Performance Analysis of Low Complexity Error Correcting Codes

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ABSTRACT

From Shannon limit it is known that, for a particular bandwidth and noise characteristics, there exists a maximum rate at which data can be transmitted with arbitrarily small number of errors. Coding schemes are utilized to improve the data transmission efficiency. The paper aims to represent the comparative performance analysis of Low Density Parity Check (LDPC) codes and Bose-Chaudhuri-Hocquenghem (BCH) codes in transmitting data over noisy channel for different parameters. The performance of LDPC block codes is simulated for different decoding schemes and code rates. Performance analysis of LDPC codes is also shown for regular and irregular codes. For fixed error correcting capability, the BCH coding scheme is further simulated for different code length with increasing code length. The simulated output is worthwhile to analyze the performance of a communication system before the physical implementation of the system.

General Terms:

Communication

Keywords:

LDPC, BCH, Code rateifx

1. INTRODUCTION

Error-correcting codes are used to correct erroneous data. The main idea is to introduce some redundant bits into the message. With the revolution of communication systems, high data rate transmission with arbitrarily small error rate is required. Errors are introduced in transmitted data due to presence of noise in transmitting medium. Therefore, an efficient coding scheme is required to reduce the impact of noise on data transmission.
Claude Shannon shows that, every communication channel has a maximum capacity. If transmission rate is less than the capacity, an arbitrarily small error rate transmission is possible [1]. The first construction of error correcting codes was done by Richard Hamming, known as Hamming code. Hamming code can detect up to two and correct up to one bit of errors. Soon after, Marcel Golay generalized Hamming's construction and constructed codes for multiple error correction [2]. In 1959, the development of BCH codes provided a precise control over the number of symbol errors correctable by the code [3]. In 1960, Robert Gallager proposed a low-density parity-check code in his doctoral dissertation, known as LDPC codes which are defined by a sparse parity-check matrix [4]. Due to the limit in computational effort in implementing the coder and decoder for such codes, LDPC codes was ignored for almost 30 years. In 1993, Turbo code makes a breakthrough in coding theory. It achieved almost the Shannon limit performance. At the same time, LDPC code was reconstructed by David J. C. Mackay and it becomes a big competitor of the Turbo code with lower complexity [5].
This paper compares the BER performance between BCH and low to high range LDPC code (code length up to 4000). Decoding complexity of LDPC codes depends on decoding algorithm. This paper also focuses on different decoding algorithms of LDPC code and compares their performance.
This paper is organized as follows. Section 2 and section 3 provides a brief description of BCH and LDPC coding scheme. Section 4 describes the performance analysis of BCH and LDPC codes for different parameters obtained from MATLAB simulation. The overall results are finally briefly described in section 5.

2. BCH CODE

The BCH codes are cyclic codes which mean that any cyclic shift of the transmitted codeword also be a valid codeword. For any integer $m \geq 3$ a BCH code has the following parameters:

$$n = 2m - 1 \tag{1}$$

$$n - k \leq mt \tag{2}$$

$$d_{min} \geq 2t + 1 \tag{3}$$

where, n is codeword length, d_{min} is minimum Hamming distance, t is correcting capability of the code and n-k are redundant bits.
In linear block code, the codeword is generated by separating total message into some pieces of equal length k. If P is the message of length K to be transmitted and G is the generator matrix, the codeword C is achieved by $C = P * G$.
There are many algorithms for decoding BCH codes. The most common ones follow this general outline [6] :

(1) Calculation of the syndrome values for the received vector.
(2) Calculation of the error locator polynomials
(3) Calculation of the roots of this polynomial to get error location positions.
(4) Calculate the error values at these error locations.

Now, if the generator matrix can be expressed as, $G = [I_k|P]$, where I is a k*k identity matrix and P is a k*(n-k) matrix that determines the code redundant bits, then its corresponding parity check matrix can be formed by $H = [I_{n-k}|P]$. It satisfies the following condition, $GH^T = 0$. Now, let V is our received codeword. Then the syndrome is $S = VH^T$. If $S = 0$, then there are no errors. And if $S \neq 0$, it means there is at least one error happened in the received codeword.

