Group Magic Labeling of Multiple Cycles

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ABSTRACT

Let G = (V, E) be a connected simple graph. For any non-trivial additive abelian group A , let $A^* = A - \{0\}$. A function f: E (G) → A* is called a labeling of G. Any such labeling induces a map f^+ : V (G) → A, defined by $f^+(v) = \sum f(uv)$, where the sum is over all uv ∈ E(G). If there exist a labeling f whose induced map on V (G) is a constant map, we say that f is an A-magic labeling of G and that G is an A-magic graph. In this paper we obtained the group magic labeling of cycles with a common vertex, a chain of three cycles and even number of times even cycles in a chain.

Key words

A-magic labeling, Group magic, cycles with a common vertex, chain of cycles.

1. INTRODUCTION

Labeling of graphs is a special area in Graph Theory. A detailed study has been done in [1]. Originally Sedlacek has defined magic graph as a graph whose edges are labeled with distinct non- negative integers such that the sum of the labels of the edges incident to a particular vertex is the same for all vertices. If those labels are from a non trivial additive abelian group A, then graph is said to be Group magic graph or A-magic. A-magic graph is introduced by J sedlacek. Recently A- magic graphs are studied and investigated in the literature [2,3,4,5,6].

An A-magic graph G is said to be Z_k -magic graph if we choose the group A as Z_k- the group of integers mod k. These Z_k- magic graphs are referred as k - magic graphs. Baskar Babujee, L.Shobana [7] have shown few graphs like cycle graphs, complete, bistar, ladder, etc. are Z_3 –magic. A detail study about zero-sum magic graphs and their null sets was done by Ebrahim salehi in [8].

It was proved in [9] that wheels, fans, cycles with a P_k chord, books are group magic. In [10] it was proved that wheels can be labeled with at most n distinct labels, where n is the number of vertices. In [11] the graph $B(n_1, n_2, \ldots, n_k)$, the k copies of C_{n_j} with a common edge or path is labeled. In [12] a biregular graph is defined and group magic labeling of few biregular graphs have been dealt with. In [13] the group magic labeling of two or more cycles with a common vertex is derived. As an extension of this result in this paper the group magic labeling of a chain of three cycles and even number of times even cycles in a chain are considered.

2. DEFINITIONS

2.1 Let G = (V, E) be a connected simple graph. For any non-trivial additive abelian group A, let $A^* = A - \{0\}$. A function f: E (G) → A* is called a labeling of G. Any such labeling induces a map f^+: V (G) → A, defined by $f^+(v) = \sum_{uv \in E(G)} f(u,v)$ If there exists a labeling f which induces a constant label c on V (G), we say that f is an A-magic labeling and that G is an A-magic graph with index c.

2.2 A A-magic graph G is said to be Z_k -magic graph if we choose the group A as Z_k- the group of integers mod k. These Z_k- magic graphs are referred as k - magic graphs.

2.3 A graph G = (V; E) is called fully magic if it is A-magic for every abelian group A. For example, every regular graph is fully magic.

2.4 A graph G = (V,E) is called non-magic if for every abelian group A; the graph is not A-magic.

The most obvious class of non-magic graphs is P_n (n ≥ 3); the path of order n. As a result, any graph with a pendant path of length n ≥ 3 would be non-magic [7].

2.5 A k-magic graph G is said to be k-zero-sum (or just zero sum) if there is a magic labeling of G in Z_k that induces a vertex labeling with sum zero.

2.6 $B_V(n_1,n_2,\ldots n_k)$ denotes the graph with k cycles C_j (j ≥ 3) of size n_j in which all C_j 's (j=1,2,…k) have a common vertex.

2.7 The chain of cycles $C(n_1,n_2,\ldots n_k)$ denotes the graph of k cycles $C_1,C_2,\ldots C_k$ of sizes $n_1,n_2,\ldots n_k$ such that for i=1,2,..k-1, C_i and C_{i+1} have a common vertex.

