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Linear Fractional Programming Procedure for Multi Objective Linear Programming Problem in Agricultural System

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ABSTRACT

This paper presents a linear fractional programming approach for solving multi objective linear programming problem. We develop a model to solve multi objective linear programming problem into fractional programming problem and proposed a method for linearization of a fractional objective. In particular, we consider the optimization of ratio of profit and cash expenditure as fractional objective and used rest of objectives as constraints by obtaining aspiration levels. As the objectives are conflicting in nature, we used the concept of conflict and non conflict between objectives for computation of appropriate aspiration level. The method is illustrated on a problem of agricultural production system to show its suitability.

General Terms

Multi objective linear programming, Agricultural cropping system.

Keywords

Fractional programming, Multi objective programming problem, Fuzzy goal programming, Conflict and non conflict, Land-use planning.

1. INTRODUCTION

In several decisions making problems in Economics and Management science, one has to optimize ratios. Such ratios arise in modeling the resource allocation, production, profit, transportation etc. In economic application these ratio like productivity, profit/cash input, return/cost etc. describe efficiency of a system. Such type of problems is inherently fractional programming problem. A variety of applications of fractional programming can be seen in a survey article Schaible [18]. Charnes and Cooper [3] developed a method in which linear fractional programming problem can be optimized by solving two linear programs. A comprehensive study of linear fractional programming can be viewed in Craven [5,6]. Kornbluth and Steuer [9,10] discussed the feasibility region for linear fractional programming problems and multi objective linear fractional programming problems and considered its solution. It was Luhandjula [12] who first proposed the fuzzy set theory approach to reach a satisfactory solution, which was later modified by Dutta et.al.[7] and further by Stancu-Minasian and Pop [19]. Further Nykowski and Zolkiewski [15] gave a compromise procedure for multiple objective linear fractional programming problems. Ohta and Yamaguchi [16], Chakraborty and Gupta [2] studied the solution of linear fractional programming problems in fuzzy environment. Many more applications of fractional programming may be obtained in literature [8,11].

A goal programming procedure for solving multi objective linear fractional programming problem has been given by Pal et.al.[17]. Stanojevic and Stancu-Minasian [20] studied a fully fuzzified linear fractional programming problem where all parameters and variables are fuzzy numbers. Mehrjerdi [13] developed a fuzzy goal programming model using a linear approximation technique for solving non linear fractional programming problem. Recently Zeng et.al [21] considered a crop area planning problem and proposed a fuzzy multi objective linear programming problem with fuzzy numbers and transformed the fuzzy multi objective programming problem in an equivalent goal programming problem to crisp ones to find its solution.

The motivation of the present study is to demonstrate that converting a multi objective cropping problem of agricultural production system into a fractional programming problem lead to a better solution and optimal profit.

2 PROBLEM DESCRIPTIONS AND MODEL DEVELOPMENT

Consider the modeling of complex system where several objectives are to be optimized at a time. For example, agricultural production system is a real life example of such complex systems comprising of multiple objectives that too of conflicting nature. In general modeling of an agricultural management system requires optimizing the profit cutting the cost of cultivation. Thus the core objective of the problem is to maximize the profit subject to minimization of resource requirements. But since situations are not ideal and have limited scope in production system. With changing scenario of capital intensive agricultural with mechanization and proper availability of resources, the problem changes to develop a model giving optimal profit in accordance of fulfilling resource goals. Thus such problems of multi objective linear programming can be better dealt with goal programming approach. Here we deal such problem as fractional programming problem in which fractional function (profit/ cash-input) is to be optimized and other objectives are to be dealt as constraints for getting optimal solution.

2.1 Multi objective linear programming problem

A multi objective optimization problem with n decision variables, m constraints and k objectives is as follows

$$\max \quad Z = \{Z_1, Z_2, \dots, Z_k\}$$

s.t. $A_i X * b_i$, $i = 1,2, \dots, m.$

$x_j \geq 0$; $j = 1, 2, \dots, n.$

(1)

Where (*) can be one of the $(\leq, \geq, =)$, and $X = \{x_1, x_2, \dots, x_n\}$.

