Matching Dominating Sets of Direct Product Graphs of Cayley Graphs with Arithmetic Graphs

S. Uma Maheswari
Lecturer
Department of Mathematics
JMJ College For Women
Tenali, AP, India

B. Maheswari
Professor
Department of
Applied Mathematics
S.P. Women's University
Tirupati, AP, India

M. Manjuri
Department of
Applied Mathematics
S.P. Women's University
Tirupati, AP, India

ABSTRACT

Graph Theory is one of the most flourishing branches of modern Mathematics finding widest applications in all most all branches of Science & Technology. It is applied in diverse areas such as social sciences, linguistics, physical sciences, communication engineering etc. Number Theory is one of the oldest branches of Mathematics, which inherited rich contributions from almost all greatest mathematicians, ancient and modern.

Every branch of mathematics employs some notion of a product that enables the combination or decomposition of its elemental structures. Product of graphs are introduced in Graph Theory very recently and developing rapidly.

In this paper, we consider direct product graphs of Cayley graphs with Arithmetic graphs and present Matching dominating set of these graphs.

Keywords
Euler totient Cayley Graph, Arithmetic V_n graph, Direct Product Graph, Matching dominating set.

AMS (MOS) Subject Classification: 6905c

1. INTRODUCTION

1.1 Euler Totient Cayley Graph $G(Z_n, \varphi)$

Madhavi [1] introduced the concept of Euler totient Cayley graphs and studied some of its properties.

For any positive integer $n \geq 1$, the number of positive integers less than n and relatively prime to n is denoted by $\varphi(n)$ and is called Euler totient function. Let S denote the set of all positive integers less than n and relatively prime to n. That is $S = \{r/1 \leq r < n \text{ and } \mathrm{GCD}(r,n) = 1\}$. Then $|S| = \varphi(n)$.

Now we define Euler totient Cayley graph as follows.

For each positive integer n, let Z_n be the additive group of integers modulo n and let S be the set of all integers less than n and relatively prime to n. The Euler totient Cayley graph $G(Z_n, \varphi)$ is defined as the graph whose vertex set V is given by $Z_n = \{0,1,2,\ldots.n-1\}$ and the edge set is $E = \{(x,y)/x - y \in S \text{ or } y - x \in S\}$.

Clearly as proved by Madhavi [1], the Euler Totient Cayley graph $G\ (Z_n, \varphi)$ is

1. a connected, simple and undirected graph,
2. $\Phi(n)-$ regular and has $n.\Phi(n)/2$ edges,
3. Hamiltonian,
4. Eulerian for $n \geq 3$,
5. Bipartite if n is even,
6. Complete graph if n is a prime.

1.2 Arithmetic V_n Graph

Vasumathi [2] introduced the concept of Arithmetic V_n graphs and studied some of its properties. Their definition of Arithmetic V_n graph is as follows.

Let n be a positive integer such that $n = p_1^{\alpha_1} p_2^{\alpha_2} \ldots \ldots . . p_k^{\alpha_k}$. Then the Arithmetic V_n graph is defined as the graph whose vertex set consists of the divisors of n and two vertices u, v are adjacent in V_n graph if and only if GCD $(u,v) = p_i$, for some prime divisor p_i of n.

In this graph vertex 1 becomes an isolated vertex. When we enumerate the parameters total domination, connected domination, the contribution of this isolated vertex is nothing. Hence we take the divisors of n excluding 1 as the vertex set of our Arithmetic V_n graph and the adjacency is defined as above.

Clearly, V_n graph is a connected graph. If n is a prime, then V_n graph consists of a single vertex. Hence it is connected. In other cases, by the definition of adjacency in V_n, there exist edges between prime number vertices and their prime power vertices and also to their prime product vertices. Therefore each vertex of V_n is connected to some vertex in V_n.