3. OBSERVATION [1]

By labeling the edges of even cycle as α, the vertex sum is 2α or if their edges are labeled as α_1 &α_2 alternatively then the vertex sum is $\alpha_1+\alpha_2$. But the edges of odd cycles can only be labeled as α with the index sum 2α.

4. MAIN RESULTS

4.1 Theorem

The graph of two cycles C_1 and C_2 with a common vertex is group magic when both cycles are either odd or even.

Proof:

G is the graph of 2 cycles C_1 and C_2 with a common vertex. Let u be the common vertex. The vertices which are adjacent with u of the two cycles C_1 and C_2 be u_1, v_1 and u_2, v_2 respectively. If the edges uu_1, uv_1, uu_2, and uv_2 are labeled as α_1, α_2, α_3 &α_4, the α´s are chosen from A* such that the edge labels are nonzero, then the vertex sum at u is $\alpha_1+\alpha_2+\alpha_3+\alpha_4$. To get this vertex sum at each of the other vertices we have to label the edges of cycle C_1 as $\alpha_2+\alpha_3+\alpha_4$ and α_1 alternatively from the edge which is adjacent with uu_1. Similarly the edges of the cycle C_2 are labeled as $\alpha_1+\alpha_2+\alpha_4$ and α_3 alternatively from the edge which is adjacent with uu_2. This labeling gives the vertex sum as $\alpha_1+\alpha_2+\alpha_3+\alpha_4$ at all vertices except at v_1 and v_2.

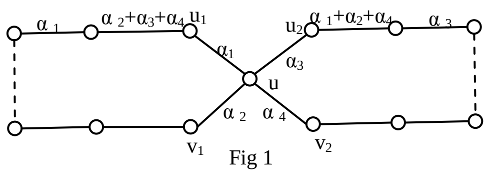

Fig 1

Case 1: Both C_1 and C_2 are odd cycles.
If C_1 and C_2 are odd cycles the edge which is adjacent with uv_1 gets the label as $\alpha_2+\alpha_3+\alpha_4$ and the edge which is incident with uv_2 gets the label as $\alpha_1+\alpha_2+\alpha_4$. So at v_1 and v_2 the magic condition requires

$\alpha_2+\alpha_3+\alpha_4+\alpha_2 = \alpha_1+\alpha_2+\alpha_3+\alpha_4$

$\alpha_1+\alpha_2+\alpha_4+\alpha_4 = \alpha_1+\alpha_2+\alpha_3+\alpha_4$

Hence $\alpha_1=\alpha_2$, and $\alpha_3=\alpha_4$.

Thus when the cycles C_1 and C_2 are odd, the edges incident with u of C_i (i=1,2) are labeled as α_i (i=1,2) the remaining edges of C_1 are labeled as $\alpha_1+2\alpha_2$ and α_1 alternatively while those of C_2 labeled as $2\alpha_1+\alpha_2$ and α_2 alternatively. This labeling gives the vertex sum $2(\alpha_1+\alpha_2)$.

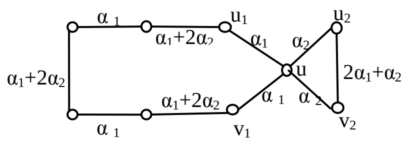

Fig 2

Case 2: Both C_1 and C_2 are even
If C_1 and C_2 are even cycles the edge which is adjacent with uv_1 gets the label as α_1 and the edge which is adjacent with uv_2 gets the label as α_3. So at v_1 and v_2 the magic condition requires

$\alpha_1+\alpha_2 = \alpha_1+\alpha_2+\alpha_3+\alpha_4$

$\alpha_3+\alpha_4 = \alpha_1+\alpha_2+\alpha_3+\alpha_4$

Hence $\alpha_1+\alpha_2=0$, and $\alpha_3+\alpha_4 = 0$

This leads to the vertex sum as zero.

Hence when the cycles C_1 and C_2 are even, by the above discussion G is zero sum magic.

Case 3: Either C_1 or C_2 is odd
Suppose C_1 is odd and C_2 is even, the edge which is adjacent with uv_1 gets the label as $\alpha_2+\alpha_3+\alpha_4$ and the edge which is adjacent with uv_2 gets the label as α_3.

