+ mE[\tilde{S}_u] + \sum_{j=1}^{n} \int_{mt_u+(j-1)t_p}^{mt_u+(j-1)t_p+t'_{pj}} E[\tilde{\tilde{C}}_u] u'(t) \\ &+ \sum_{j=1}^{n} \int_{mt_u+(j-1)t_p}^{mt_u+jt_p} E[\tilde{h}_S]\, x_{Sj}(t) dt \\ &+ nE[\tilde{S}_p] + \sum_{j=1}^{n} (t_p - t'_{pj}) E[\tilde{I}d_p] \quad \text{(by lemma-3)} \quad (12) \end{aligned}$$

In plant-II, the expected cost function $E[\tilde{\tilde{J}}_2]$ (by using lemma-3) is given below:

$$\begin{aligned} MinE[\tilde{\tilde{J}}_2] &= \sum_{i=1}^{m} \int_{(i-1)t_u}^{it_u} E[\tilde{h}_R]\, x_{Ri}(t)\} dt \\ &+ \sum_{j=1}^{n} \int_{mt_u+(j-1)t_p}^{mt_u+jt_p} E[\tilde{h}_R]\, x_{Rj}(t) dt \\ &+ \sum_{j=1}^{n} \int_{mt_u(j-1)t_p}^{mt_u+(j-1)t_p+t'_{pj}} C_p p(t) dt \\ &+ \int_0^T \Big[C_r(\alpha_0 + \alpha_1 t) d(t) + C_z\, z(t) \Big] dt \quad (13) \end{aligned}$$

Subject to (8)-(11).

6. SOLUTION METHODOLOGY

Using (12)-(14), from (8) the serviceable stock function for $i^{th}(i = 1,2,...,m)$ production cycle is given by

$$x_{Si}(t) = \begin{cases} (u_0 - d_0)(t - (i-1)t_u) + u_1 \frac{t^2 - ((i-1)t_u)^2}{2} \\ -d_1 \frac{(e^{-\beta t} - e^{-\beta(i-1)t_u})}{\beta} \\ \text{if}(i-1)t_u \le t \le (i-1)t_u + t'_{ui} \\ x_{Si}((i-1)t_u + t'_{ui}) - d_0(t - (i-1)t_u - t'_{ui}) \\ -d_1 \frac{(e^{-\beta t} - e^{-\beta((i-1)t_u + t'_{ui})})}{\beta} \\ \text{if}(i-1)t_u + t'_{ui} \le t \le it_u \end{cases}$$

and using (12)-(14), from (9) the serviceable stock function for $j^{th}(j = 1,2,...,n)$ production and recycling cycle is given by

$$x_{Sj}(t) = \begin{cases} (u'_0 + p_0 - d_0)(t - mt_u - (j-1)t_p) \\ +u'_1 \frac{t^2 - (mt_u + (j-1)t_p)^2}{2} \\ -d_1 \dfrac{(e^{-\beta t} - e^{-\beta(mt_u + (j-1)t_p)})}{\beta} \\ \text{if} mt_u + (j-1)t_p \le t \le mt_u \\ +(j-1)t_p + t'_{pj} x_{Sj}(mt_u + (j-1)t_p) \\ -d_0(t - mt_u - (j-1)t_p - t'_{pj}) \\ -d_1 \dfrac{(e^{-\beta t} - e^{-\beta(mt_u + (j-1)t_p + t'_{pj})})}{\beta} \\ \text{if} mt_u + (j-1)t_p + t'_{pj} \le t \le mt_u + jt_p \end{cases} \quad (14)$$

Again using (12)-(14), from (10) non serviceable stock only production cycles is given by

$$x_R(t) = \begin{cases} (\alpha_0 d_0 - z_0)t + (\alpha_1 d_0 - z_1)\frac{t^2}{2} - \alpha_0 d_1 \frac{(1 - e^{-\beta t})}{\beta} \\ +\alpha_1 d_1 \dfrac{te^{-\beta t}}{\beta} - \alpha_1 d_1 \dfrac{(1 - e^{-\beta t})}{\beta^2} \quad \text{if } 0 \le t \le mt_u \end{cases} \quad (15)$$

and also using (12)-(14), from (11) the non serviceable stock for $j^{th}(j = 1,2,...,n)$ production and recycling cycle is given by