1.3 Direct Product Graphs

Products of graphs occur naturally in discrete mathematics as tools in combinatorial constructions. They give rise to important classes of graphs and deep structural problems. In the beginning the emphasis was on the structure of finite and infinite products, but later it shifted to recognition algorithms for classes of isometric subgraphs of products of graphs, which are referred as product graphs. The methods recently developed for recognizing them have been extremely fruitful, have led to a better perception of the earlier work, and have helped to simplify many of the original proofs, both for finite and infinite graphs.

The structure and applicability of these products are quite interesting. For example, large networks such as the Internet graph, with several hundred million hosts, can be efficiently modeled by subgraphs of powers of small graphs with respect to the direct product. This is one of many examples of the dichotomy between the structure of products and that of their subgraphs. For a detailed description on product graphs refer [3, 4].

There are four standard products of graphs, namely those of the Cartesian, the strong, the direct and the lexicographic product.

In this paper, we consider direct product graphs of Cayley graphs with Arithmetic graphs and present Matching dominating set of these graphs.

In the literature, the direct product is also called as the tensor product, categorical product, cardinal product, relational product, Kronecker product, weak direct product, or conjunction.

Let G_1 and G_2 be two simple graphs with their vertex sets as $V_1 = \{u_1, u_2, \dots, u_l\}$ and $V_2 = \{v_1, v_2, \dots, v_m\}$ respectively. Then the direct product of these two graphs denoted by $G_1 \times G_2$ is defined to be a graph with vertex set $V_1 \times V_2$, where $V_1 \times V_2$ is the Cartesian product of the sets V_1 and V_2 such that any two distinct vertices (u_1, v_1) and (u_2, v_2) of $G_1 \times G_2$ are adjacent if u_1u_2 is an edge of G_1 and v_1v_2 is an edge of G_2.

The cross symbol $\times$, shows visually the two edges resulting from the direct product of two edges.

Now we consider the direct product graph of Euler totient Cayley graphs with Arithmetic V_n graph. The properties and some dominating parameters of these graphs are presented respectively in [5, 6, 7, 8, 9].

We state the following theorem without proof and the proof can be found in [7].

Theorem 1.3.1: Suppose n is neither a prime nor $2p$. Let $n = p_1^{\alpha_1} p_2^{\alpha_2} \dots\dots . p_k^{\alpha_k}$, where $p_1, p_2, \quad \dots p_k$ are primes and $\alpha_1, \alpha_2, \dots . \alpha_k$ are integers ≥ 1. Then the domination number of $G(Z_n, \varphi)$ is given by $\gamma(G(Z_n, \varphi)) = \lambda + 1$, where λ is the length of the longest stretch of consecutive integers in V, each of which shares a prime factor with n.

We state the following theorem without proof and the proof can be found in [9].

Theorem 1.3.2: If $n \neq p^{\alpha}, n \neq 2p$ and $n = p_1^{\alpha_1} p_2^{\alpha_2} \dots\dots . p_k^{\alpha_k}$ where $\alpha_i \geq 1$, then the domination number of $G_1 \times G_2$ is given by

$\gamma(G_1 \times G_2) = (\lambda + 1)k$, where λ is the length of the longest stretch of consecutive integers in V_1 of G_1 each of which shares a prime factor with n and k is the core of n.

1.4 Matching Dominating Set

A matching in a graph $G(V, E)$ is a subset M of edges of E such that no two edges in M are adjacent. A matching M in G is called a perfect matching if every vertex of G is incident with some edge in M.

A dominating set D of G is said to be a matching dominating set if the induced subgraph $\langle D \rangle$ admits a perfect matching. The cardinality of the smallest matching dominating set is called matching domination number and is denoted by γ_m.

The matching domination number of Euler totient Cayley graph is presented in [5] and we state the theorem without proof and the proof can be found in [5].

