On Fuzzy Soft Right Ternary Near-Rings

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ABSTRACT

The first step towards near-rings was an axiomatic research done by Dickson in 1905. In 1936, it was Zassenhaus who used the name near-ring. Many parts of the well established theory of rings are transferred to near-rings and new specific features of near-rings have been discovered. To deal with the idea of near-rings using ternary product Warud Nakkhasen and Bundit Pibaljommee have applied the concept of ternary semiring to define left ternary near- rings, ternary subnear-rings and their ideals and investigated some properties of L-fuzzy ternary near subrings in 2012. In this paper, we consider right ternary near-rings and their ideals and apply fuzzy soft set technology initiated by Maji et al in 2001 to introduce fuzzy soft right ternary near-rings, fuzzy soft ideals and study their basic algebraic properties.

Keywords

Fuzzy soft set, ideals, homomorphism, level set of fuzzy soft set, near-ring.

1. INTRODUCTION

The concept of fuzzy set was introduced by Zadeh [22] in 1965. Since then many researchers are exploring the generalisation of the notion of fuzzy sets. In 1999, Molodtsov [16] introduced the soft set to deal with the uncertainties present in most of our real life situations. The parametrization tools of soft set theory enhance the flexibility of its application to different problems. In 2001, Maji et al [13] expanded soft set theory to fuzzy soft set theory. In recent times, researches have contributed a lot towards fuzzification of soft set theory. Maji et al introduced some properties regarding fuzzy soft union, intersection, complement of a fuzzy soft set, DeMorgan Law etc. These results were further revised and improved by Ahmad and Kharal [2]. They defined arbitrary fuzzy soft union and intersection and proved DeMorgan inclusions and DeMorgan laws in fuzzy soft set theory.

Fuzzy soft sets combine the strengths of both soft sets and fuzzy sets. They have applications in Medical diagnosis [6], decisionmaking ([14]), knowledge representation and retrieval [15] etc. Aktas and Cagman [3] introduced the notion of soft groups and Aygungolu and Aygun [4] generalized their concept and introduced fuzzy soft groups in 2009.Thereafter many reaserchers are applying fuzzy soft tools to other algebraic structures viz., rings, modules, semigroups ([7, 8, 21]).

The first step towards near-ring was an axiomatic research done by Dickson in 1905. In 1936, it was Zassenhaus who used the name near-ring. Near-rings appear to have an application in characterising endomorphism of a group. Near-rings are algebraic structures which arise in a natural way in the study of mappings from a group into itself where addition is defined pointwise and multiplication is defined as composition of mappings. Many parts of the well established theory of rings are transferred to near-rings and new specific features of near-rings have been discovered. Near-rings have a number of interesting applications ranging from Geometry, Combinatorics and Interpolation theory to the study of polynomial mappings. The notion of fuzzy subnear-ring, fuzzy left (resp. right) ideals and prime ideals in near-ring was introduced by Abou-Zaid([1]).Fuzzy ideals in near-rings are further discussed by Kim et al ([9]).

The notion of ternary algebraic system was introduced by Lehmer [12] in 1932. Ternary semigroups[19,18], ternary semirings [10]are some of the algebraic structures which involve ternary product. To deal with the concept of near-rings using ternary product Warud Nakkhasen and Bundit Pibaljommee [20] have applied the concept of ternary semiring to define left ternary near- rings, ternary subnear-rings and their ideals and investigated some properties of *L*-fuzzy ternary near subrings in 2012 .

In this paper, we consider right ternary near-rings and their ideals and apply fuzzy soft set technology to introduce fuzzy soft right ternary near-rings, their fuzzy soft ideals and study their basic algebraic properties.

2. PRELIMINARIES

In this section we give the basic definitions that are necessary for the following sections of this paper.

Definition 2.1 [17] A *right near-ring* N is a non-empty set N together with two binary operations + and . such that

(i) (N, +) is a group (not necessarily abelian)

(ii) (N, .) is a semigroup.