$$x_{Rj}(t) = \begin{cases} x_R(mt_u + (j-1)t_p) + (\alpha_0 d_0 - p_0 - z_0) \\ (t - mt_u - (j-1)t_p) \\ +(\alpha_1 d_0 - z_1)\frac{t^2 - (mt_u + (j-1)t_p)^2}{2} \\ -\alpha_0 d_1 \frac{(e^{-\beta(mt_u + (j-1)t_p)} - e^{-\beta t})}{\beta} \\ +\alpha_1 d_1 \dfrac{(te^{-\beta t} - (mt_u + (j-1)t_p)e^{-\beta(mt_u + (j-1)t_p)})}{\beta} \\ -\alpha_1 d_1 \dfrac{(e^{-\beta(mt_u + (j-1)t_p)} - e^{-\beta t})}{\beta^2} \\ \text{if } mt_u + (j-1)t_p \le t \le mt_u + (j-1)t_p + t'_{pj} \\ x_{Rj}(mt_u + (j-1)t_p + t'_{pj}) + (\alpha_0 d_0 - z_0) \\ (t - mt_u - (j-1)t_p - t'_{pj}) \\ +(\alpha_1 d_0 - z_1)\dfrac{t^2 - (mt_u + (j-1)t_p + t'_{pj})^2}{2} \\ -\alpha_0 d_1 \frac{(e^{-\beta(mt_u + (j-1)t_p + t'_{pj})} - e^{-\beta t})}{\beta} \\ +\alpha_1 d_1 (te^{-\beta t} - (mt_u + (j-1)t_p + t'_{pj}) \\ e^{-\beta(mt_u + (j-1)t_p + t'_{pj})})\frac{1}{\beta} \\ -\alpha_1 d_1 \dfrac{(e^{-\beta(mt_u + (j-1)t_p + t'_{pj})} - e^{-\beta t})}{\beta^2} \\ \text{if } mt_u + (j-1)t_p + t'_{pj} \le t \le mt_u + jt_p \end{cases} \quad (16)$$

In plant-I, the expected cost function $E[\tilde{\tilde{J}}_1]$ is given below:

$$
\begin{aligned}
E[\tilde{\tilde{J}}_1] \;=\; & \sum_{i=1}^{m}\Bigg[E[\tilde{\tilde{C}}_{u0}]\Bigg\{u_0 t'_{ui} + \frac{u_1}{2}\Bigg(((i-1)t_u + t_{ui})^2 \\
- \; & ((i-1)t_u)^2\Bigg)\Bigg\} + E[\tilde{\tilde{C}}_{u1}]t'_{ui} \qquad (17)\\
+ \; & \frac{E[\tilde{\tilde{C}}_{u2}]}{u_1(\gamma_1+1)}\Bigg\{(u_0 + u_1(i-1)t_u + t'_{ui})^{\gamma_1+1} \\
- \; & (u_0 + u_1(i-1)t_u + t'_{ui})^{\gamma_1+1}\Bigg\} \\
+ \; & \frac{E[\tilde{\tilde{C}}_{u2}]}{u_1(\gamma_2+1)}\Bigg\{(u_0 + u_1(i-1)t_u + t'_{ui})^{\gamma_2+1} \\
- \; & (u_0 + u_1(i-1)t_u + t'_{ui})^{\gamma_2+1}\Bigg\} + \qquad (18)\\
& E[\tilde{h}_S]((u_0 - d_0)(\frac{((i-1)t_u + t'_{ui})^2 - ((i-1)t_u)^2}{2} \\
+ \; & \frac{u_1}{2}(\frac{((i-1)t_u + t'_{ui})^3 - ((i-1)t_u)^3}{3} \\
& -((i-1)t_u)^2 t'_{ui}) + \sum_{i=1}^{m}(t_u - t'_{ui})E[\tilde{I}d_u] \\
+ \; & d_1\frac{(e^{-\beta((i-1)t_u + t'_{ui})} - e^{-\beta(i-1)t_u})}{\beta^2} \\
& +d_1\frac{e^{-\beta(i-1)t_u}t'_{ui}}{\beta} + x_{Si}((i-1)t_u + t'_{ui})(t_u - t'_{ui}) \\
- \; & d_0\frac{(it_u)^2 - ((i-1)t_u + t'_{ui})^2}{2} \\
& +d_0\Bigg((i-1)t_u + t'_{ui})(t_u - t'_{ui}) + \sum_{j=1}^{n}(t_p - t'_{pj})E[\tilde{I}d_p]
\end{aligned}
$$