2.2 Linear Fractional Programming Approach

Let there be two objectives out of k objectives such that their ratio is to be maximized as core of the problem. Let these two linear objectives are $Z_p(X) > 0$ and $Z_q(X) > 0$ whose ratio forms a new objective function, giving rise to linear fractional programming as

Maximize $\frac{Z_p(X)}{Z_q(X)}$

Assuming that $Z_p(X) > 0$ *and* $Z_q(X) > 0$, therefore it is equivalent to $Maximize\ Z_p(X) - Z_q(X)$ and $Z_p(X) - rZ_q(X) \geq 0$

where r is a positive real number, which is a restriction that the ratio should always be greater than to a level r .

The rest of objectives may be converted into fuzzy goals by obtaining the maximum and minimum values for constructing the membership function with a method given by Zimmermann [22]. Consider a problem under situation that aspiration levels for each of the objective with respective tolerances are available with the decision maker. Then the rest of the objectives can be treated as fuzzy goals having fuzzy aspiration level. Thus the problem (1) can be equivalently written in a fuzzy goal programming problem as

Maximize $Z_p(X) - Z_q(X)$

s.t. $Z_p(X) - rZ_q(X) \geq 0$

$Z_k(X) \gtrsim b_k$,

$Z_s(X) \lesssim b_s$,

$Z_t(X) \approx b_t$,

$x_j \geq 0$; $j = 1, 2, \dots, n.$

(2)

$where\ k, s, t \in (1,2, \dots k) - \{p, q\}$

2.3 Fuzzy goal programming approach

For solution of the above fuzzy goal programming problem (5), we consider the fuzzy goal programming approach given by Zimmermann [22] where membership function for various goal in the solution set is can be obtained as

For the goal of type maximization:

$$\mu_{Z_k}(X) = \begin{cases} 1 & Z_k(X) \geq b_k \\ \frac{Z_k(X) - l_k}{(b_k - l_k)} & l_k \leq Z_k(X) \leq b_k \\ 0 & Z_k(X) \leq l_k \end{cases} \quad (3)$$

and for the goal of type minimization:

$$\mu_{Z_k}(X) = \begin{cases} 1 & Z_k(X) \leq b_k \\ \frac{u_k - Z_k(X)}{(u_k - b_k)} & b_k \leq Z_k(X) \leq u_k \\ 0 & Z_k(X) \geq u_k \end{cases} \quad (4)$$

Where b_k is aspiration level for k^{th} goal and $u_k\ and\ l_k$ are upper and lower tolerance limits for b_k.

One of the major problems in solution of multi objective programming problem by fuzzy goal programming approach is to obtain appropriate priorities to various goals. As a matter of fact in agricultural planning system various goals are conflicting in nature like : production and expenditure, profit and cost of cultivation etc. Thus computation of appropriate aspiration levelsb_k', b_s', b_t' corresponding to b_k, b_s, b_t for the fuzzy goals need to consider the conflict and non conflict among various goals for realistic modeling of the problem. We consider the measure of conflict and non conflict between objectives by Cohon [4], Mohanty and Vijayaraghavan [14] to compute degree of non-conflict between objectives.

2.3.1 Computation of conflict and non conflict between objectives

Let gradients of the jective $Z_k\ be\ (C_{k1}, C_{k2}, \dots, C_{kn})$, and θ_{ij} be angle between gradients of objective $Z_i\ and\ Z_j$, now the simultaneous achievement of objectives $Z_i\ and\ Z_j$ is possible if $\theta_{ij} = 0$ and a situation of conflict arises when $\theta_{ij} \neq 0$; $with\ \theta_{ij} \in \left[0, \frac{\pi}{2}\right]$. Let $(C_{i1}, C_{i2}, \dots, C_{in})\ and\ (C_{j1}, C_{j2}, \dots, C_{jn})$ be the gradients of the objectives $Z_i\ and\ Z_j$ respectively, and then angle θ_{ij} between them is given by

$$\cos\theta_{ij} = \frac{\sum_{k=1}^{n} C_{ik} C_{jk}}{\sqrt{(\sum_{k=1}^{n} C_{ik}^2 \sum_{k=1}^{n} C_{jk}^2)}} \quad (5)$$

