The PSD of QPSK Modulation passed through a Pass band Chebeshev Filter

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ABSTRACT

The demand for high bit rate data transmission in digital data communication. In a multi carrier operation, the problem of interference between carrier spectrums appears. Several modulation techniques for digital transmission have recently been investigated for the purpose of achieving narrow signal spectra with power concentrated within a given bandwidth. QPSK modulation is best suited for digital signal transmission. In this paper, an analytical PSD estimation with periodogram, periodogram with a data window and segmented periodogram are discussed and finally, PSD estimation with non averaged and averaged will be simulated with the help of MATLAB codes. The performance of periodogram like spectral leakage and resolution were also simulated in MATLAB.

Index terms--- Quadrature phase shift keying (QPSK), Power spectral density (PSD):

1. Introduction

The spectral density of a signal characterized the distribution of the signal's energy or power in the frequency domain. This concept is particularly important when considering filtering in communication system. We need to be able to evaluate signal and noise at filter output. The PSD is used in evaluation. The conventional quadriphase modulation technique suffers from the fact that the limiting bandwidth and frequency sidelobes, remove by proper filtering. Using the in phase and quardrature phase. Channel with arbitrary pulse shaping analytical results are established for generalize quadriphase modulation.The spectral efficiency requirements are satisfied by conventional pulse-shaped, four phase, modulation schemes such as QPSK symmetric differential phase-shift keying, commonly known as π/4 – DQPSK , has several advantages when used in a mobile radio environment[1][2][6][7].

2. Analytical approach

A. The periodogram

The simplest, fastest and most frequently used PSD estimation algorithm is the periodogram. It is defined by

$$\hat{S}(k f\Delta) = (1/N) \left| \sum_{n=0}^{N-1} x[n] \exp(-j2k\pi f_\Delta n) \right|^2$$

$$= (1/N) \left| I_N (k f_\Delta) \right|^2$$

n=0

….k=0, 1, 2, 3,-------N-1; (1)

In which N is total number of sample of the data record, and $I_N (k f_\Delta)$ is the N-point FFT of the data for which the PSD estimate at frequency $f = k f \Delta$ is to be computed. The computational efficiency of the periodogram comes from the use of the FFT to form the PSD estimate. The result provides us with N frequency domain estimates having a resolution of $f\Delta = f_s / N$, where f_s is sampling frequency associated with the time domain point for which the spectral estimate is being performed. Note that f_s in his context is not always the sampling frequency associated with the simulation. For example, if a given signal in a simulation is significantly oversampled, the samples may be decimated prior to forming the PSD estimate. The difficulty with periodogram is that it is biased and is not a consistent Estimator of the PSD at a frequency f. For many applications, the variation of the periodogram is unacceptably high. The bias results from the unavoidable fact that the data record is finite. However for significantly large N, the bias can be neglected. Therefore, The main difficulty results from the high variance. Assuming that the data sample x [n] are independent, the variance of the spectral estimate at frequency f is [3][4][5][11][12].

$$\mathrm{Var}(\hat{S}(f)) = \sigma_x^4 \{1 + (\sin[2fn\pi] / N \sin[2f\pi])^2\} \qquad (2)$$

Where σ_x^4 is the variance of the data sample x [n]. We observe that the variation of the $\hat{S}(f)$ does not tend to zero as $N \rightarrow \infty$ and, for large N, the variance of the spectral estimate is independent of frequency. The periodogram, however, despite this serious flaw, is useful for "quick look" at the PSD [8][9][10].

B. The periodogram with a Data window

If a data window is not explicitly specified, the default rectangular window is used. For a rectangular window each sample value x[n] is multiplied by w[n] =1 for $0 \leq n \leq N-1$. The impact of rectangular window is to convolve the data sample x[n], with the Fourier transform of w[n], which has amplitude spectrum Sin [f N π] / N sin [f π]. The sidelobe structure of this data window, when viewed in the frequency domain, results in considerable spectral leakage [ ]. This spectral leakage distorts and reduces the dynamic range of the estimated spectrum.