Theorem 1.4.1: Let n be neither a prime nor $2p$ and $n = p_1^{\alpha_1} p_2^{\alpha_2} \dots\dots . p_k^{\alpha_k}$, where $p_1, p_2, \quad \dots p_k$ are primes and $\alpha_1, \alpha_2, \dots . \alpha_k$ are integers ≥ 1. Then the matching domination number of $G(Z_n, \varphi)$ is given by

$$\gamma_m(G(Z_n, \varphi)) = \begin{cases} \lambda + 1 & \text{if } \lambda \text{ is odd} \\ \lambda + 2 & \text{if } \lambda \text{ is even} \end{cases}$$

where λ is the length of the longest stretch of consecutive integers in V each of which shares a prime factor with n.

2. RESULTS

2.1Matching Dominating Sets in Arithmetic V_n Graph

We now present the matching domination number of Arithmetic V_n graph.

Theorem 2.1.1: Suppose $n = p_1^{\alpha_1} p_2^{\alpha_2} \dots\dots . p_k^{\alpha_k}$, where $\alpha_i \geq 1$. Then the matching domination number of $G(V_n)$ is given by $\gamma_m(V_n) = \begin{cases} k, & \text{if k is even} \\ k+1, & \text{if k is odd} \end{cases}$

where k is the core of n.

Proof: Let $n = p_1^{\alpha_1} p_2^{\alpha_2} \dots\dots . p_k^{\alpha_k}$, where $\alpha_i \geq 1$.

Case 1: Suppose k is an even number.

Let $D_m = \{p_1, p_1p_2, p_3, p_3p_4, \dots\dots\dots\dots, p_{k-1}, p_{k-1}p_k\}$ and let $S = V - D_m$. All the vertices $u \in S$ for which GCD $(u, p_i) = p_i$ for $i = 1, 3, \dots, k-1$ are adjacent with the vertex p_i in D_m. All the vertices $v \in S$ for which *GCD* $(v, p_i . p_{i+1}) = p_{i+1}$ for $i = 1, 3, \dots, k-1$ are adjacent with the vertex $p_i . p_{i+1}$. Since every vertex in S has atleast one prime factor viz., $p_1, p_2, \dots\dots, p_k$ (as they are the divisors of n) every vertex in S is adjacent to atleast one vertex in D_m. Thus D_m becomes a dominating set of $G(V_n)$.

Also the induced subgraph $\langle D_m \rangle$ admits a perfect matching with the pairs of vertices given by $\{p_1, p_1p_2; p_3, p_3p_4; \dots\dots\dots; p_{k-1}, p_{k-1}p_k\}$ as follows.

Any pair of vertices say, p_i, p_ip_{i+1} in D_m are adjacent since GCD $(p_i, p_ip_{i+1}) = p_i$.

Also there is no edge between the vertices p_ip_{i+1} *and* p_{i+2} as GCD $(p_ip_{i+1}, p_{i+2}) \neq p_i$ for any i. Further by the selection of pairs of vertices in D_m, no two pairs of vertices say, p_i, p_ip_{i+1} *and* $p_{i+2}, p_{i+2}p_{i+3}$ are adjacent.

Thus $\langle D_m \rangle$ admits a perfect matching. Hence D_m becomes a matching dominating set of $G(V_n)$.

We now claim that D_m is minimal. Suppose we remove any vertex p_i from D_m. Then the paired vertex of p_i, viz., $p_i p_{i+1}$ is not adjacent to any other vertices of D_m and becomes isolated in $\langle D_m \rangle$. Similar is the case if we delete any vertex $p_i p_{i+1}$ from D_m.

Again if we remove two vertices say p_j and $p_j p_{j+1}$ from D_m then an edge is removed in $\langle D_m \rangle$. But this does not make D_m a dominating set.

Thus D_m becomes minimal. If we construct matching dominating set in any other manner, then clearly it will be of larger size than D_m, or same size of D_m because of properties of prime numbers.

Hence $\gamma_m\left(G(V_n)\right) = |D_m| = k$.

Case 2: Let k be an odd number. Similar to case 1, it can be easily seen that $D_m' =$

$$\{ p_1, p_1p_2, p_3, \quad p_3p_4, \ldots\ldots\ldots, p_{k-2}, \; p_{k-2}p_{k-1}, \; p_k, \; p_k\, p_1 \}$$

is a minimum matching dominating set where the induced subgraph $\langle D_m' \rangle$ admits a perfect matching given by

$\{ p_1, p_1p_2 \,;\; p_3, p_3p_4 \,; \ldots, p_{k-2}, p_{k-2}p_{k-1};\; p_k, p_k\, p_1 \}$.