$where\ i, j = 1,2\ \dots, k$

A function of non – conflict between objectives $Z_i\ and\ Z_j$ is defined as

$$\eta_{Z_i Z_j} = \begin{cases} 1 & \theta_{ij} = 0 \\ \frac{\pi - \theta_{ij}}{\pi} & 0 \leq \theta_{ij} \leq \pi \\ 0 & \theta_{ij} = \pi \end{cases} \quad (6)$$

And the computed degree of non-conflict among objectives can be arranged in form of a symmetric matrix as a non conflict matrix follows

$$\begin{array}{c c} & \begin{array}{cccc} Z_1 & Z_2 & \cdots & Z_n \end{array} \\ \begin{array}{c} Z_1 \\ Z_2 \\ \vdots \\ Z_n \end{array} & \begin{bmatrix} \eta_{Z_1 Z_1} & \eta_{Z_1 Z_2} & \cdots & \eta_{Z_1 Z_n} \\ \eta_{Z_2 Z_1} & \eta_{Z_2 Z_2} & \cdots & \eta_{Z_2 Z_n} \\ \vdots & \vdots & \ddots & \vdots \\ \eta_{Z_n Z_1} & \eta_{Z_n Z_2} & \cdots & \eta_{Z_n Z_n} \end{bmatrix} \end{array}$$

Here, $\eta_{Z_i Z_j}$ represents extent to which the objective Z_i non-conflicts with objective Z_j and vice-versa. From the non conflict matrix we can find total amount of support that objective Z_i gets from the remaining objectives, as

$$W_i = \frac{\sum_{j=1}^{k} \eta_{z_i z_j}}{k} \quad (7)$$

2.3.2 Appropriate aspiration level

Using membership function defined in (3) and (4), we assign aspiration level to each goal in view of its non-conflicting nature with other goals. Depending on values of W_i associated with each goal; aspiration level can be defined by using membership function as

$$b_k{}' = \left[\mu_{Z_k^{-1}}(W_k)\right], \quad k \in (1,2,\dots k) - \{p,q\} \quad (8)$$

Similarly, we can calculate $b_s{}'$ *and* $b_t{}'$. Now the goals in (2) can be equivalently written in crisp form as

$$Z_k(X) \geq b_k', \quad Z_s(X) \leq b_s', \quad Z_t(X) = b_t{}'$$
$$where\ k,s,t \in (1,2,\dots K) - \{p,q\} \quad (9)$$

2.4 Equivalent linear programming problem

Thus using (9), we obtain linear programming problem equivalent to (2) as

$$\begin{aligned} &Optimize \quad Z_p(X) - Z_q(X) \\ &\text{s.t.} \quad Z_k(X) \geq b_k',\ Z_s(X) \leq b_s{}', Z_t(X) = b_t{}' \\ &\quad Z_p(X) - rZ_q(X) \geq 0 \\ &\quad X \geq 0 \\ &where\ k,s,t \in (1,2,\dots k) - \{p,q\} \end{aligned} \quad (10)$$

This equivalent Linear Programming Problem can be easily solved by any standard method of solving LPP.

3 NUMERICAL ILLUSTRATIONS

In view of proper illustration of developed method, we consider the problem of land use planning for production of principal crops as undertaken by Biswas and Pal [1], in which different types of seasonal crops and decision variables are given in Table-1. The data for productive resource utilization, production rate and market price are given in Table-2. The data for aspiration levels of fuzzy goals and their respective tolerance limits are given in Table-3.

Table-1 The seasonal crops and the associated decision variables

Season(s)	Crop(c)	Variable (x_{cs})
Prekharif(1)	Jute(1)	x_{11}
	Sugarcane(2)	x_{21}
	Aus(3)	x_{31}
Kharif(2)	Aman(4)	x_{42}
	Boro(5)	x_{53}
Rabi(3)	Wheat(6)	x_{63}
	Mustard(7)	x_{73}
	Potato(8)	x_{83}

Here in the Table-2 MH=machine hour (in hrs./ha). MD=mandays (days/ha). WC=water consumption (inch/ha). FR=fertilizer (kg/ha). PA=production achievement (kg/ha). CE=cash expenditure (Rs./ha). MP=market price (Rs./qtls.) and table 3 contains data for aspiration level for various goals.