When an arbitrary data window is used, $\hat{S}(k f\Delta)$ takes the form,

$$\hat{S}(k f\Delta) = [1/U] \left| \sum^{N-1} x[n] \, w[n] \exp(-j2k\pi f_\Delta n) \right|^2 \qquad (3)$$

Where U is the energy in the data window, which is given by

$$U = \sum_{n=0}^{N-1} w^2[n] \quad (4)$$

Note for rectangular window, for which w[n] = 1 for all n, U = N and (1) results. The choice of data window represents a number of tradeoff. The ideal data window must have finite duration in the time domain so that I_N (k f $_\Delta$), the Fourier transform of the data, can be accuratelly estimated using a finite data record. In addition the estimated Fourier transform of data record must not be adversely affected by window function. Since multiplication in time domain is convolution in frequency domain and only convolution with an impulse function leaves the transform unchanged the ideal window function an impulse in frequency domain. Since Fourier transform of an impulse function is not a finite extent, these are conflicting requirements. We therefore seek a data window that, in the frequency domain, exhibits a narrow main lobe about f = 0, and side lobe that are greatly attenuated. A variety of window functions are discussed in the classic paper by Harris [10][11][14][15].

C. Segmented periodograms

A common technique for reducing the variance associated with the periodogram is to divide the N-sample data record into K segments, with each segment consisting of M samples. The FFT is computed for each segment and the results are averaged. The averaging process reduces the variance of spectral estimate. The segment or may not be overlapping. If the segment do not overlap, K = M/N ; otherwise K> M/N. The periodogram of the ith data segment is given by

$$I_m^{(i)}(kf\Delta) = [1/U] \left| \sum_{n=0}^{N-1} x^{(i)}[n]\, w[n] \exp(-j2k\pi f_\Delta n) \right|^2$$

$$\ldots\ldots\ldots i = 1, 2, \quad (5)$$

Where $x^{(i)}[n]$ is represent the samples in the ith data record and $f_\Delta = f_s/M$. The K peridogram are then averaged to produce the PSD estimator

$$\hat{S}(k f \Delta) = (1/K) \sum_{i=0}^{K-1} I_m^{(i)}(k f_\Delta) \;,$$

$$\ldots k = 0,1,2,\ldots M-1; \quad (6)$$

This estimator is biased, of course, since the data record is finite. Assuming that the K periodogram are independent

$$\mathrm{Var}\,[(\hat{S}(k f \Delta)] = (1/K)\; S^2(k f_\Delta) \quad (7)$$

3. Performance of the Periodogram

The following sections discuss the performance of the periodogram with regard to the issues of leakage and resolution.

Spectral Leakage: Consider the power spectrum or PSD of a finite-length signal $x_L[n]$, as discussed in the Periodogram. It is frequently useful to interpret $x_L[n]$ as the result of multiplying an infinite signal, x[n], by a finite-length rectangular window, $w_R[n]$:

$$x_L[n] = x[n] \cdot w_R[n] \quad (8)$$

Because multiplication in the time domain corresponds to convolution in the frequency domain, the Fourier transform of the expression above is

$$x_L(f) = \frac{1}{f_s} \int_{-fs/2}^{fs/2} X(\rho) W_R(f - \rho)\, d\rho \quad (9)$$

The expression developed earlier for the periodogram,

$$P_{xx}(f) = \frac{|X_L(f)^2|}{f_s L} \quad (10)$$

illustrates that the periodogram is also influenced by this convolution.

The effect of the convolution is best understood for sinusoidal data. Suppose that x[n] is composed of a sum of M complex sinusoids:

$$x[n] = \sum_{k=1}^{M} A_k\, e^{jw_k n} \quad (11)$$

Its spectrum is

$$X(f) = f_s \sum_{k=1}^{M} A_k\, \delta(f - f_k) \quad (12)$$

which for a finite-length sequence becomes

$$X_L(f) = \int_{-f_{s/2}}^{f_{s/2}} \{ \sum_{k=1}^{M} A_k\, \delta(\rho - f_k) W_R(f - \rho) d\rho = \sum_{k=1}^{M} A_k W_R(f - f_k) \quad (13)$$

So in the spectrum of the finite-length signal, the Dirac deltas have been replaced by terms of the form $W_R(f - f_k)$, which corresponds to the frequency response of a rectangular window centered on the frequency f_k.

Resolution: Resolution refers to the ability to discriminate spectral features, and is a key concept on the analysis of spectral estimator performance.

In order to resolve two sinusoids that are relatively close together in frequency, it is necessary for the difference between the two frequencies to be greater than the width of the mainlobe of the leaked spectra for either one of these sinusoids. The mainlobe width is defined to be the width of the mainlobe at the point where the power is half the peak mainlobe power (i.e., 3 dB width). This width is approximately equal to f_s / L.

The above discussion about resolution did not consider the effects of noise since the signal-to-noise ratio (SNR) has been relatively high thus far. When the SNR is low, true spectral features are much harder to distinguish, and noise artifacts appear in spectral estimates based on the periodogram.