So at v_1 and v_2 the magic condition requires

$\alpha_2+\alpha_3+\alpha_4+\alpha_2 = \alpha_1+\alpha_2+\alpha_3+\alpha_4$

$\alpha_3+\alpha_4 = \alpha_1+\alpha_2+\alpha_3+\alpha_4$

Hence $\alpha_1=\alpha_2$, and $\alpha_1+\alpha_2=0$.

Which in turn $\alpha_1 = 0$ which is impossible. □

This result can be extended to $B_V(n_1,n_2,\ldots n_k)$.

4.2 Theorem

$B_V(n_1,n_2,\ldots n_k)$ for $k \geq 3$ is group magic.

Proof :

Denote the common vertex in $B_V(n_1,n_2,\ldots n_k)$ as u and the vertices of C_j which are adjacent to u as u_j and v_j for every j = 1,2,…k.

Case 1: Among C_j's (j=1,2,…k) at least two are even cycles.
For our convenience let us take $C_1, C_2, \ldots, C_s$ are the odd cycles and the remaining k-s cycles are even. In each C_j, label the edges uu_j and uv_j as α_{2j-1} and α_{2j}. At u the vertex sum is $\sum_{i=1}^{2k}\alpha_i$. Choose α′s from A* such that the edge labels are nonzero.

In C_1 the remaining edges are labeled $\sum_{i=1}^{2k}\alpha_i - \alpha_1$ and α_1 alternatively from the edge which is incident with u_1. At v_1 the magic condition requires

$\sum_{i=1}^{2k}\alpha_i - \alpha_1+\alpha_2 = \sum_{i=1}^{2k}\alpha_i$.

That is $\alpha_1=\alpha_2$

Similarly we can do for the cycles C_j for j=2,…,s.

we have $\alpha_{2j-1} = \alpha_{2j}$ for j=2,…s .

In each C_j for j = s+1,s+2,…k, the remaining edges are labeled $\sum_{i=1}^{2k}\alpha_i - \alpha_{2j-1}$ and α_{2j-1} alternatively from the edge which is incident with u_j. At v_j the magic condition requires

$\alpha_{2j-1}+\alpha_{2j} = \sum_{i=1}^{2k}\alpha_i$.

That is $\sum_{i=1, i\neq 2j-1, i\neq 2j}^{2k}\alpha_i = 0$ for each j = s+1,s+2,…k

These k-s equations can be written as,

$$2\sum_{i=1}^{s}\alpha_{2i-1} + \sum_{i=s+1, i\neq j}^{k}(\alpha_{2i-1}+\alpha_{2i}) = 0 \qquad (1)$$

Taking $M = -2\sum_{i=1}^{s}\alpha_{2i-1}$

Equation (1) gives

$\sum_{i=s+1, i\neq j}^{k}(\alpha_{2i-1}+\alpha_{2i}) = M$

From these k-s equations we get $\alpha_{2j-1}+\alpha_{2j} = \alpha_{2i-1}+\alpha_{2i}$ for every i and j = s+1,s+2,…k.

Substituting in (1) we get for each j = s+1,s+2,…k

$2\sum_{i=1}^{s}\alpha_{2i-1} + (k-s-1)(\alpha_{2j-1}+\alpha_{2j}) = 0$

$$\text{That is } \alpha_{2j-1}+\alpha_{2j} = \frac{1}{k-s-1}M \qquad (2)$$

Provided k-s ≠1, that is $B_V(n_1,n_2,\ldots n_k)$ contains at least two even cycles.

Thus choosing α_j for j = s+1,s+2,…k in such a way that it satisfies (2) will give the group magic labeling with the vertex sum

$$\sum_{i=1}^{2k}\alpha_i = -M + (k-s)(\alpha_{2j-1}+\alpha_{2j})$$
$$= -M + \frac{k-s}{k-s-1}M$$
$$= \frac{1}{k-s-1}M \qquad (3)$$

If all the cycles are even then M takes the value zero. So $B_V(n_1,n_2,\ldots n_k)$ is zero sum magic when all n′s are even.

