A Quantum Differential Evolutionary Algorithm for the Independent Set Problem

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ABSTRACT

The Independent Set problem consists to find a maximum cardinality subset of vertices of a given graph such that no two vertices are adjacent. In this paper, we propose a quantum evolutionary algorithm which uses a differential operator to update the quantum angles of the superposition state of Q-bits for solving this problem. Simulation results on some graph examples show that this approach is effective.

General Terms

Graph theory, Stochastic Algorithm.

Keywords

Independent Set, Quantum evolutionary algorithm.

1. INTRODUCTION

In graph theory, the Maximum Independent Set is a maximum cardinality subset of vertices in which no two vertices are adjacent. This problem which has many applications in computer science is computationally NP-hard in general [1], but can be solved in polynomial time on some special graph classes [2] [3]. For large and hard instances of this problem, approximation algorithms and heuristic approaches are required to obtain near optimal solutions within reasonable amount of computation time.

Quantum computing, based on the concepts and principles of quantum theory, such as superposition of quantum states, entanglement and intervention, was first proposed by Benioff [4]. It was successfully applied later to many optimization and combinatorial problems [5] [6] [7].

Recently, an evolutionary algorithm was proposed for tackling the Independent Set Problem [8] and another approach based on ellipsoids was presented in [9] in order to provide exact solutions to this problem. In this work, we propose a quantum evolutionary algorithm using a differential operator to update the quantum angles of the superposition state of Q-bits for solving this problem.

Given an undirected graph G = (V, E) with vertex set V = V (G) of cardinality |V(G)| = n, and edge set E = E(G) of cardinality |E(G)| = m.

The neighborhood of a vertex $v \in V$ is the set $N(v) = \{u \in V: vu \in E\}$.

A set $S \subseteq V(G)$ is independent if no two vertices from S are adjacent; by Ind(G) we mean the set of all the independent sets of G.

This paper is organized as follows: In section II we describe the proposed approach for solving this problem, section III provides some examples of the application of the proposed algorithm. Finally, concluding remarks are given in section IV.

II Quantum Evolutionary Algorithm is inspired from the concepts of quantum computing, where the smallest unit of information is called Q-bit and is defined as $[\alpha,\beta]^T$, where α and β are complex numbers that specify the probability amplitude of the respective Q-bit states such that $|\alpha|^2+|\beta|^2 =1$. $|\alpha|^2$ is the probability that the Q-bit will be in state '0' and $|\beta|^2$ represents the probability that the Q-bit will be in state '1'.

The representation for an individual q in the population with m-bit is given as follows:

$$q = \left(\begin{array}{c|c|c} \alpha_1, & \dots, & \alpha_m \\ \beta_1, & \dots, & \beta_m \end{array} \right)$$

where $|\alpha_i|^2+|\beta_i|^2 =1$, i=1,2,...m.

and $\theta_i=\arctan(|\beta_i|/|\alpha_i|)$ is the phase of the ith qubit.

2. Description of the algorithm:

First, the population is initialized with the α_i and β_i of all bits of all individuals set to $1/\sqrt{2}$. At each generation, the binary strings are generated from the respective Q-bit strings by observing the Q-bit states using the following rule:

$P_{i,j} =1$, if rand (0,1) < $\sin(\theta_{i,j})^2$ and $P_{i,j} =0$ else. (1)

where $P_{i,j}$ is represents the jth bit of ith individual in the population and $\theta_{i,j}$ its phase.

The fitness value of the population strings generated is evaluated and the best solutions are stored separately.

The mutant Q-bits $\theta_i^{\,m}$ are generated for all the individuals in the population in every generation.

Mutant Q-bits of the i individual in generation t are determined as follows:

$$\theta_i^{mt} = \theta_{r1} + F^t .(\theta_{r2} - \theta_{r3}) \quad (2)$$

where r_1,r_2,r_3 and i are mutually distinct and F^t is the mutation control parameter which is determined in every generation by :

F^t = rand(0,1)

where rand is a random number generated from a uniform distribution on [0,1].

The crossover operation transform the original Q-bits and the respective mutant Q-bits by using the following rule:

$$\theta_{i,j}^{ct} = \begin{cases} \theta_{i,j}^{mt} & \text{if } (rand_i(0,1) \le CR) \text{ or } (i = I_{rand}) \\ \theta_{i,j}^{t} & \text{if } (rand_i(0,1) > CR) \text{ and } (i \ne I_{rand}) \end{cases} \quad (3)$$

where $\theta_{i,j}^{ct}$ is the ith Q-bit of jth individuals after the

crossover operation. I_{rand} is an integer randomly chosen from {1,2,...n} which ensures that at least one Q-bit is different from the original set in each individual. CR = 0.5, is the control parameter which is found to be the best value experimentally.

The population and the Q-bits are updated by observing the state of the newly obtained Q-bits $\theta_{i,j}^{ct}$, a new set of individuals are generated which replace the corresponding individual in the population if their fitness values are higher. The replacement is done by using the following rules:

$$P_j^{t+1} = \begin{cases} P_j^{ct} & \text{if } f(P_j^{ct}) > f(P_j^{t}) \\ P_j^{t} & \text{otherwise} \end{cases} \quad (4)$$

and

$$\theta_{i,j}^{t+1} = \begin{cases} \theta_{i,j}^{ct} & \text{if } f(P_j^{ct}) > f(P_j^{t}) \\ \theta_{i,j}^{t} & \text{otherwise} \end{cases}$$

where P_j^{c} is the jth individual by observing the Q-bits modified after crossover ($\theta_{i,j}^{ct}$) and f(P_j) is the fitness value of the corresponding individual.