In other words, for two sinusoids of frequencies f_1 and f2, the resolvability condition requires that

In the example above, where two sinusoids are separated by only 10 Hz, the data record must be greater than 100 samples to allow resolution of two distinct sinusoids by a periodogram.

$$\Delta f = (f_1 - f_2) > \frac{f_s}{L}$$

4. RESULT

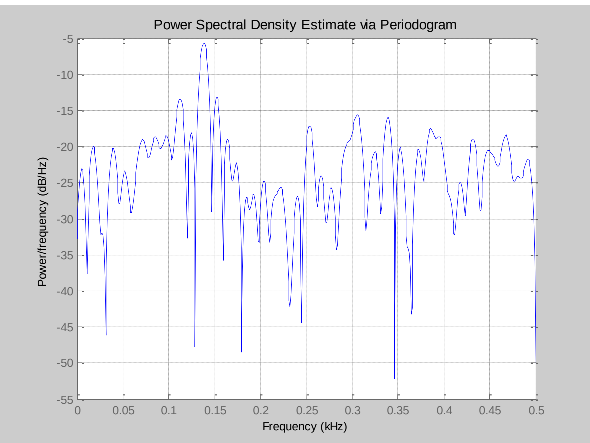
Following parameters are considered for simulation results:

1. sampling frequency f_s = 1000 Hz;
2. size of sample data record N = 50000;
3. number of segment K = 25;

The PSD estimator was developed by using Basic MATLAB command. The MATLAB signal processing toolbox contains a number of routines for PSD estimation. Here we consider periodogram from signal processing tool box. The

performance of periodogram as a spectral leakage shown in figure [1] and resolution shown in figure [2].

Spectral Leakage:



Resolution

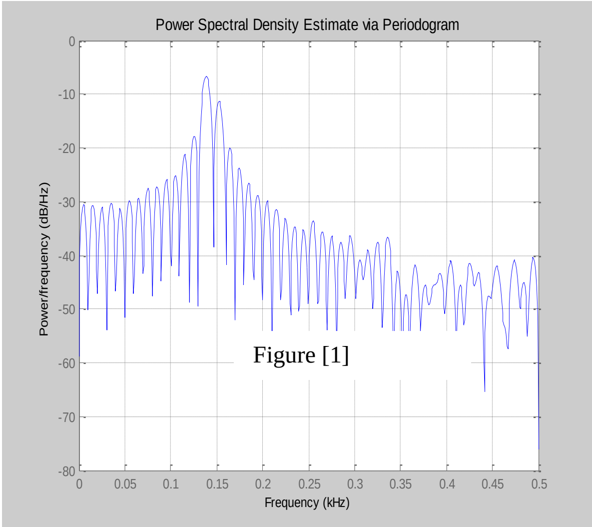


Figure [2]

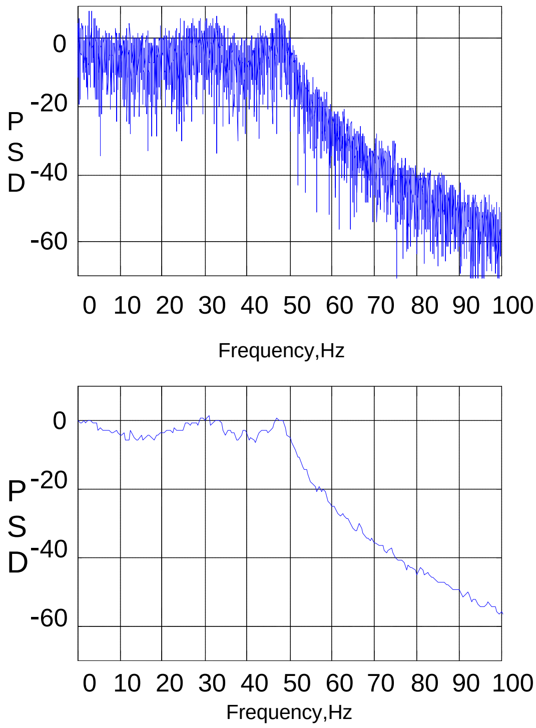


Figure (3): Power spectral density estimates, non averaged (Top Frame) and averaged (bottom frame) [3][4][5].

5. CONCLUSION

Refer figure [3] for non-overlapped case (top frame), the variance is large and independent of frequency. Using K = 25 segments (bottom frame) results in much smaller variance at the cost of reduced frequency resolution. The 5db pass band ripple of Chebeshev filter is much more obvious with K = 25 than for K = 1.

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