Case 2: Among C_j's (j=1,2,…k) only one C_j is even cycle.
Let C_k be the even cycle. Label the edges uu_j and uv_j as α_j (j=1,2,…k-1) and the remaining edges of those C_j's are labeled $T-\alpha_j$ and α_j alternatively, where T is the vertex sum.

Label the edges uu_k and uv_k as α_k and $\alpha_{k'}$. Here the vertex sum is

$T = 2\sum_{i=1}^{k-1}\alpha_i + \alpha_k + \alpha_{k'}$

Since C_k is even cycle, the remaining edges of C_k are labeled as $T-\alpha_k$ and α_k alternatively from the edge which is incident with u_k. At v_k the magic condition requires

$\alpha_k+\alpha_{k'} = 2\sum_{i=1}^{k-1}\alpha_i+\alpha_k+\alpha_{k'}$

Shows $\sum_{i=1}^{k-1}\alpha_i=0$ (4)

Thus choosing α_j for j = 1,2,…k-1 in such a way that it satisfies (4) will give the group magic labeling with the vertex sum $T = \alpha_k+\alpha_{k'}$. Here k>2. If k=2 the condition (4) shows $\alpha_1=0$ which is impossible. This one is derived in case 3 of theorem 4.1.

Case 3: All C_j's (j=1,2,…k) are odd.

Label the edges uu_j and uv_j as α_j (j=1,2,…k) and the remaining edges of C_j are labeled alternatively as $2\sum_{i=1}^{k}\alpha_i-\alpha_j$ and α_j. This labeling induces a vertex sum $2\sum_{i=1}^{k}\alpha_i$. □

4.3 Illustrations

For case 1

Consider k=4 and s=2 and choose the edge labels $\alpha_1=\alpha_2=1$, $\alpha_3=\alpha_4=2$, then M = -2(1+2) = -6 and k-s-1 = 1

Now choose α_5, α_6, α_7, and α_8 such that

$\alpha_5+\alpha_6 = -6$ and $\alpha_7+\alpha_8 = -6$, here the vertex sum is -6.

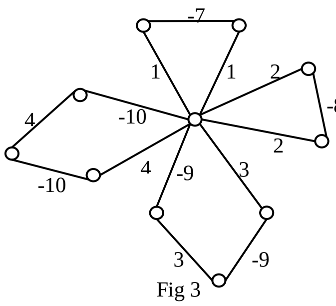

Fig 3

For case 2

Let k = 4 and s = 1

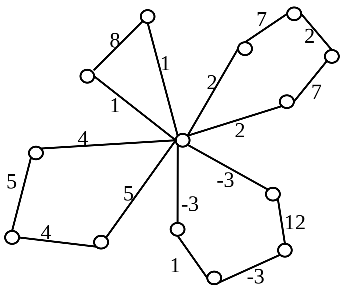

Fig 4

4.4 Corollary

$B_V(n_1, n_2,\ldots n_k)$ for $k\geq 3$ is h - magic for h > k where k is the maximum of all edge labels.

4.5 Theorem

The chain of three cycles $C(n_1,n_2,n_3)$ is group magic.

Proof:

Consider 3 cycles C_1, C_2, and C_3. Let u′ be the vertex common to C_1 and C_2 and the vertex common to C_2 and C_3 be u″. Let the vertices adjacent to u′ in C_1 be u_1, v_1, those in C_2 be u_2, v_2 and the vertices adjacent to u″ in C_2 be u_2', v_2', those in C_3 be u_3, v_3.

We label the edges $u'u_1$, $u'v_1$, $u'u_2$ and $u'v_2$ as α_1, α_2, α_3, and α_4 respectively. Choose α′s from A* such that the edge labels are nonzero.

To get the vertex sum as $\alpha_1+\alpha_2+\alpha_3+\alpha_4$ at the vertices of C_1 we label the other edges of C_1 as $\alpha_2+\alpha_3+\alpha_4$ and α_1 alternatively from the edge which is adjacent with $u'u_1$.