Hence $\gamma_m\left(G(V_n)\right) = \left|D_m'\right| = k+1$.

2.2 Matching Dominating Sets in Direct Product Graph

Theorem 2.2.1: If $n = p_1^{\alpha_1} p_2^{\alpha_2} \ldots\ldots . p_k^{\alpha_k}$, where $\alpha_i \geq 1$, then the matching domination number of $G_1 \times G_2$ is given by

$$\gamma_m(G_1 \times G_2) = \begin{cases} (\lambda+1)\cdot k, & \text{if k is even} \\ (\lambda+1)\cdot(k+1), & \text{if k is odd} \end{cases}$$

where λ is the length of the longest stretch of consecutive integers in V_1 of G_1 each of which shares a prime factor with *n* and k is the core of *n*.

Proof: Suppose $n = p_1^{\alpha_1} p_2^{\alpha_2} \ldots\ldots . p_k^{\alpha_k}$, where $\alpha_i \geq 1$. Consider the graph $G_1 \times G_2$. By Theorem 1.3.2, we have $\gamma\,(G_1 \times G_2) = (\lambda+1)\cdot k$ where λ is the length of the longest stretch of consecutive integers in V_1 of G_1 each of which shares a prime factor with *n* and k is the core of *n*. Hence $\gamma_m(G_1 \times G_2) \geq (\lambda+1)\cdot k$. …………. (1)

We know that $\gamma\,(G_1) = \lambda + 1$ (by Theorem 1.3.1) and let $D_1 = \{ u_{d_1}, \ldots, u_{d_{\lambda+1}} \}$ be a dominating set of G_1 with minimum cardinality, where $u_{d_1}, u_{d_2}, \ldots\ldots, u_{d_{\lambda+1}}$ are consecutive integers. By Theorem 2.1.1, we know that matching dominating set of G_2 is

$$\gamma_m\,(G_2) = \begin{cases} k, & \text{if k is even} \\ k+1, & \text{if k is odd} \end{cases}$$

Hence two cases arise.

Case 1: Suppose k is an even number. Then by Theorem 2.1.1, it can be easily seen that

$D_2 = \{ p_1, p_1p_2 \,;\, p_3, p_3p_4 \,; \ldots\ldots\ldots ; p_{k-1}, p_{k-1}p_k \}$ is a minimum matching dominating set of G_2, as the induced sub graph $\langle D_2 \rangle$ admits a perfect matching by the pairs of vertices given by $\{ (p_i, p_ip_{i+1}) \,/\, i = 1, 3, 5, \ldots, k-1 \}$.

Now to obtain matching dominating set of $G_1 \times G_2$, we proceed as follows.

Let $D = D_1 \times D_2$

$$= \{ u_{d_1}, u_{d_2}, \ldots\ldots, u_{d_{\lambda+1}} \} \times \{ p_1, p_1p_2; \; p_3, p_3p_4; \ldots\ldots\ldots ; p_{k-1}, p_{k-1}p_k \}.$$

Let (u,v) be any vertex in $\langle V - D \rangle$ in $G_1 \times G_2$. Then $u \in G_1$ is adjacent to u_{d_i} for some i, where $1 \leq i \leq \lambda + 1$, as D_1 is a dominating set of G_1. Vertex $v \in G_2$ is adjacent to any of p_j or $p_j\, p_{j+1}$, for $j = 1, 3, 5, \ldots, k-1$ as D_2 is a dominating set of G_2.

Hence every vertex (u,v) of $\langle V - D \rangle$ is adjacent to at least one vertex (u_{d_i}, p_j) or $(u_{d_i}, p_j\, p_{j+1})$ in D. This implies that D is a dominating set of $G_1 \times G_2$.