(iii) $(n_1 + n_2) . n_3 = n_1 . n_3 + n_2 . n_3$ for every $n_1, n_2, n_3 \in N$ (right distributive law).

Definition 2.2 [19] Let T be a non-empty set and [ ] be an operation defined from $N\times N \times N$ to N called a ternary operation. Then (N , [ ]) is a *ternary semigroup* if for every $x, y, z, u, v \in N$ $[[xyz]uv] = [x[yzu]v] = [xy[zuv]]$.

Definition 2.3 [19] Let A,B,C be non-empty subsets of a ternary semigroup N. Then

$$[ABC] = \{[abc] \in N \mid a \in A, b \in B, c \in C\}.$$

Definition 2.4 [20] A tri-tuple (N , + , .) consisting of a non-empty set N, a binary operation + and a ternary operation . on N is called a *ternary near-ring* if

(i) (N , +) is a group,

(ii) (N , .) is a ternary semigroup and

(iii) $ab(c + d) = abc + abd$ for all $a, b, c, d \in N$.

Definition 2.5 [20] A non-empty subset S of a ternary near-ring is called a *ternary subnear-ring* if (i) $x - y \in S$ if $x, y \in S$ (ii) $[SSS] \subseteq S$.

Definition 2.6 [20] Let N and N ′ be any two right ternary near rings. Then a mapping h : N → N′ is called a *right ternary near ring homomorphism* if (i) h(x+ y) = h(x)+h(y), (ii) h([x y z]) = [h(x) h(y) h(z)], for every x, y, z ∈ N.

Definition 2.7 [22] If X is a universal set then a *fuzzy subset* of X is a map μ : X → [0,1] denoted by μ = {(x, μ(x))|x ∈ X }.

Definition 2.8 [16] Let U be a universal set. Let A be a subset of a set of parameters E. Then (F, A) is called a *soft set* over U where F: A→℘(U) and ℘(U) is the set of subsets of U.

In other words, a soft set is a parameterized family of subsets of the set U. Every set F (e), for every e ∈ E, from this family may be considered as the set of e − elements of the soft set (F, E), or as the set of e – approximate elements of the soft set.

According to this manner, a soft set (F, E) is given as a collection of approximations: (F, E) = {F(e) : e ∈ E}.

Definition 2.9 [16] Let U be a universal set and let A be a subset of a set of parameters E. Let I^U (where I = [0, 1]) be the set of fuzzy subsets of U. Then

(f , A) is called a *fuzzy soft set* over U where f : A → I^U and f(a) = f_a :U→ I is a fuzzy subset of U .

In other words, a fuzzy soft set is a parameterized family of fuzzy subsets of the set U.

Every fuzzy subset f_e, for every e ∈ E may be considered as the fuzzy subset of e − approximate elements of the fuzzy soft set. According to this manner, a fuzzy soft set (f, E) is given as a collection of approximations : (f , E) = { f_e: e ∈ E}.

Remark 2.10 From the above two definitions we notice that if x ∈ F(a) then f_a(x) = 1, for each a ∈ A and vice-versa.

Definition 2.11 [2, 5, 13] Let U be a universal set and let A and B any two non-empty subsets of a set of parameters E. Let (f , A) , (g , B) be any two fuzzy soft sets over U. Then

(i) (f , A) is a *fuzzy soft subset* of (g , B) i.e.,(f , A) $\widetilde{\subseteq}$ (g , B) if A⊆ B and $f_a(x) \le g_a(x)$ for every a ∈ A and x ∈ U .

(ii) The *complement* of a fuzzy soft set denoted by $(f , A)^c$ is defined by $(f , A)^c = (f^c , A)$ where $f_a^c(x) = 1 – f_a(x)$,for every a ∈ A and x ∈ U.