According to [10], we can choose the fitness function f to maximize, as follows :

$$f(x)= e^T x - x^T A_G x/2 \quad (5)$$

where A_G is the adjacency matrix of the input graph and $x \in \{0,1\}^n$ is the indicator vector of a subset of vertices of G and $e^T=(1,...,1)$.

We summarize the pseudo code of the proposed algorithm as follows:

input: A_G, the adjacency matrix of graph G

output: an independent set S

initialize Q(t);

make P(t) from Q(t) by using (1)

t← 1

CR← 0.5

evaluate the fitness function f of P(t) by using (5)

while t<T do

    t← t+1

    F^t ←rand (0,1)

    apply mutation on Q(t) by using (2)

    make $Q^C(t)$ by crossover by using (3)

    make $P^C(t)$ from $Q^C(t)$ by using (1)

    evaluate fitness of P(t)

    update P(t+1) and Q(t+1) by (4)

end while

output P=ArgMax(f) as the indicator vector of the independent set S.

3. Examples

In order to assess the performance of this method, tests have been conducted on some graph examples taken from the literature. The test environment is P4 3G, Windows 7, Matlab [11].

The population sizes was set to 20.The number of generations was set to 100. The obtained results are reported in the following table:

Graph	**n**	**independent set found**	***Elapsed time (s.)***
Octahedron	6	1 4	*8.89*
Cube	8	2 4 6 8	10.57
Grötzsch	11	2 3 4 5 6	18.32
Herschel	11	2 4 5 7 9 11	18.006
Icosahedron	12	3 5 7	31.99
Wheel	8	3 5 7	21.65
Petersen	10	3 5 7 8	24.97
Ramsey graph R(4,4)	17	4 11 16	58.63

Table 1. The independent set found by the proposed method on the tested graph examples.

In the following figures where tested graphs are presented, white vertex belongs to the independent set found by the proposed method.

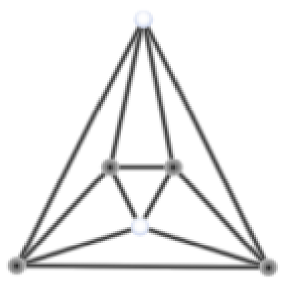

Figure 1: The Octahedron

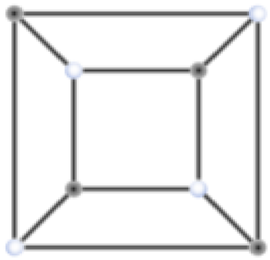

Figure 2: The Cube

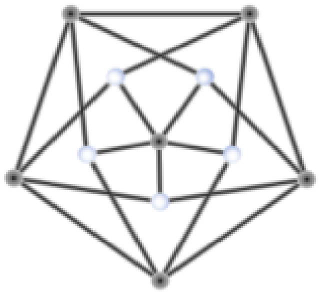

Figure 3: The Grötzsch graph [12]

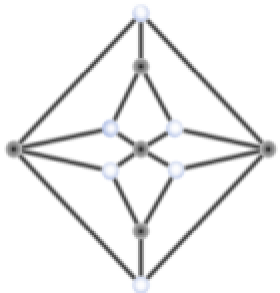

Figure 4: The Herschel graph [13]

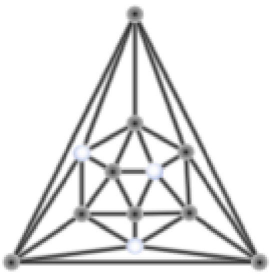

Figure 5: The Icosahedron

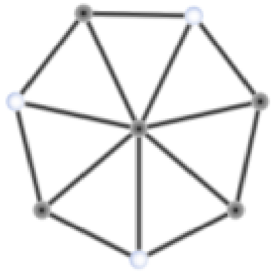

Figure 6: The Wheel graph W8

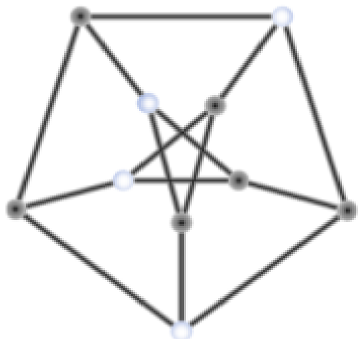

Figure 7: The Petersen graph [14]

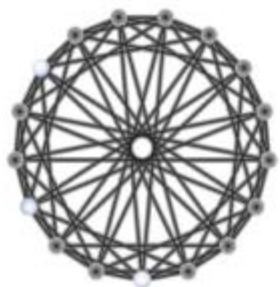

Figure 8: The Ramsey graph [15]

4. CONCLUSIONS

In this paper, we have described a quantum differential evolutionary algorithm in attempt to solve the Independent Set problem. We have tested this method on some graph examples. Simulation results show the effectiveness of the proposed approach. Further research will be concerned with the improvement of this method. To enhance its performance in both speed and quality, it would be interesting to find some adequate initialization phase instead of random initialization and to test this approach on large graph instances.

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