$$
\begin{aligned}
+ \; & d_1\frac{(e^{-\beta(it_u)} - e^{-\beta((i-1)t_u + t'_{ui})})}{\beta^2} \\
+ \; & d_1\frac{e^{-\beta((i-1)t_u + t'_{ui})}(t_u - t'_{ui})}{\beta}\Bigg)\Bigg] + mE[\tilde{s}_u] + nE[\tilde{s}_p] \\
+ \; & \sum_{j=1}^{n}\Bigg[E[\tilde{\tilde{C}}_{u0}]\Bigg\{u'_0 t'_{pj} + \frac{u'_1}{2}\Bigg((t_u + (j-1)t_p + t'_{pj})^2 \\
- \; & (mt_u + (j-1)t_p)^2\Bigg)\Bigg\} + E[\tilde{\tilde{C}}_{u1}]t'_{pj} \\
+ \; & \frac{E[\tilde{\tilde{C}}_{u2}]}{u_1(\gamma+1)}\Bigg\{\Bigg(u'_0 + u'_1(t_u + (j-1)t_p + t'_{pj}\Bigg)^{\gamma+1} \\
- \; & \Bigg(u'_0 + u'_1 mt_p + (j-1)t_u + t'_{pj}\Bigg)^{\gamma+1}\Bigg\} \\
+ \; & E[\tilde{h}_S]((u'_0 + p_0 - d_0)(\frac{(mt_u + (j-1)t_p + t'_{pj})^2}{2} \\
- \; & (mt_u + (j-1)t_p)t'_{pj}) + \frac{u'_1}{2}(\frac{(mt_u + (j-1)t_p + t'_{pj})^3}{3} \\
- \; & ((mt_u + (j-1)t_p)^2 t'_{pj}) + d_1\frac{(e^{-\beta(mt_u + (j-1)t_p + t'_{pj})})}{\beta^2} \\
+ \; & d_1\frac{e^{-\beta(mt_u + (j-1)t_p)}t'_{pj}}{\beta} + x_{Sj}(mt_u + (j-1)t_p + t'_{pj}) \\
- \; & d_0\frac{(mt_u + jt_p)^2 - (mt_u + (j-1)t_p + t'_{pj})^2}{2} \\
+ \; & d_0(mt_u + (j-1)t_p + t'_{pj})(t_p - t'_{pj}) \\
+ \; & d_1\frac{(e^{-\beta(mt_u + jt_p)} - e^{-\beta(mt_u + (j-1)t_p + t'_{pj})})}{\beta^2} \\
+ \; & d_1\frac{e^{-\beta(mt_u + (j-1)t_p + t'_{pj})}(t_p - t'_{pj})}{\beta})\Bigg]
\end{aligned}
$$

In plant-II, the expected cost function $E[\tilde{\tilde{J}}_2]$ is given below:

$$
\begin{aligned}
& Min\, E[\tilde{\tilde{J}}_2] = E[\tilde{h}_R]\Bigg[(\alpha_0 d_0 - z_0)\frac{(mt_u)^2}{2} \\
+ \; & (\alpha_1 d_0 - z_1)\frac{(mt_u)^3}{6} - \frac{\alpha_0 d_1}{\beta}(mt_u - \frac{1 - e^{-\beta mt_u}}{\beta})\Bigg] \\
+ \; & E[\tilde{h}_R]\sum_{j=1}^{n}[\, x_{Rj}(mt_u + (j-1)t_p)t'_{pj} + (\alpha_0 d_0 - p_0 \\
& -z_0)(\frac{(mt_u + (j-1)t_p + t'_{pj})^2 - (mt_u + (j-1)t_p)^2}{2} \\
- \; & (mt_u + (j-1)t_p)t'_{pj}) + (\frac{(\alpha_1 d_0 - z_1)}{2} \\
& (\frac{(mt_u + (j-1)t_p + t'_{pj})^3 - (mt_u + (j-1)t_p)^3}{3} \\
- \; & (mt_u + (j-1)t_p)^2 t'_{pj}) \\
- \; & \alpha_0 d_1 \beta^{-2}(e^{-\beta(mt_u + (j-1)t_p + t'_{pj})} - e^{-\beta(mt_u + (j-1)t_p)}) \\
- \; & \alpha_0 d_1\frac{e^{-\beta(mt_u + (j-1)t_p)}t'_{pj}}{\beta} \\
- \; & \alpha_1 d_1\frac{(mt_u + (j-1)t_p)e^{-\beta(mt_u + (j-1)t_p)}}{\beta}t'_{pj} \\
- \; & \alpha_1 d_1((mt_u + jt_p)e^{-\beta(mt_u + jt_p)} \\
& -(mt_u + (j-1)t_p + t'_{pj})e^{-\beta(mt_u + (j-1)t_p + t'_{pj})}) \\
- \; & \alpha_1 d_1\frac{e^{-\beta(mt_u + (j-1)t_p)}t'_{pj}}{\beta^2} \\
& -\alpha_1 d_1(e^{-\beta(mt_u + (j-1)t_p + t'_{pj})} - e^{-\beta(mt_u + (j-1)t_p)}) \\
+ \; & x_{Rj}(mt_u + (j-1)t_p + t'_{pj})(t_p - t'_{pj}) \\
& -(\alpha_0 d_0 - z_0)(\frac{(mt_u + jt_p)^2 - (mt_u + (j-1)t_p + t'_{pj})^2}{2} \\
- \; & (mt_u + (j-1)t_p + t'_{pj})(t_p - t'_{pj})) \\
& -\frac{(\alpha_1 d_0 - z_1)}{2}(\frac{(mt_u + jt_p)^3 - (mt_u + (j-1)t_p + t'_{pj})^3}{3} \\
- \; & (mt_u + (j-1)t_p + t'_{pj})^2(t_p - t'_{pj})) \\
& -\alpha_0 d_1(e^{-\beta(mt_u + jt_p)} - e^{-\beta(mt_u + (j-1)t_p + t'_{pj})}) \\
- \; & \alpha_0 d_1 e^{-\beta(mt_u + (j-1)t_p + t'_{pj})}(t_p - t'_{pj}) \\
& -\alpha_1 d_1\frac{(mt_u + (j-1)t_p + t'_{pj})e^{-\beta(mt_u + (j-1)t_p + t'_{pj})}}{\beta}(t_p - t'_{pj}) \\
- \; & \alpha_1 d_1 \beta^{-2}((mt_u + jt_p)e^{-\beta(mt_u + jt_p)} \\
& -(mt_u + (j-1)t_p + t'_{pj})e^{-\beta(mt_u + (j-1)t_p + t'_{pj})}) \\
- \; & \alpha_1 d_1\frac{(e^{-\beta(mt_u + jt_p)} - e^{-\beta(mt_u + (j-1)t_p + t'_{pj})})}{\beta^3}
\end{aligned}
$$

$$-\alpha_1 d_1 \frac{e^{-\beta(mt_u+(j-1)t_p+t'_{pj})}(t_p - t'_{pj})}{\beta^2} + s_p j^{b_2}\,]$$
$$+\ C_s(d_0T - d_1\frac{1-e^{-\beta T}}{\beta})$$
$$-C_r\left(\alpha_0 d_0 T + \frac{\alpha_1 d_0 T^2}{2} - \frac{\alpha_0 d_1(1-e^{-\beta T})}{\beta}\right.$$
$$+\ \left.\frac{\alpha_1 d_1 T e^{-\beta T}}{\beta} - \frac{\alpha_1 d_1(1-e^{-\beta T})}{\beta^2}\right) - C_z(z_0T + \frac{z_1T^2}{2})$$
$$+\ \sum_{j=1}^{n} C_p t_p$$

Where (By using Lemma-1)

$$E[\tilde{h}_j] = \frac{(h_{R1} + 2h_{R2} + h_{R3})}{4}, j = R, S$$
$$E[\tilde{Id}_k] = \frac{(Id_{k1} + 2Id_{k2} + Id_{k3})}{4}, k = u, p$$
$$E[\tilde{S}_k] = \frac{(S_{k1} + 2S_{j2} + S_{k3})}{4}, k = u, p$$

and (By using Lemma-2)

$$E[\tilde{\tilde{C}}_{ui}] = C_{ui} + \frac{(C_{ui2} + C_{ui4}) - (C_{ui1} + C_{ui3})}{4}, i = 0,1,2$$
$$E[\tilde{\tilde{C}}'_{ui}] = C'_{ui} + \frac{(C'_{ui2} + C'_{ui4}) - (C'_{ui1} + C'_{u03})}{4}, i = 0,1,2$$

The total expected cost function is given by

$$\text{Max } J = E[\tilde{\tilde{J}}_1] + E[\tilde{\tilde{J}}_2] \quad (19)$$
$$\text{and (8) -(14)} \quad (20)$$

The objective function (23) with constraints (24) is minimized using the following GA optimization technique.