(iii) (f , A) **AND** (g, B) denoted by (f, A) $\widetilde{\wedge}$ (*g*, B) = (h, *C*) where *C* = A × B is defined by , h(a,b)= $h_{(a,b)} = f_a \wedge g_b$ where (a, b) ∈ C. That is h(a, b)(x) = min{$f_a(x)$, $g_b(x)$}, for every x ∈ U .

(iv) (f , A) **OR** (g , B) denoted by (f , A) $\widetilde{\vee}$ (g , B) is defined by (f, A) $\widetilde{\vee}$ (g , B) = (h, C), where C = A × B and h((a,b)) = $h_{(a,b)} = f_a \vee g_b$, h(a, b)(x) = max{$f_a(x)$, $g_b(x)$}, for every x ∈ U.

(v) The *union* of (f, A) and (g , B) denoted by (f, A) $\widetilde{\cup}$ (g, B) = (h, C), where C = A ∪ B and

$$h(c) = h_c = \begin{cases} f_c & \text{if } c \in A - B \\ g_c & \text{if } c \in B - A \\ f_c \vee g_c & \text{if } c \in A \cap B \end{cases}$$

(vi) The *intersection* of (f , A) and (g , B) such that A∩B ≠ ∅ is defined to be the fuzzy soft set (h, C), where C = A∩B and $h(c) = h_c = f_c \cap g_c$ for all c ∈ C . This is denoted by (h, C) = (f, A) $\widetilde{\cap}$ (g, B).

Definition 2.12 [13] Let U and V be any two non-empty universal sets. Let E_1 and E_2 be parameter sets for U and V respectively and A ⊆ E_1, B ⊆ E_2. Let (f, A) and (g, B) be any two non-empty fuzzy soft sets over U and V respectively. Then their *cartesian product* over U × V is defined by (f , A) × (g , B) = (h , C), where C = A × B and h(a,b) = $h_{(a,b)} = f_a \wedge g_b$ for every (a, b) ∈ C. That is, h(a, b)(u, v) = $h_{(a, b)}(u, v)$ = min{$f_a(u)$, $g_b(v)$}.

Definition 2.13 [11] Let X and Y be any two non-empty sets and E_1 and E_2 be their parameter sets. Let A⊆ E_1, a ∈ A, and t ∈ Imf_a . Let (f, A) and (g, B) be any two non-empty fuzzy soft sets over U and V respectively. Let φ: X → Y and $\psi: A \to B$.Then (φ , ψ):(f , A) → (g, B)is called a *fuzzy soft function* and the image set (φ (f) , B) of (f , A) under (φ, ψ) is defined as follows.

For every y ∈ Y and b ∈ B

$$(\varphi (f)_b(y)$$

$$= \begin{cases} \bigvee_{x\in\varphi^{-1}(y)} \left(\bigvee_{a\in\psi^{-1}(b)\cap A} f_a(x)\right), & if\,\varphi^{-1}(y) \neq \emptyset and \\ & for\ all\ b \in \psi(A) \\ 0 & otherwise \end{cases}$$

Definition 2.14 [11] Inverse image of fuzzy soft set(g, B) is defined by $(\varphi,\psi)^{-1}((g , B))=(\varphi^{-1}(g) , \psi^{-1}(B))$, where

$(\varphi^{-1}(g))_a(x) = g_{\psi(a)}(\varphi(x))$,for every a ∈ ψ^{-1}(B) and x ∈ X .

3. FUZZY SOFT RIGHT TERNARY NEAR-RINGS AND FUZZY SOFT IDEALS

In this section we first consider the right ternary near-ring, right ternary subnear-ring and the right, lateral and left ideals and then define fuzzy soft right ternary near-ring and fuzzy soft ideal over a right ternary near-ring and study their basic algebraic properties.