Result:1

If C_1 is odd cycle the edge incident with v_1 will receive the label as $\alpha_2+\alpha_3+\alpha_4$. To satisfy the magic condition at v_1 we require

$\alpha_2+\alpha_3+\alpha_4+\alpha_2 = \alpha_1+\alpha_2+\alpha_3+\alpha_4$.

Hence $\alpha_2 = \alpha_1$. Here the vertex sum is $2\alpha_1+\alpha_3+\alpha_4$.

Result : 2

If C_1 is even, the edge incident with v_1 will receive the label as α_1.

The magic condition requires

$\alpha_1+\alpha_2 = \alpha_1+\alpha_2+\alpha_3+\alpha_4$.

Hence $\alpha_3+\alpha_4 = 0$

This shows the edges $u'u_2$ and $u'v_2$ in C_2 receives labels α_3 and $-\alpha_3$. Here the vertex sum is $\alpha_1+\alpha_2$.

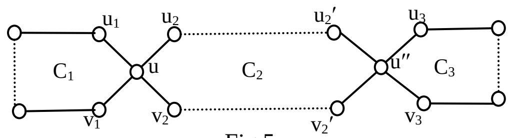

Fig 5

The above discussed results also hold for the cycle C_3. In C_2 there are 2 paths joining u′ and u″. Let them be P and Q. Now we see the labeling of G in the following cases.

Case 1: C_2 even C_1 and C_3 are odd

By the result (1) the edges $u''u_3$ and $u''v_3$ in C_3 receives the same label α_1.

To get the vertex sum at the vertices of C_2 as $2\alpha_1+\alpha_3+\alpha_4$, along P the edges of C_2 can be labeled as $2\alpha_1+\alpha_4$ and α_3 alternatively from the edge which is incident with u_2 and along Q the edges of C_2 labeled as $2\alpha_1+\alpha_3$ and α_4 alternatively from the edge which is incident with v_2.

As C_2 is even, both P and Q contain either odd or even number of edges. If both P and Q contain odd number of edges, along P, $u_2'u''$ and along Q, $v_2'u''$ gets the label as α_3 and α_4. The remaining edges in C_3 receive $\alpha_1+\alpha_3+\alpha_4$ and α_1 alternatively. This labeling gives the vertex sum $2\alpha_1+\alpha_3+\alpha_4$.

As P and Q are the paths joining u′ and u″ and since G is a simple graph both P and Q cannot contain exactly one edge. So C_2 must contain more than four edges. Refer figure 6.

If both P and Q contain even number of edges, along P, $u_2'u''$ and along Q, $v_2'u''$ gets the label as $2\alpha_1+\alpha_4$ and $2\alpha_1+\alpha_3$.

To attain the magic condition at u″, the edges of C_3 which are incident with u″ must take the label as $-\alpha_1$ and $-\alpha_1$. The remaining edges in C_3 receive $3\alpha_1+\alpha_3+\alpha_4$ and $-\alpha_1$ alternatively.

In this case also the vertex sum is $2\alpha_1+\alpha_3+\alpha_4$. Refer figure 7.

Case 2 : All C_1,C_2 and C_3 are even.

From result (2) $\alpha_4 = -\alpha_3$. If both P and Q contain odd number of edges, along P, $u_2'u''$ and along Q, $v_2'u''$ gets the label as α_3 and $-\alpha_3$. Here α_3 should be chosen in such a way that $\alpha_3 \neq$

$\alpha_1+\alpha_2$. The edges of C_1 and C_3 receive the labels α_1 and α_2 alternatively. Here the vertex sum is $\alpha_1+\alpha_2$. Refer figure 8.

If both P and Q contain even number of edges, along P, $u_2'u''$ and along Q, $v_2'u''$ gets the label as $\alpha_1+\alpha_2-\alpha_3$ and $\alpha_1+\alpha_2+\alpha_3$. Since C_3 is even from result (2) we have $\alpha_1+\alpha_2-\alpha_3 = -(\alpha_1+\alpha_2+\alpha_3)$.

This shows $\alpha_1+\alpha_2 = 0$. In this case the graph is zero sum magic.