3. LDPC CODE

Low-Density Parity-Check (LDPC) codes are a class of linear block codes that are characterized by sparse parity check matrices H. LDPC codes have better block error performance than turbo codes, because the minimum distance of an LDPC code increases proportionately to the code length with a high probability [7]. LDPC codes can be represented both in matrix form and in graphical way. A parity check matrix H of dimension $m * n$ defines a LDPC code as (n,k), where $k = n - m$ are message bits. The code rate is given by $r = k/n$. For a matrix to be called low-density the two conditions, $w_c \leq n$ and $w_r \leq m$ must be satisfied where w_r and w_c defining the number of 1's in each row and the number of 1's in each column respectively [8]. LDPC codes can be classified as regular and irregular. A LDPC code is called regular if w_c is constant for every column and $w_r = w_c * (n/m)$ is also constant for every row [8]. Being low density, if the no. of 1's in rows and columns isn't constant, then it is called irregular LDPC code. For a generalized linear block code

$$C = XG \quad (4)$$

$$CH^T = 0 \quad (5)$$

When X is a message sequence and C is valid codeword.

4. SIMULATION RESULT

Simulation was performed using MATLAB V7.6. All the simulations were performed in the AWGN channel.
Here the performance of BCH and LDPC coding scheme is compared. Table 1 shows the simulation parameters for BCH and LDPC codes.Fig. 1 on page 2 shows performance analysis among LDPC, BCH and uncoded system.The simulated output in Fig. 1 represents that the LDPC code outperforms the performance of BCH coding scheme for same signal to noise ratio.
Fig. 2 on page 2 represents the simulated performance of LDPC code for different decoding methods. Here four different decoding

Table 1. Simulation parameters for LDPC and BCH codes

Code	(n,k)	Rate
BCH	(31,16)	.51
LDPC	(4000,2000)	.5

methods namely Bit-Flipping algorithm, Belief Propagation, Sum-product algorithm and Min-sum algorithm are used. The simulated output in Fig. 2 shows that the performance of the Bit-Flipping algorithm is worst. The belief propagation and sum product
algorithm are both same. They only differ in computational complexity, where the sum product algorithm is implemented using logarithm. Due to this, both the curves of these two algorithms overlap. The Min-Sum algorithm is slightly worse compared to the sum-product algorithm. Therefore, its curve is in the close proximity of the sum product curve.
Fig. 3 on page 3 shows the simulated output of LDPC code for different code rate. The performance is best for code rate 0.5 and it deteriorates as the code rate increases. The result is desired because increase in code rate means the number of redundant bits