Definition 3.1 Let N be a non-empty set together with a binary operation + and a ternary operation [ ]: N×N×N → N. Then (N, + , [ ]) is a *right ternary near-ring*(a right ternary near ring is written as RTNR) if

(**RTNR-1**) (N, +) is a group (not necessarily abelian)

(RTNR-2) (N, [ ]) is a ternary semigroup

(RTNR-3) [(a + b) c d] = [a c d] + [b c d], for every a, b, c, d ∈ N. Similarly we can define a *left ternary near-ring* and a *lateral ternary near ring*.

Example 3.2 (i) Let N = {0, x, y, z}. Define + as in table (i) and [ ] on N by [x y z] = (x.y).z for every x, y, z ∈ N where . is defined as in table (ii). Then (N , + , [ ]) is a right ternary near-ring.

+	0	x	y	z
0	0	x	y	z
x	x	0	z	y
y	y	z	0	x
z	z	y	x	0

Table (i)

.	0	x	y	z
0	0	0	0	0
x	0	0	0	0
y	0	0	0	0
z	0	x	y	x

Table (ii)

(ii) Let N = {0, x, y, z}. Define + as in table (iii) and [ ] on N by [x y z] = (x.y).z for every x, y, z in N where . is defined as in table (iv). Then (N , + , [ ]) is a right ternary near-ring.

+	0	x	y	z
0	0	x	y	z
x	x	0	z	y
y	y	z	0	x
z	z	y	x	0

Table (iii)

.	0	x	y	z
0	0	0	0	0
x	x	x	x	x
y	0	x	y	z
z	x	0	z	y

Table (iv)

(iii) Let Γ be a group under +. Let M(Γ) = { θ | θ: Γ→ Γ}. Define + and [ ] on M(Γ) by (θ + η)(x) = θ (x) + η (x) and [θ ξ η](x) = θ (ξ (η (x))), for every x ∈ Γ. Then (M(Γ) , + , [ ]) is a right ternary near-ring.

Definition 3.3 Let N be a right ternary near-ring. Let J be a normal subgroup of (N, +). Then J is called (i) a *right ideal* of N if [J N N] ⊆ J (ii) a *left ideal* if [t t ′ (t″ +i)] – [t t′ t″] ∈ J (iii) a *lateral ideal* if [t (t′ + i) t″] - [t t′ t″] ∈ J where t, t′, t″ ∈ N , i ∈ J.

J is an *ideal* of N if it is a right, lateral and left ideal of N.

We note that {0,x} is an ideal of N in Example 3.2 (i) but {0,y} is not an ideal of N as [yxx] = x ∉ {0, y} . In Example 3.2 (ii) both {0, x} and {0, y} are ideals of N but their union is not an ideal of N.

Definition 3.4 Let N be a right ternary near-ring and A be a subset of a parameter set E. Let (F, A) be a soft set over N. i.e. F: A → ℘(N) where ℘(N) is the set of subsets of N . Then (F, A) is a *soft right ternary subnear-ring* (*ideal*) over N if for every a ∈A, F(a) is right ternary subnear-ring (ideal) of N.

Definition 3.5 A fuzzy soft set (f, A) over N is a *fuzzy soft right ternary near-ring* if

(i) f_a (x + y) ≥ min { f_a (x), f_a (y)},

(ii) f_a (- x) ≥ f_a (x) ∀ a ∈A, x,y ∈X and

(iii) f_a([xyz]) ≥ min{ f_a x), f_a(y), f_a(z)} for every a ∈ A and x,y,z ∈ N.

Remark 3.6 Conditions (i) and (ii) are normally combined together to get the condition f_a(x – y) ≥ min{ f_a(x), f_a(y) }.

Example 3.7 (i) Let N be as in Example 3.2(i).Consider a fuzzy soft set (f , A) over N where A = N and we define for every a ∈ A, f_a(0) =1, f_a(x) = 0.7 , f_a(z) = 0.6, f_a(y) = 0.6 Then (f, A) is a fuzzy soft right ternary near-ring over N.