Case 3: C_2 even and either C_1 or C_3 is odd.
Suppose C_1 is even from the result (2) $\alpha_4 = -\alpha_3$. If both P and Q contain odd number of edges, along P, $u_2'u''$ and along Q, $v_2'u''$ gets the label as α_3 and $-\alpha_3$. To attain the magic condition at u'', the edges of C_3 which are incident with u'' must take the label as α_1 and α_2. Since C_3 is odd by the result (1) we have $\alpha_2 = \alpha_1$. Hence the edges of C_2 along P receive labels $2\alpha_1-\alpha_3$ and α_3 alternatively from the edge which is incident with u_2 and those along Q receives $2\alpha_1+\alpha_3$ and $-\alpha_3$ alternatively from the edge which is incident with v_2. Here α_3 should be chosen in such a way that $\alpha_3 \neq 2\alpha_1$. The remaining edges in C_3 receive α_1. This labeling gives the vertex sum $2\alpha_1$. Refer figure 9.

If both P and Q contain even number of edges, discussing as before in this case also the vertex sum is $2\alpha_1$. Refer figure 10.

Case 4: All C_1, C_2 and C_3 are odd.
By the result (1) the edges in C_1 which are incident with u' receives the same label α_1. Since C_2 is odd either P or Q contains odd number of edges. Suppose P contain odd number of edges, the edge in P which is incident with u'' receives α_3. The edge in Q which is incident with u'' receives $2\alpha_1+\alpha_3$. The magic condition at u'' requires the labels of edges of C_3 incident with u'' as α_4 and $-\alpha_3$. Since C_3 is odd from result (1) we have $\alpha_4 = -\alpha_3$. In this case the vertex sum is $2\alpha_1$ and α_3 should be chosen in such a way that $\alpha_3 \neq 2\alpha_1$. Suppose Q contain odd number of edges, same result derived from the similar argument. Refer figure 11.

Case 5: C_2 is odd and both C_1 and C_3 are even
Since C_1 is even, from the result (2) $\alpha_4 = -\alpha_3$. If P contains odd number of edges, the edge in P which is incident with u'' receives α_3. The edge in Q which is incident with u'' receives $\alpha_1+\alpha_2+\alpha_3$. Since C_3 is even from result (2) we have $\alpha_1+\alpha_2+\alpha_3=-\alpha_3$.
Shows $\alpha_3 = -(\alpha_1+\alpha_2)/2$.

Hence choosing α_3 as $-(\alpha_1+\alpha_2)/2$ the labeling is possible and it gives the vertex sum $\alpha_1+\alpha_2$. Suppose Q contain odd number of edges, same result derived from the similar argument.

Case 6: C_1, C_2 are odd and C_3 even
By the result (1) the edges in C_1 which are incident with u' receives the same label α_1. If P contains odd number of edges, the edge in P which is incident with u'' receives α_3. The edge in Q which is incident with u'' receives $2\alpha_1+\alpha_3$. Since C_3 is even from result (2) we have

$$2\alpha_1+\alpha_3 = -\alpha_3.$$

And therefore $\alpha_3 = -\alpha_1$.

Hence choosing α_3 as $-\alpha_1$ labeling is possible and it gives the vertex sum $\alpha_1+\alpha_4$. □