We now prove that D is a matching dominating set of $G_1 \times G_2$. Again two subcases arise.

Subcase 1: Suppose λ is an even number. Then the induced subgraph $\langle D \rangle$ admits a perfect matching with the pairs of vertices given by

$\{ \langle (u_{d_i}, p_j), (u_{d_{i+1}}, p_j . \, p_{j+1}) \rangle \} \cup \{ \langle (u_{d_{\lambda+1}}, p_j), (u_{d_1}, p_j . \, p_{j+1}) \rangle \}$ for i = 1, 2, 3,……., λ and j = 1, 3, 5, …., $k-1$.

For i = 1, 2 and j = 1, (u_{d_1}, p_1), $(u_{d_2}, p_1 . \, p_2)$ and $(u_{d_2}, p_1), (u_{d_3}, p_1 . \, p_2)$ are two pairs of vertices in D. Since u_{d_1}, u_{d_2} and u_{d_3} are all distinct and $p_1 \neq p_1 . \, p_2$, no two vertices among these four vertices are equal.

Further u_{d_1}, u_{d_2} and u_{d_2}, u_{d_3} are adjacent vertices because GCD $(u_{d_1} - u_{d_2}, n) = 1$ and GCD $(u_{d_2} - u_{d_3}, n) = 1$. Also p_1, $p_1 . \, p_2$ are adjacent vertices since GCD $(p_1, p_1 . \, p_2) = p_1$. Therefore there exists edges between the pair of vertices $(u_{d_1}, p_1), (u_{d_2}, p_1 . \, p_2)$ and $(u_{d_2}, p_1), (u_{d_3}, p_1 . \, p_2)$. This is true for all pairs of other vertices as these four vertices are arbitrary. Thus $\langle D \rangle$ admits a perfect matching and hence $\langle D \rangle$ becomes a matching dominating set of $G_1 \times G_2$, with cardinality$(\lambda+1)k$. Then it follows by equation (1) that $\gamma_m(G_1 \times G_2) = (\lambda+1)\cdot k$

Subcase 2: Suppose λ is an odd number. Then the induced subgraph $\langle D \rangle$ admits a perfect matching with the pairs of vertices as follows.

$\{\langle (u_{d_i}, p_j), (u_{d_{i+1}}, p_j.\ p_{j+1}) \rangle\} \ \cup$
$\{\langle (u_{d_{i+1}}, p_j), (u_{d_i}, p_j.\ p_{j+1}) \rangle\}$for i = 1, 3, 5……., λ and j = 1, 3, 5, …., $k-1$.

Consider any two pairs of vertices in$D$. Let them be $(u_{d_1}, p_1), (u_{d_2}, p_1.\ p_2)$ and $(u_{d_2}, p_1), (u_{d_1}, p_1.\ p_2)$.

Since u_{d_1}, u_{d_2} are consecutive integers, GCD $(u_{d_1} -$ *ud2, n =1* and GCD *p1, p1. p2 = p1.* Hence there exist edges between the pairs of vertices (u_{d_1}, p_1), $(u_{d_2}, p_1.\ p_2)$ and $(u_{d_2}, p_1), (u_{d_1}, p_1.\ p_2)$.

Since $u_{d_1} \neq u_{d_2}$and $p_1 \neq p_1.\ p_2$, no two vertices among these four are equal. This is true for all pairs of other vertices as these four vertices are arbitrary. Thus $\langle D \rangle$ admits a perfect matching and hence $\langle D \rangle$ becomes a matching dominating set of $G_1 \times G_2$, with cardinality $(\lambda + 1)k$. Then it follows by equation (1) that $\gamma_m(G_1 \times G_2) = (\lambda + 1) \cdot k$

Case 2: Suppose k is an odd number. Then by Theorem 2.1.1, we can see that $D_2' =$