Using above data Table 1-3, the complete mathematical formulation of all 19 fuzzy goals of the under taken cropping problem of agricultural system are as follows

1. Land utilization goals

Z_1: $x_{11} + x_{21} + x_{31} \lesssim 272.135$ (11)

Z_2: $x_{21} + x_{42} \lesssim 272.135$ (12)

Z_3: $x_{21} + x_{53} + x_{63} + x_{73} + x_{83} \lesssim 272.135$ (13)

2. Productive resource goals

(a) Machine hour goal

Z_4: $61.02(x_{11} + x_{31}) + 40.52(x_{21} + x_{42}) + 38.51x_{53} + 36.36(x_{63} + x_{73} + x_{83}) \gtrsim 37843.75$ (14)

(b) Man power goal

Z_5: $124x_{11} + 247x_{21} + 84x_{31} + 89x_{42} + 111x_{53} + 74x_{63} + 47x_{73} + 119x_{83} \gtrsim 46510.66$ (15)

(c) Water consumption goals

Z_6: $60x_{11} + 30x_{21} + 25x_{31} \gtrsim 2727.84$ (16)

Z_7: $12\, x_{42} \gtrsim 1490.40$ (17)

Z_8: $48x_{53} + 12x_{63} + 6x_{73} + 20x_{83} \gtrsim 5675$ (18)

(d) Fertilizer requirement goals

Z_9: $20(x_{11} + x_{42}) + 200x_{21} + 40x_{31} + 100(x_{53} + x_{63}) + 80x_{73} + 150x_{83} \gtrsim 44500$ (19)

Z_{10}: $20(x_{11} + x_{31} + x_{42}) + 100x_{21} + 50(x_{53} + x_{63}) + 40x_{73} + 75x_{83} \gtrsim 23000$ (20)

Z_{11}: $20(x_{11} + x_{31} + x_{42}) + 100x_{21} + 50(x_{53} + x_{63}) + 40x_{73} + 75x_{83} \gtrsim 19000$ (21)

3. Cash expenditure goal

Z_{12}: $8577.98x_{11} + 23031.57x_{21} + 6700.94x_{31} + 6811.57x_{42} + 10508.44x_{53} + 7685.76x_{63} + 5093.10x_{73} + 22527.05x_{83} \lesssim 6441015.80$ (22)

4. Production achievement goals

Z_{13}: $2538x_{11} \gtrsim 306000$ (jute) (23)

Z_{14}: $59283x_{21} \gtrsim 259000$ (sugarcane) (24)

Z_{15}: $2076x_{31} + 1885x_{42} + 3401x_{53} \gtrsim 870000$ (rice) (25)

Z_{16}: $2301x_{63} \gtrsim 136260$ (wheat) (26)

Z_{17}: $795x_{73} \gtrsim 60540$ (mustard) (27)

Z_{18}: $17779x_{83} \gtrsim 110000$ (potato) (28)

5. Profit goal

Z_{19}: $24872.4x_{11} + 889245x_{21} + 13410.96x_{31} + 10179x_{42} + 19198.645x_{53} + 16107x_{63} + 9142.5x_{73} + 33780.1x_{83} \gtrsim 12500000$ (29)

Here we solve the above multi objective fuzzy goal programming problem (11)-(29) by using fractional programming approach as discussed in section-2. In this particular problem, ratio of profit to expenditure has been taken as fractional goal. Rather to maximize profit goal and to minimize the expenditure goal, now our objective is to maximize ratio of profit to expenditure goal. Here $Z_{19}(X)$ is the profit goal and $Z_{12}(X)$ is the cash expenditure goal, and the fractional function to maximize is $\frac{Z_{19}(X)}{Z_{12}(X)}$. Thus the problem is equivalently written as

$maximize \quad Z_{19}(X) - Z_{12}(X)$

s.t. $\quad Z_k(X) \gtrsim b_k \text{ or } Z_k(X) \lesssim b_k$

$Z_{19}(X) - rZ_{12}(X) \geq 0$

$k = (1,2, \ldots 19) - \{12,19\}$

$X \geq 0$ (30)

In order to obtain appropriate aspiration level for rest of the goals except profit goal and expenditure goal, we apply concept of non-conflicting of one goal with respect to other goals as discussed in section 2. The method proceeds as

The gradients C_{in} are used to compute angle between goals. For illustration the computations of angle for the goal Z_1 with other goals are given in Table 4.