7. GENETIC ALGORITHM(GA) FOR SINGLE-OBJECTIVE PROGRAMMING PROBLEM

Genetic Algorithms are exhaustive search algorithms based on the mechanics of natural selection and genesis (crossover, mutation etc.) and have been developed by Holland (cf. Holland (1975)), his colleagues and his students at the University of Michigan (cf. Goldberg (1989)).

A GA for a particular problem must have the following six components.

(a) A genetic representation for potential solutions(**chromosomes**) to the problem
(b) A way to create an **initial population** of potential solutions (chromosomes).
(c) A way to **evaluate fitness** of each solution.
(d) An evolution function that plays the role of environment, rating solutions in term of their fitness, i.e., **selection process** for mating pool.
(e) Genetic operators- **crossover,mutation** that alter the composition of children
(f) Values of different parameters that the genetic algorithm uses (**Population size, probabilities of applying genetic operators etc**).

Procedures for different GA components

(a) Chromosome representation: The concept of chromosome is normally used in the GA to stand for a feasible solution to the problem. A chromosome has the form of a string of genes that can take on some value from a specified search space. The specific chromosome representation varies based on the particular problem properties and requirements. Normally, there are two types of chromosome representation – (i) the binary vector representation based on bits and (ii) the real number representation. In this research work, the real number representation scheme is used.
Here, a 'K dimensional real vector' X=(x_1, x_2, ... x_K) is used to represent a solution, where x_1, x_2, ... x_K represent different decision variables of the problem.

(b) Initialization: A set of solutions (chromosomes) is called a population. N such solutions X_1, X_2, X_3, ... X_N are randomly generated from search space by random number generator such that each X_i satisfies the constraints of the problem. This solution set is taken as initial population and is the starting point for a GA to evolve to desired solutions. At this step, probability of crossover p_c and probability of mutation p_m are also initialized. These two parameters are used to select chromosomes from mating pool for genetic operations- crossover and mutation respectively.

(c) Fitness value: All the chromosomes in the population are evaluated using a fitness function. This fitness value is a measure of whether the chromosome is suited for the environment under consideration. Chromosomes with higher fitness will receive larger probabilities of inheritance in subsequent generations, while chromosomes with low fitness will more likely be eliminated. The selection of a good and accurate fitness function is thus a key to the success of solving any problem quickly. In this thesis, value of a objective function due to the solution X, is taken as fitness of X. Let it be f(X).

(d)Selection process to create mating pool: Selection in the GA is a scheme used to select some solutions from the population for mating pool. From this mating pool, pairs of individuals in the current generation are selected as parents to reproduce offspring. There are several selection schemes, such as roulette wheel selection, local selection, truncation selection, tournament selection, etc. Here, roulette wheel selection process is used in different cases. This process consists of following steps-

(i) Find total fitness of the population F=$\sum_{i=1}^{N} f(X_i)$
(ii) Calculate the probability of selection pr_i of each solution X_i by the formula pr_i=f(X_i)/F .
(iii) Calculate the cumulative probability qr_i for each solution X_i by the formula $qr_i=\sum_{j=0}^{i} pr_j$
(iv) Generate a random number 'r' from the range [0..1].
(v) If r<qr_1 then select X_1 otherwise select X_i(2≤i≤N) where qr_{i-1}≤r<qr_i.
(vi) Repeat step (iv) and (v) N times to select N solutions from current population. Clearly one solution may be selected more than once.
(vii) Let us denote this selected solution set by $P^1(T)$.

(e)Crossover: Crossover is a key operator in the GA and is used to exchange the main characteristics of parent individuals and pass them on the children. It consists of two steps:

(i) Selection for crossover: For each solution of $P^1(T)$ generate a random number r from the range [0..1]. If r<p_c then the solution is taken for crossover, where p_c is the probability of crossover.
(ii) Crossover process: Crossover taken place on the selected solutions. For each pair of coupled solutions Y_1, Y_2 a random number c is generated from the range [0..1] and Y_1, Y_2 are replaced by their offspring's Y_{11} and Y_{21} respectively

where Y_{11}=cY_1+(1-c)Y_2 , Y_{21}=cY_2+(1-c)Y_1, provided Y_{11}, Y_{21} satisfied the constraints of the problem.