$\{p_1,\ p_1p_2\ ;\ p_3,\ p_3p_4\ ; \dots\dots\dots;\ p_{k-2}, p_{k-2}p_{k-1}\ ;\ p_k, p_kp_1\}$ is a matching dominating set of G_2, with minimum cardinality $k+1$. Proceeding in the same way as in case 1, we can prove that

$D' = D_1 \times D_2' \quad = \{u_{d_1}, u_{d_2}, \dots\dots, u_{d_{\lambda+1}}\} \times$

$\{p_1,\ p_1p_2\ ;\ p_3, p_3p_4\ ; \dots\dots\dots;\ p_{k-2}, p_{k-2}p_{k-1}\ ;\ p_k, p_kp_1\}$

is a dominating set with minimum cardinality. Further the induced sub graph $\langle D' \rangle$ admits a perfect matching as shown in the following two subcases.

Subcase 3: Suppose λ is an even number, then

$$\{\langle (u_{d_i}, p_j), (u_{d_{i+1}}, p_j.\ p_{j+1}) \rangle\} \cup \{\langle (u_{d_i}, p_k), (u_{d_{i+1}}, p_k.\ p_1) \rangle\} \cup \{\langle (u_{d_{\lambda+1}}, p_j), (u_{d_1}, p_j.\ p_{j+1}) \rangle\} \cup \{\langle (u_{d_{\lambda+1}}, p_k), (u_{d_1}, p_k.\ p_1) \rangle\}$$

for $i = 1, 2, \dots\dots, \lambda$ and $j = 1, 3, 5, \dots, k-2$ gives a perfect matching.

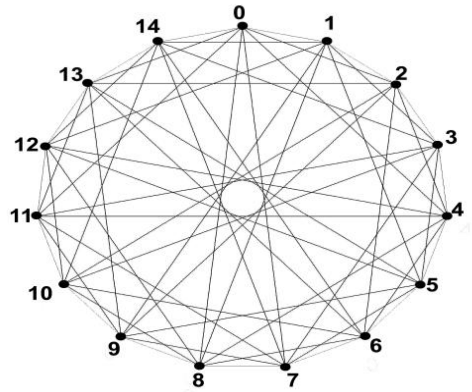
For $i = 1$ and $j = 1$, consider two pairs of vertices (u_{d_1}, p_1), $(u_{d_2}, p_1.p_2)$ and $(u_{d_1}, p_k), (u_{d_2}, p_k.p_1)$. Since $u_{d_1} \neq u_{d_2}$ and $p_1, p_1.p_2, p_k, p_k.p_1$, are all distinct, it follows that no two vertices among these four vertices are equal. Also GCD$(p_1, p_1.\ p_2) = p_1 =$ GCD$(p_k, p_k.\ p_1)$ and GCD $(u_{d_1} - u_{d_2}, n) = 1$ it follows that there exist edges between the pairs of vertices (u_{d_1}, p_1), $(u_{d_2}, p_1.p_2)$ and $(u_{d_1}, p_k), (u_{d_2}, p_k.p_1)$. Thus D' becomes a matching dominating set with minimum cardinality $(\lambda + 1) \cdot (k+1)$. Therefore $\gamma_m(G_1 \times G_2) = (\lambda + 1) \cdot (k+1)$.

Subcase 4: Suppose λ is an odd number. Then the induced subgraph $\langle D' \rangle$ admits a perfect matching given by the pairs of vertices

$\{\langle (u_{d_i}, p_j), (u_{d_{i+1}}, p_j.\ p_{j+1}) \rangle\} \cup$
$\{\langle (u_{d_{i+1}}, p_j), (u_{d_i}, p_j.\ p_{j+1}) \rangle\} \cup$
$\{\langle (u_{d_i}, p_k), (u_{d_{i+1}}, p_k\, p_1) \rangle\} \cup$
$\{\langle (u_{d_{i+1}}, p_k), (u_{d_i}, p_k.\ p_1) \rangle\}$

for $i = 1, 3, \dots\dots, \lambda$ and $j = 1, 3, 5, \dots, k-2$ as in subcase 3. Thus D' becomes a matching dominating set with minimum cardinality $(\lambda + 1) \cdot (k+1)$.