4.6 Illustrations

Case 1

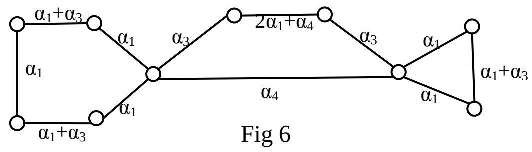

Fig 6

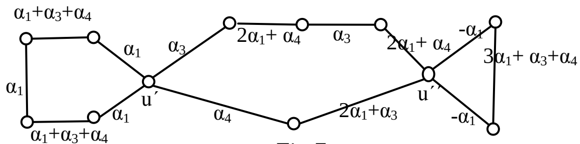

Fig 7

Case 2

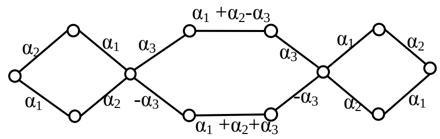

Fig 8

Case 3

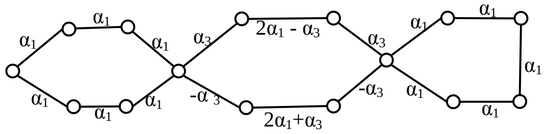

Fig 9

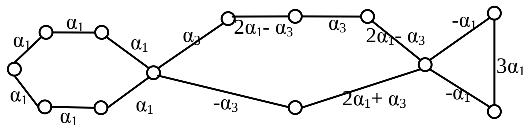

Fig 10

Case 4

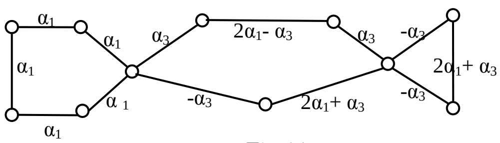

Fig 11

4.7 Corollary
G is h - magic for h > k where k is the maximum of all edge labels.

4.8 Theorem
The chain of cycles $C(n_1,n_2,\ldots n_k)$ is zero sum magic when k and all C_j's are even.

Proof:
The case k = 2 is in the case- 2 of theorem 4.1.

For i=1,2,…,k-1, let u_i be the vertex common to C_i and $C_{(i+1)}$ and let P_i and Q_i are the paths joining u_i and $u_{(i+1)}$.

Consider C $(n_1, n_2,\ldots n_k)$ for k=4. The labeling of the graph is discussed as in theorem 4.5. From result (2) of theorem-4.5 the edges of C_1which are incident with u_1 gets the labels α_1

and α_2 and the edges of P_1 and Q_1 which are incident with u_1 gets labels α_3 and $-\alpha_3$.Since all cycles are even, for i=1,2 and 3 both P_i and Q_i contains either even or odd number of edges.

Consider both P_i and Q_i contains even number of edges. In C_2 if the edge of P_1 and Q_1 gets the label α_3 and $-\alpha_3$ then the edge of P_1 and Q_1 which is incident with u_2 gets the label $\alpha_1+\alpha_2-\alpha_3$ and $\alpha_1+\alpha_2+\alpha_3$. To get the vertex sum at u_2 as $\alpha_1+\alpha_2$, the edges of C_3 incident with u_2 gets $-\alpha_1$ and $-\alpha_2$. The edges of C_3 which are incident with u_3 receive the label $2\alpha_1+\alpha_2$ and $\alpha_1+2\alpha_2$. As C_4 is even from result (2), $2\alpha_1+\alpha_2+\alpha_1+2\alpha_2=0$. This proves the vertex sum as zero.

Consider both P_i and Q_i contains odd number of edges. In C_2 if the edge of P_1 and Q_1 gets the label α_3 and $-\alpha_3$ then the edge of P_1 and Q_1 which is incident with u_2 also gets the label α_3 and $-\alpha_3$ respectively. To get the vertex sum at u_2 as $\alpha_1+\alpha_2$, the edges of C_3 incident with u_2 gets α_1 and α_2. The edges of C_3 which are incident with u_3 receive the label α_1 and α_2. As C_4 is even from result (2), $\alpha_1+\alpha_2=0$. This proves the vertex sum as zero.

Suppose both P_1 and Q_1 contains even number of edges and both P_2 and Q_2 contains odd number of edges. Discussing the edge labels as above the edge of P_1 and Q_1 which is incident with u_2 gets the label $\alpha_1+\alpha_2-\alpha_3$ and $\alpha_1+\alpha_2+\alpha_3$. To get the vertex sum at u_2 as $\alpha_1+\alpha_2$, the edges of C_3 incident with u_2 gets $-\alpha_1$ and $-\alpha_2$. The edges of C_3 which are incident with u_3 receive the label $-\alpha_1$ and $-\alpha_2$.