(ii) Let N = M(Γ) as in Example 3.2(iii). Consider a fuzzy soft set (f , A) over N where A = N and we define for every a ∈ A,

(f_a) (θ) = $\begin{cases} 0.9 & if\ \theta = 0 \\ 0.6 & if\ \theta \neq 0 \end{cases}$ for every a ∈ A. Then (f , A) is a fuzzy soft right ternary near-ring over N.

Definition 3.8 A fuzzy soft set (f , A) over a right ternary near-ring N is *fuzzy soft ideal* over N if

(i) f_a(x – y) ≥ min{ f_a(x), f_a(y) }

(ii) f_a([xyz]) ≥ min{ f_a(x), f_a(y), f_a(z)}

(iii) f_a(y + x - y) ≥ f_a(x)

(iv) f_a([xyz]) ≥ f_a(x)

(v) f_a([x y (z +i)] - [x y z]) ≥ f_a(i)

(vi) f_a([x (y + i) z] - [x y z]) ≥ f_a(i) ,for every a ∈A and x,y,z ,i ∈N.

A fuzzy soft set (f , A) is called a *fuzzy soft right ideal* if it satisfies (i), (ii), (iii), (iv), (f , A) is called a *fuzzy soft left ideal* if it satisfies(i), (ii), (iii),(v) and (f , A) is called a *fuzzy soft lateral ideal* if it satisfies (i), (ii), (iii),(vi).

Example 3.9 (i) If N is as given in Example 3.2(i) and A = {0,x}and f : A→ I^N is defined by f_a(0) =1 and f_a(x) = 0.6 and f_a(y) = f_a(z)=0 ,for every a ∈ A then (f , A) is a fuzzy soft ideal over N.

(ii) If N is as given in Example 3.2(ii) and A = {0,x} and f : A→ I^N is defined by f_a(0) = 1 and f_a(x) = 0.7 and f_a(y) = f_a(z) = 0 , for every a ∈ A then (f , A) is a fuzzy soft ideal over N.

(iii) If N is as given in Example 3.2(iii) and B = {0,y} and g : B→ I^N is defined by g_b(0) =1 and g_b(x) = 0 and g_b(y) = 0.9, g_b(z) = 0 , for every b ∈ B then (g , B) is a fuzzy soft ideal over N.

Lemma 3.10 If (f, A) is a fuzzy soft right ternary near-ring over N then

(i) f_a(- x) = f_a(x),

(ii) f_a(0) ≥ f_a(x),

(iii) if f_a(x – y) = f_a(0), then f_a(x) = f_a(y), for every a ∈A and x,y ∈ N.

Proof: (i) Consider f_a(x) = f_a(-(-x)) ≥ f_a(-x) ≥ f_a(x), using condition (ii) in Definition 3.5 and hence (i)

(ii) Consider, f_a(0)=f_a(x–x) ≥ min{f_a(x),f_a(-x)} ≥ f_a(x),using (i)

(iii) Consider

f_a(x) = f_a(x+0) = f_a(x-y+y) ≥ min{ f_a(x-y), f_a(y)}

= min{ f_a(0), f_a(y) }= f_a(y),using (ii).

Also,

f_a(y) = f_a(y - x + x) ≥ min{f_a(y-x),f_a(x)}

≥ min{f_a(-(x-y)),f_a(x)} = min{f_a((x-y)),f_a(x)}

= min{f_a(0),f_a(x)} = f_a(x). Hence f_a(x) = f_a(y).

Theorem 3.11 If (f, A) is a fuzzy soft set over N and if for a ∈ A, S = $N_{f_a}^{f_a(0)} = \{x \in N | f_a(x) = f_a(0)\}$ then

(i) S is a right ternary subnear-ring of N if (f , A) is a fuzzy soft right ternary near-ring over N.

(ii) S is an ideal of N if (f , A) is a fuzzy soft ideal over N.