Therefore $\gamma_m(G_1 \times G_2) = (\lambda + 1) \cdot (k+1)$.



3. ILLUSTRATIONS

Let n = 15

Fig. 1
$\boldsymbol{G_1 = G(Z_{15}, \varphi)}$

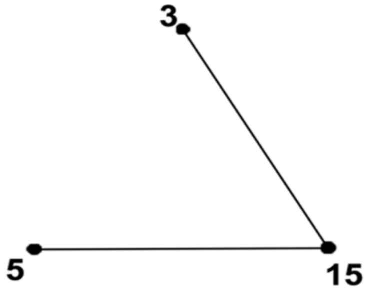


Fig. 2

$G_2 = G(V_{15})$

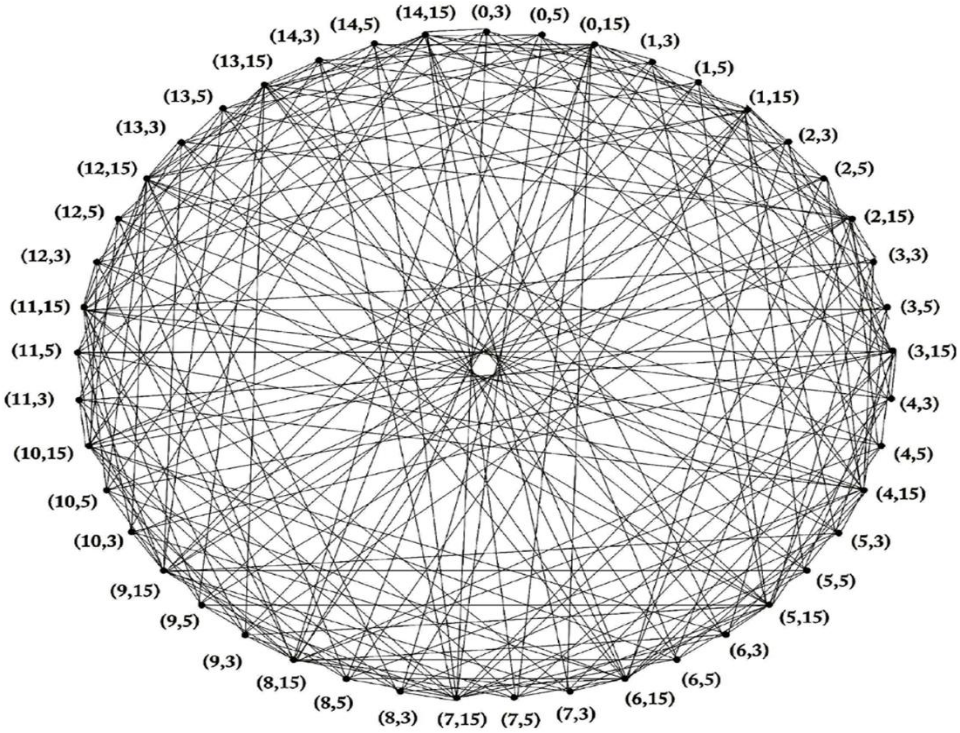


Fig. 3

G_1 X G_2

Table 1. Matching Dominations in Direct Product Graph $G_1 \times G_2$

n Value	Dominating sets	$G_1 = G(Z_n, \varphi)$	$G_2 = G(V_n)$	$G_1 \times G_2$	Domination Number in $G_1 \times G_2$
n = 15	Minimum Dominating set	$\{5, 6, 7\}$	$\{15\}$	$\{(5,3), (5,15), (6,3), (6,15), (7,3), (7,15)\}$	$\gamma = 6$
	Minimum Matching Dominating set	$\{5,6,7,8\}$	$\{\langle 3,15 \rangle\}$	$\{\langle (5,3), (6,15) \rangle;$ $\langle (6,3), (7,15) \rangle;$ $\langle (7,3), (5,15) \rangle\}$	$\gamma_m = 6$

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