(f) Mutation: The mutation operation is needed after the crossover operation to maintain population diversity and recover possible loss of some good characteristics. It is also consist of two steps:

(i) Selection for mutation: For each solution of $P^1(T)$ generate a random number r from the range [0..1]. If r< p_m then the solution is taken for mutation, where p_m is the probability of mutation.

(ii) Mutation process: To mutate a solution X=($x_1, x_2, ., x_K$) select a random integer r in the range [1..K]. Then replace x_r by randomly generated value within the boundary of r^{th} component of X.

Following selection, crossover and mutation, the new population is ready for it's next iteration, i.e., $P^1(T)$ is taken as population of new generation. With these genetic operations a simple genetic algorithm takes the following form. In the algorithm T is iteration counter, P(T) is the population of potential solutions for iteration T, Evaluate(P(T)) evaluate fitness of each members of P(T).

GA Algorithm

1. Set iteration counter T=0.
2. Initialize probability of crossover p_c and probability of mutation p_m.
3. Initialize P(T).
4. Evaluate(P(T)).
5. Repeat
   a. Select N solutions from P(T), for mating pool using Roulette-wheel selection process. Let this set be $P(T)^1$.
   b. Select solutions from $P(T)^1$, for crossover depending on p_c .
   c. Made crossover on selected solutions for crossover to get population $P(T)^2$.
   d. Select solutions from $P(T)^2$, for mutation depending on p_m .
   e. Made mutation on selected solutions for mutation to get population $P(T+1)$.
   f. $T \leftarrow T+1$.
   g. Evaluate $P(T)$.
6. Until(Termination condition does not hold).
7. Output: Fittest solution(chromosome) of P(T).

8. NUMERICAL EXPERIMENT

To illustrate the production-recycling model numerically, we consider input data in Tables-1,-2 & -3 for crisp data, fuzzy data and bi-fuzzy data respectively. For these input data and by using the above single objective genetic algorithm technique §7 and using Lemma-2, we solve the problem (23)-(24) and we obtained the optimal productions, optimal recycling and optimal disposal which are $u(t) = 25.19 + 0.6t, u'(t) = 17.2 + 0.4t, p(t) = 11$ and $z(t) = 0.71 + 0.16t$. Also the optimal values of $x_{Si}(t), x_{Sj}(t)$, $x_R(t)$, $u(t)$, $u'(t)$, $d(t)$ and $p(t)$ are evaluated using (23)-(24) for different values of t. We have shown the optimum results of $x_{Si}(t)$, $x_R(t)$, $u(t)$ and $d(t)$ of production-cycle in Table-4. Similarly, the optimum results of $x_{Sj}(t)$, $x_{Rj}(t), u'(t)$, $p(t)$ and $d(t)$ for the production and recycling-cycle are presented in Table-5. We get optimal profit by using GA technique as 871.349\$.

Table-1
Input crisp Data:

d_0	d_1	β	C_p in \$	C_z in \$	
15	0.2	0.05	1.6	0.15	
C_r	γ_1	γ_2	α_0	α_1	T
0.11	0.1	0.12	0.12	0.1	15

Table-2
Input fuzzy Data: (in \$)

$\tilde{h}_R$	$\tilde{h}_S$	$\tilde{Id}_u$
$(1.1, 1.3, 1.5)$	$(1.3, 1.5, 1.7)$	$(3, 4, 5)$
$\tilde{Id}_p$	$\tilde{S}_u$	$\tilde{S}_p$
$(2.5, 3.5, 4.6)$	$(3, 4, 5)$	$(3, 5, 5)$

Table-3
Input bi-fuzzy Data:(in \$)