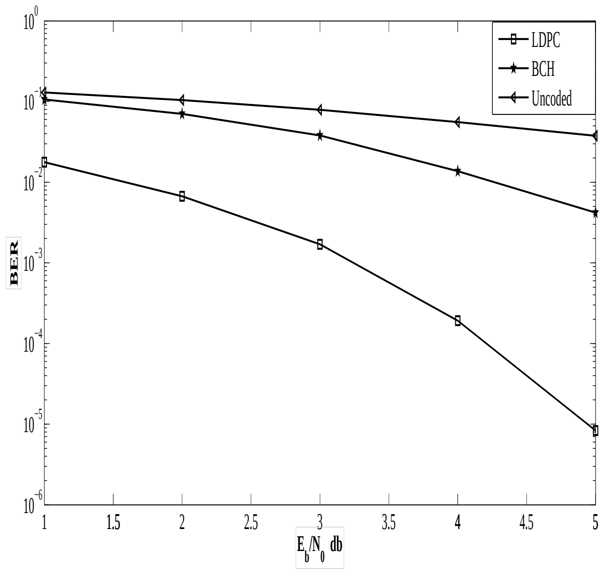

Fig. 1. Performance analysis of BCH and LDPC coding scheme

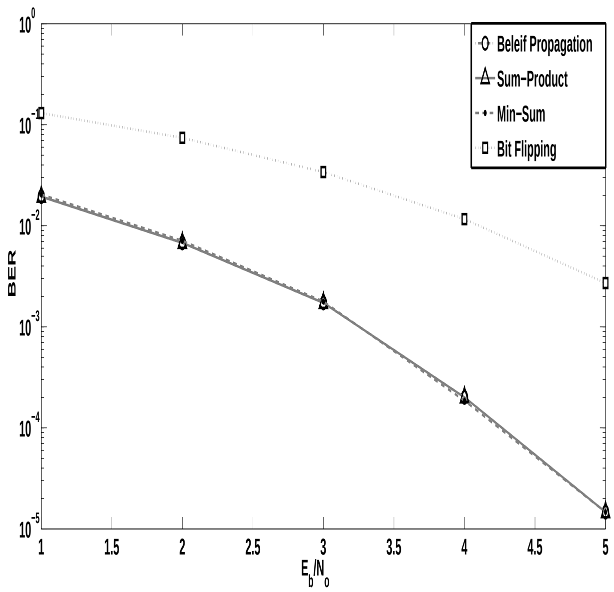

Fig. 2. Performance analysis of LDPC codes for different decoding algorithms

are reduced which in turn reduces the error correction capability.
Fig. 4 on page 3shows the decoding speed of LDPC code for different decoding methods. Simulation shows, Log domain algorithm takes maximum time for decoding whereas bit flipping algorithm takes least time. The output is reasonable since, bit flipping algorithm involves only flipping the received bit depending on hard decision, where Log domain algorithm involves soft decision based on received bit probability using logarithmic algebra.
The simulated output in Fig. 5 on page 3 is performed for bch code (15, 5), (31, 16), (63, 45) and (127, 106). The simulated output shows that for a fixed error correction capability, as the code length of the BCH code is increased, the performance deteriorates. The output is reasonable, because for a fixed error correcting capability

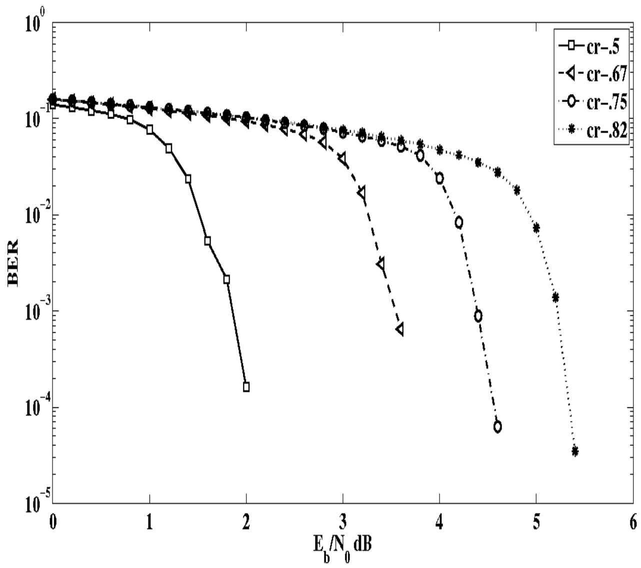

Fig. 3. Performance analysis of LDPC codes for different code rates

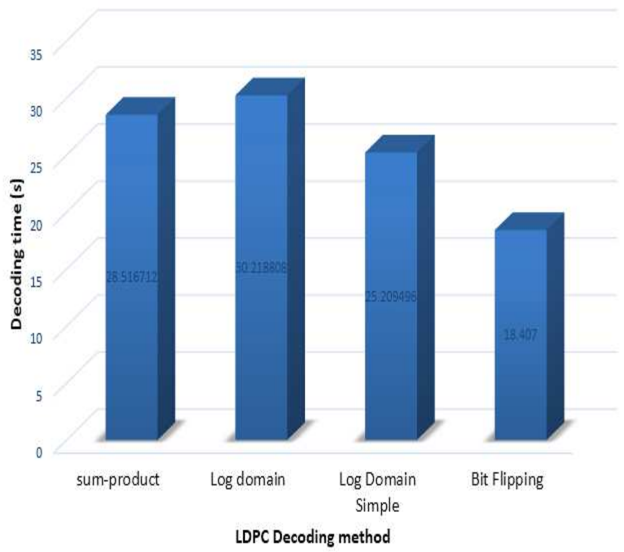

Fig. 4. Decoding speed for different LDPC decoding methods

when the length of the BCH code is increased the code rate increases. This increase in code rate means that the no. of redundant bits are reduced, which is responsible for performance degradation.

In Fig. 6 on page 3, the simulated output shows that in the low E_b/N_o region irregular LDPC codes perform better than regular ones. This is because the nodes with higher degrees converge faster and assist the nodes with lower degrees. However on the other hand drawback is that irregularity of the code results in a more complex hardware architecture. Fig. 7 on page 4 shows the simulated output for the decoding speed of LDPC and BCH code for handling same amount of data. The simulated output shows that for handling the same amount of data LDPC requires much more time compared to BCH coding scheme. However, for when code length becomes very long, practical implementation using BCH coding scheme becomes impractical due to increased decoding complexity of BCH code.