Table - 4 Angle between goal Z_1 and all other goals.

$cos\ \theta_{1,2}$ 5.91	$cos\ \theta_{1,3}$ 75.02	$cos\ \theta_{1,4}$ 42.46	$cos\ \theta_{1,5}$ 42.15	$cos\ \theta_{1,6}$ 21.96
$cos\ \theta_{1,7}$ 90	$cos\ \theta_{1,8}$ 90	$cos\ \theta_{1,9}$ 60.21	$cos\ \theta_{1,10}$ 58.12	$cos\ \theta_{1,11}$ 58.12
$cos\ \theta_{1,12}$ 53.74	$cos\ \theta_{1,13}$ 54.74	$cos\ \theta_{1,14}$ 54.74	$cos\ \theta_{1,15}$ 74.22	$cos\ \theta_{1,16}$ 90
$cos\ \theta_{1,17}$ 90	$cos\ \theta_{1,18}$ 90	$cos\ \theta_{1,19}$ 53.05		

and similarly , we can compute for all other goals.

Now, we calculate the function of non conflict for the goal Z_1 with other goals by method given in section 3.1.2 and the results obtained are in Table-5

Table – 5 Degree of non-conflict of goal Z_1 with other goals

$\eta_{Z_1 Z_1}$ 1	$\eta_{Z_1 Z_2}$ .6338	$\eta_{Z_1 Z_3}$ .5832	$\eta_{Z_1 Z_4}$ .7641	$\eta_{Z_1 Z_5}$ .7658
$\eta_{Z_1 Z_6}$ 0.878	$\eta_{Z_1 Z_7}$ 0.5	$\eta_{Z_1 Z_8}$ 0.5	$\eta_{Z_1 Z_9}$ 0.6655	$\eta_{Z_1 Z_{10}}$ 0.6771
$\eta_{Z_1 Z_{11}}$ 0.6771	$\eta_{Z_1 Z_{12}}$ .7014	$\eta_{Z_1 Z_{13}}$ 0.6959	$\eta_{Z_1 Z_{14}}$ 0.6959	$\eta_{Z_1 Z_{15}}$ .5877
$\eta_{Z_1 Z_{16}}$ 0.5	$\eta_{Z_1 Z_{17}}$ 0.5	$\eta_{Z_1 Z_{18}}$ 0.5	$\eta_{Z_1 Z_{19}}$ 0.7053	

Further, we can calculate weight of goal Z_1 ,

$W_1 = 0.6595$ and Similarly, we calculated weight of each of goal corresponding to all other goals and computed weights are:

$W_2 = 0.6475, W_3 = 0.6924, W_4 = 0.7098,$

$W_5 = 0.7421, W_6 = 0.6445, W_7 = 0.5653,$

$W_8 = 0.6220, W_9 = 0.7358, W_{10} = 0.7448,$

$W_{11} = 0.7448, W_{12} = 0.7372,\ W_{13} = 0.5775,$

$W_{14} = 0.6556, W_{15} = 0.6092, W_{16} = 0.5668,$

$W_{17} = 0.5589, W_{18} = 0.5884, W_{19} = 0.6583$.