$\tilde{\tilde{C}}_{u0}$	$\tilde{\tilde{C}}_{u1}$
$(\tilde{C}_{u0} - 5, \tilde{C}_{u0}, \tilde{C}_{u0} + 7)$ $\tilde{C}_{u0} = (3.5, 6.0, 7.5)$	$(\tilde{C}_{u1} - 14, \tilde{C}_{u1}, \tilde{C}_{u1} + 15)$ $\tilde{C}_{u1} = (11.5, 15.6, 20.8)$
$\tilde{\tilde{C}}_{u2}$	$\tilde{\tilde{C}}_{u3}$
$(\tilde{C}_{u2} - 1, \tilde{C}_{u2}, \tilde{C}_{u2} + 2)$ $\tilde{C}_{u2} = (1.4, 1.7, 2.9)$	$(\tilde{C}_{u3} - 0.9, \tilde{C}_{u3}, \tilde{C}_{u3} + 1.4)$ $\tilde{C}_{u3} = (0.6, 1, 2.1)$

Table-4
Optimal values of $x_{Sj}(t), x_{Rj}(t), u'(t), p(t)$ and $d(t)$, $j = 1, 2, 3$.

t	0	1.28	3	3.01	4.01	6
$x_{Si}(t)$	0	15.33	0	0.16	16.2	0
$x_R(t)$	0	1.73	6.78	6.86	8.28	15.4
$u(t)$	29.13	29.77	–	30.53	31.14	–
$d(t)$	15.7	15.32	16.63	15.67	15.42	14.97

Table-5
Comparison of the results obtained with different GA parameters.

No.	$N_{pop-size}$	P_c	P_m	Generations	Objective
$\alpha = 0.6$					
1	60	0.3	0.2	500	871.349
2	80	0.4	0.3	500	883.542
3	100	0.5	0.4	500	883.214
4	60	0.3	0.2	1000	885.856
5	80	0.4	0.3	1000	887.652
6	100	0.5	0.4	1000	890.214
$\alpha = 0.8$					
1	60	0.3	0.2	500	891.124
2	80	0.4	0.3	500	891.142
3	100	0.5	0.4	500	893.124
4	60	0.3	0.2	1000	894.214
5	80	0.4	0.3	1000	894.541
6	100	0.5	0.4	1000	895.214
$\alpha = 1.0$					
1	60	0.3	0.2	500	897.124
2	80	0.4	0.3	500	897.252
3	100	0.5	0.4	500	898.214
4	60	0.3	0.2	1000	899.214
5	80	0.4	0.3	1000	899.124
6	100	0.5	0.4	1000	905.124

Table-5 shows the experimental results obtained by a GA with different GA parameters. The tested GA parameters contain the population size $N_{pop-size}$, the probability of crossover Pc and the probability of mutation P_m. We compare these results when different parameters are put with the same or different generations as a stop criterion. It appears that almost all the objective values differ little from each other, which implies that the algorithm is robust to the GA parameters setting and effective to solve multiple objective programming problem. Figure-2 pictorially represents the optimum result for production, recycling, demand and serviceable stock. Figure-4.3 pictorially also represents the optimum result for non serviceable stock. The increasing demand rate is very small.

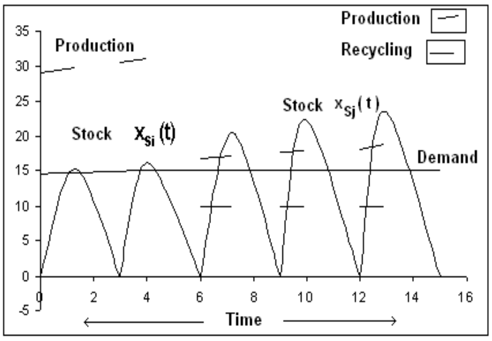

Fig. 4. Optimal production, demand, recycling and serviceable stock

9. CONCLUSION

In this paper, we develop a two plants production, recycling-disposal system over a finite time horizon in fuzzy and bi-fuzzy environment. The holding cost, setup cost, idle cost are fuzzy in nature. But the production cost is bi-fuzzy in nature as the purchasing of raw materials faces how to make purchasing decisions, in order to obtain required raw materials at a lower price and at the same time meet production demand in terms of item, quality, quantity, due date, and so on. Here, the dynamic demand is satisfied by production and recycling. Recycling products can be used as new products which are sold again. The rate of production, recycling and disposal are assumed to be control variables. The cost is expenditure due to growing environmental concern and according to the rule of environmental regulations like 'Kyoto Protocol' for Industry. At the beginning, production satisfies the demand. After sometime, production and recycling fill up the demand. The total cost is minimized as an optimal control problem. It is solved by single objective genetic algorithm technique. The model is illustrated through numerical examples and results are also presented both in tabular form only. The model can be extended for imperfect production, recycling-disposal optimization problem in uncertain environment.

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