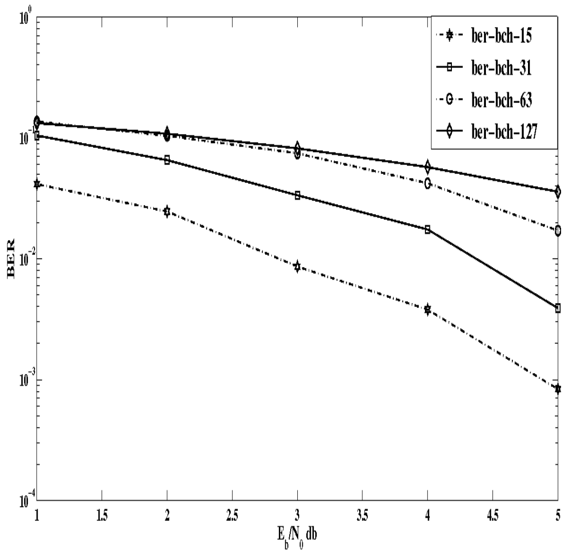

Fig. 5. Performance comparison of BCH code for different code length and fixed error correction capability

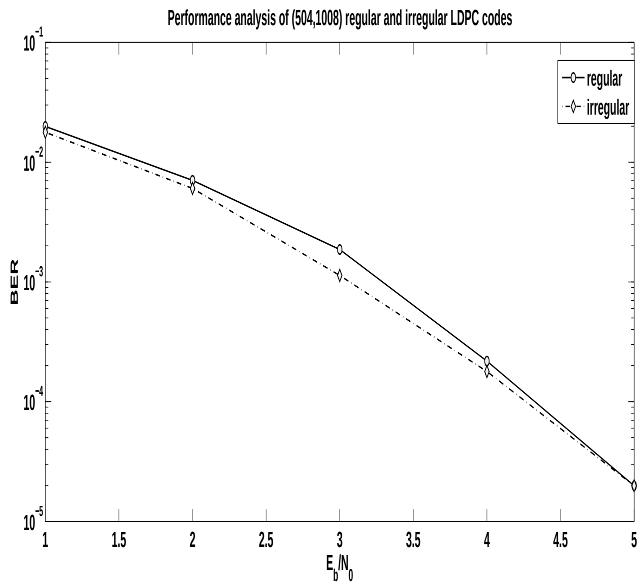

Fig. 6. Performance comparison of regular and irregular LDPC code

5. CONCLUSION

This paper is focused on BCH and LDPC coding scheme. Here, different comparisons between BCH and LDPC code are made. On the basis of the results obtained in the present simulation based study, it can be concluded that the LDPC coding scheme is very much effective for retrieval of transmitted data in noisy

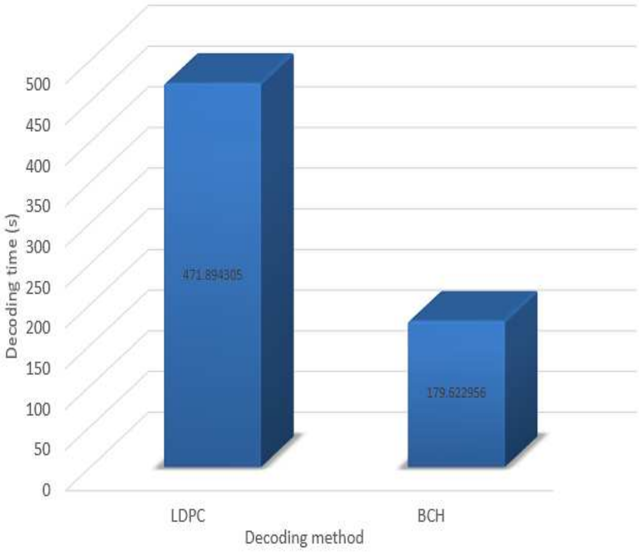

Fig. 7. Comparison of decoding speed between LDPC and BCH method

transmitting medium compared to BCH coding scheme. Though the BCH coding scheme provides a definite error correcting capability, the complexity of this code increases as the code length is increased. Further, for a very large code length BCH coding scheme becomes impractical.In order to achieve the same error probabilities, LDPC codes can tolerate lower signal-to-noise-ratio conditions than the conventional codes, which can imply higher densities and lower media costs. Moreover, from simulation it can also be concluded that the belief propagation decoding algorithm of LDPC codes provides the best decoding performance. The decreasing performance of LDPC code with increasing code rate is also verified by the computer simulation. From simulation it was shown that the performance of irregular LDPC code is better than regular LDPC code. However, the irregular LDPC code results in a complex hardware architecture which constitutes the tradeoff between regular and irregular LDPC code. Moreover, it is found that the bit flipping algorithm requires minimum time to decode while sum-product algorithm requires maximum decoding time.

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