Thus we compute aspiration level of all goals using values of W_i as above. The respective aspiration levels for goals are obtained as:

$b_1 = 284.8, b_2 = 285.25, b_3 = 283.58,$

$b_4 = 35542.19, b_5 = 45759.02,$

$b_6 = 2655.5, b_7 = 1467.45, b_8 = 5648.62,$

$b_9 = 42571.34, b_{10} = 22183.36, \quad b_{11} = 17468.8, b_{13} = 304669.1,$

$b_{14} = 197869, b_{15} = 859721.96,$

$b_{16} = 125967.17, b_{17} = 57831.65,$

$b_{18} = 105307.76,$

As our objective is to maximize ratio of profit goal to expenditure goal with general case that profit must be greater than cash expenditure, we take $r = 1$ without any loss of generality. Thus, we have the equivalent linear programming problem

$maximize$

$f(X) = 16294.42x_{11} + 866213.4x_{21} + 6710.02x_{31} + 3367.43x_{42} + 8690.21x_{53} + 8421.24x_{63} + 4049.4x_{73} + 11253.05x_{83}$

$such\ that$

$x_{11} + x_{21} + x_{31} \leq 284.8$

$x_{21} + x_{42} \leq 285.25$

$x_{21} + x_{53} + x_{63} + x_{73} + x_{83} \leq 283.58$

$61.02(x_{11} + x_{31}) + 40.52(x_{21} + x_{42}) + 38.51x_{53} + 36.36(x_{63} + x_{73} + x_{83}) \geq 35542.19$

$124x_{11} + 247x_{21} + 84x_{31} + 89x_{42} + 111x_{53} + 74x_{63} + 47x_{73} + 119x_{83} \geq 45759.02$

$60x_{11} + 30x_{21} + 25x_{31} \geq 2655.5$

$12\, x_{42} \geq 1467.45$

$48x_{53} + 12x_{63} + 6x_{73} + 20x_{83} \geq 5648.62$

$20(x_{11} + x_{42}) + 200x_{21} + 40x_{31} + 100(x_{53} + x_{63}) + 80x_{73} + 150x_{83} \geq 42571.34$

$20(x_{11} + x_{31} + x_{42}) + 100x_{21} + 50(x_{53} + x_{63}) + 40x_{73} + 75x_{83} \geq 22183.36$

$20(x_{11} + x_{31} + x_{42}) + 100x_{21} + 50(x_{53} + x_{63}) + 40x_{73} + 75x_{83} \geq 17468.8$

$2538x_{11} \geq 304669.13$

$59283x_{21} \geq 197869$

$2076x_{31} + 1885x_{42} + 3401x_{53} \geq 859721.96$
$2301x_{63} \geq 125967.17$

$795x_{73} \geq 57831.65$

$17779x_{83} \geq 105307.76$

$16294.42x_{11} + 866213.4x_{21} + 6710.02x_{31} + 3367.43x_{42} + 8690.21x_{53} + 8421.24x_{63} + 4049.4x_{73} + 11253.05x_{83} \geq 0$ (31)

The above linear programming problem (31) has been solved by MATLAB®, and the solution obtained is

$x_{11} = 188.901, x_{21} = 40.284, x_{31} = 55.615,$

$x_{42} = 244.966, x_{53} = 101.049,$
$x_{63} = 54.744,\ x_{73} = 72.744, x_{83} = 14.758.$

4. CONCLUSIONS

The result obtained by our proposed method of using fractional programming clearly shows the superiority of achievement of various goals under taken in the study. Thus the proposed method of fractional programming using the simple linearization of fractional function into linear function, we get much better result. Here using the proposed method we obtain a profit/ expenditure ratio of 7.05 against a ratio of 2.78 as obtained by Biswas and Pal [1]. Hence the developed method is suitable method for dealing such cropping problem of agricultural system for optimal profits. The results of attainability of different goal by our proposed method and of Biswas and Pal are given in Table 6

5. ACKNOWLEDGMENTS

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Table-2 Data description for utilization of productive resources, production rate, cash expenditure and market price.

Production activity	MH	MD	WC	FR			PA	CE	MP
				N	P	K			
Jute(x_{11})	61.02	124	60	20	20	20	2538.00	8577.98	980.00
Sugarcane(x_{21})	40.52	247	30	200	100	100	59283.0 0	23031.57	1500.00
Aus(x_{31})	61.02	84	25	40	20	20	2076.00	6700.94	646.00
Aman(x_{42})	40.52	89	12	20	20	20	1885.00	6811.57	540.00
Boro(x_{53})	38.51	111	48	100	50	50	3401.00	10508.44	564.50