Similarly we can prove for any even number k, the labeling of the graph $C(n_1,n_2,\ldots n_k)$ as discussed in the above two cases will give the vertex sum as zero. □

4.9 Theorem

Let C $(n_1,n_2,\ldots n_k)$ be a chain of k cycles C_j with k is odd and all C_j's are even. For i = 1,2,…k-2, let P_i's and Q_i's are the paths in the cycle $C_{(i+1)}$ connecting the vertices u_i and $u_{(i+1)}$ where the vertex u_i is common to C_i and $C_{(i+1)}$ and the vertex $u_{(i+1)}$ is common to $C_{(i+1)}$ and $C_{(i+2)}$. If P's and Q's contains odd number of edges then C $(n_1,n_2,\ldots n_k)$ is group magic and if P's and Q's contains even number of edges then $C(n_1,n_2,\ldots n_k)$ is zero sum magic.

Proof:

As the end cycle C_1 is even by result (1) of theorem 4.5 the edges of C_1which are incident with u_1 gets the labels α_1 and α_2 and the edges of P_1 and Q_1 which are incident with u_1 gets labels α_3 and $-\alpha_3$. Since all cycles are even, for i=1, 2,..., k-2, both P_i and Q_i contains either even or odd number of edges.

Consider all P_i's and Q_i's contains odd number of edges. The edges of P_1 and Q_1 incident with u_2 get the labels α_3 and $-\alpha_3$ and the edges of P_2 and Q_2 incident with u_2 gets the labels α_1 and α_2. Since P_2 and Q_2 contains odd number of edges, the edges of P_2 and Q_2 incident with u_3 get the labels α_1 and α_2 and the edges of P_3 and Q_3 incident with u_3 gets the labels α_3 and $-\alpha_3$. Continuing in the same way it can be observed that the edges of P_{k-1} and Q_{k-1} incident with u_{k-1} gets the labels α_3 and $-\alpha_3$ and the edges of C_k incident with $u_{(k-1)}$ gets the labels α_1 and α_2 since k is odd. Thus at all vertices the vertex sum is $\alpha_1+\alpha_2$.

Consider all P_i's and Q_i's contains even number of edges. The edges of P_1 and Q_1 incident with u_2 get the labels $\alpha_1+\alpha_2-\alpha_3$ and $\alpha_1+\alpha_2+\alpha_3$ and the edges of P_2 and Q_2 incident with u_2 gets the labels $-\alpha_1$ and $-\alpha_2$. Since P_2 and Q_2 contains even number of edges, the edges of P_2 and Q_2 incident with u_3 get the labels $2\alpha_1+\alpha_2$ and $\alpha_1+2\alpha_2$ and the edges of P_3 and Q_3 incident with u_3 gets the labels $-2\alpha_1$ and $-2\alpha_2$. Continuing in the same way it can be observed that the edges of P_{k-1} and Q_{k-1} incident with u_{k-1} gets the labels $(k-2)\alpha_1+\alpha_2$ and $\alpha_1+(k-2)\alpha_2$ since k is odd. Since C_k is even from result (2) of theorem 4.5, we have $(k-2)\alpha_1+\alpha_2+\alpha_1+(k-2)\alpha_2=0$.
This shows the vertex sum $\alpha_1+\alpha_2$ is zero. □

5. CONCLUSION AND FUTURE WORK

In theorems 4.1 and 4.2 it was derived that in group magic labeling of $B_V(n_1, n_2,\ldots n_k)$ the vertex sum contains as many distinct labels as many odd cycles. The vertex sum is zero when all cycles are even. The chain of cycles $C(n_1,n_2,\ldots n_k)$ is group magic when there are odd number of cycles in $C(n_1,n_2,\ldots n_k)$ also the path connecting the common vertices have odd number of edges. The chain of cycles $C(n_1,n_2,\ldots n_k)$ is zero sum magic when there are even number of even cycles in $C(n_1,n_2,\ldots n_k)$ and also in the case that there are odd number of cycles in $C(n_1,n_2,\ldots n_k)$ and the path connecting the common vertices have even number of edges. We further work to derive the group magic labeling to the chain of cycles $C(n_1,n_2,\ldots n_k)$ in the remaining cases.

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