Table3. Aspiration levels for fuzzy goals

Sl .No.	Goals	Aspiration level	Tolerance limit	
			Lower	upper
1.	Land utilization (in'000'ha)			
	I. Prekharif season (Z_1)	272.135	-	309.33
	II. Kharif season (Z_2)	272.135	-	309.33
	III. Rabi season (Z_3)	272.135	-	309.33
2.	Productive Resource Goals			
	(a)Machine-hr.(MH) (in hrs.) (Z_4)	37843.75	29912.80	-
	(b)Man-days(MD) (in days) (Z_5)	46510.66	43596.18	-
	(c)Water consumption(WC) (in inch)			
	I. Prekharif season (Z_6)	2727.84	2524.34	-
	II. Kharif season (Z_7)	1490.40	1437.60	-
	III. Rabi season (Z_8)	5675.00	5605.20	-
	(d)Fertilizer requirement (in metric ton)			
	I. Nitrogen(N) (Z_9)	44.50	37.20	-
	II. Phosphate(P) (Z_{10})	23.00	19.80	-
	III. Potash(K) (Z_{11})	19.00	13.00	-
3.	Cash expenditure (CE) (in Rs.) (Z_{12})	64,41,015.80	-	9400113.90
4.	(a) Jute (production) (Z_{13})	306.00	302.85	-
	(b) Sugarcane (production) (Z_{14})	259.00	81.5	-
	(c) Rice (production) (Z_{15})	870.00	843.70	-
	(d) Wheat (production) (Z_{16})	136.26	112.50	-
	(e) Mustard (production) (Z_{17})	60.54	54.40	-
	(f) Potato (production) (Z_{18})	110.00	98.60	-
5.	Profit (MP*PA) (in Rs.) (Z_{19})	1,25,00,000.0	1,10,86,621.61	-

Table-4 Optimal solution

	Goals		Goals set by Decision Maker			Achievements by Biswas and Pal [2005]	Achievements byproposed method
			Aspiration level	Tolerance limit			
				Lower	upper		
1.	Land utilization (in'000'ha)						
	IV.	Prekharif season (Z_1)	272.135	-	309.33	223.39	284.8
	V.	Kharif season (Z_2)	272.135	-	309.33	128.596	285.25
	VI.	Rabi season (Z_3)	272.135	-	309.33	272.163	283.759
2.	Productive Resource Goals						
	(a)Machine-hr.(MH) (in hrs.) (Z_4)		37843.75	29912.80	-	28582.48	35542.16
	(b)Man-days(MD) (in days) (Z_5)		46510.66	43596.18	-	58108.06	80290.17
	(c)Water consumption (WC) (in inch)						
	IV.	Prekharif season (Z_6)	2727.84	2524.34	-	9826.55	13932.96
	V.	Kharif season (Z_7)	1490.40	1437.60	-	1490.4	2939.59
	VI.	Rabi season (Z_8)	5675.00	5605.20	-	7377.91	6238.904
	(d) Fertilizer requirement (in ton)						
	IV.	Nitrogen(N) (Z_9)	44500	37200	-	35288.19	42571.26
	V.	Phosphate(P) (Z_{10})	23000	19800	-	20091.77	25624.3
	VI.	Potash(K) (Z_{11})	19000	13000	-	20091.77	25624.3
3.	Cash expenditure (CE) (in Rs.) (Z_{12})		64,41,015.80	-	9400113.9	4952850.88	6775031.87
4.	Jute (production) (Z_{13})		306000	302850	-	305999	479430.7
	Sugarcane (production) (Z_{14})		259000	81500	-	260608.1	238815.6
	Rice (production) (Z_{15})		870000	843700	-	870002.3	920885.3
	Wheat (production) (Z_{16})		136260	112500	-	136260.6	125965.9
	Mustard (production) (Z_{17})		60540	54400	-	60000.24	57831.48
	Potato (production) (Z_{18})		110000	98600	-	109998.7	262382.5
5.	Profit (MP*PA) (in Rs.) (Z_{19})		1,25,00,000	1,10,86,621.6	-	13758194.4	47745480.5
6.	Ratio (profit/ cash expenditure)		1.94	1.18	-	2.78	7.05

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