Proof (i) Let x,y ∈ S. Then f_a(x) = f_a(0) and f_a(y) = f_a(0). As

(f, A) is a fuzzy soft right ternary near-ring over N and x, y ∈ N, f_a(x-y) ≥ min{ f_a(x), f_a(y) }= f_a(0). But by (ii) of the above lemma f_a(0) ≥ f_a(x-y). Hence f_a(x-y)= f_a(0) which implies that x-y ∈ S.

Now, let x, y, z ∈ S. Then x, y, z ∈ N and since (f , A) is a fuzzy soft right ternary near-ring over N , f_a([xyz]) ≥ min{ f_a(x), f_a(y), f_a(z)} = f_a(0). But by (ii) of the above lemma f_a(0) ≥ f_a([xyz]).Hence f_a([xyz])= f_a(0) which implies that [xyz] ∈S. Thus S is right ternary subnear-ring of N.

(ii) By (i) x-y ∈ S and [xyz] ∈ S. Now let x ∈ S and y ∈ N. Then x,y ∈ N and since (f , A) is a fuzzy soft ideal f_a(y + x - y)

$\geq f_a(x)$ and hence $f_a(y + x - y) \geq f_a(0)$. But $f_a(0) \geq f_a(y + x - y)$, by (ii) of the above lemma . Hence $f_a(y + x - y) = f_a(0)$ which implies that $y + x - y \in S$.

Let $x \in S$ and y, $z \in N$.Then $x,y,z \in N$ and since (f , A) is a fuzzy soft ideal $f_a([xyz]) \geq f_a(x) = f_a(0)$. But by (ii) of the above lemma $f_a(0) \geq f_a([x\ y\ z])$ and hence $f_a([x\ y\ z]) = f_a(0)$ which implies that $[SNN] \subseteq S$.

Now let $i \in S$ and $x,y \in N$. Then $x,y,i \in N$ and since (f , A) is a fuzzy soft ideal $f_a([x\ y\ (z+i)] - [x\ y\ z]) \geq f_a(i) = f_a(0)$.

But by (ii) of the above lemma $f_a(0) \geq f_a([xy(z+i)] - [xyz])$. Hence $f_a([xy(z+i)] - [xyz]) = f_a(0)$ which implies that $[xy(z+i)] - [xyz] \in S$. Similarly it can be proved that $[x(y+i)z] - [x\ y\ z] \in S$ and hence S is an ideal of N.

Corollary 3.12 Let (f , A) be a fuzzy soft set over N and

$T = N_{f_a}{}^1 = \{x \in N \mid f_a(x) = 1\}$. Then

(i) T is a right ternary subnear-ring (ideal) of N if (f , A) is a fuzzy soft right ternary near-ring over N.

(ii) T is an ideal of N if (f , A) is a fuzzy soft ideal over N.

Proof: By taking $f_a(0) = 1$ in $N_{f_a}{}^{f_a(0)}$ in the above theorem the proof follows.

Remark 3.13 In general, the converse of Theorem 3.11 is not true. The converse holds if $|Im\ f_a| = 2$ and $f_a(0) = 1$ which is established in the following theorem.

Theorem 3.14 Let (f , A) be a fuzzy soft set over N and $|Im\ f_a| = 2$ with $f_a(0) = 1$. Then

(i) if $N_{f_a}{}^1 = \{x \in N \mid f_a(x) = 1\}$, for each $a \in A$ is a right ternary subnear-ring of N, then (f , A) is a fuzzy soft right ternary near-ring over N.

(ii) if $N_{f_a}{}^1 = \{x \in N \mid f_a(x) = 1\}$ is an ideal of N, then (f , A) is a fuzzy soft ideal over N.

Proof:

To prove (i)

Let $Im\ f_a = \{\alpha, 1\}$, as $|Im\ f_a| = 2$ where $\alpha \in [0,1)$. Let $x,y \in N$. Then we have the following three cases. (i) $x,y \in N_{f_a}{}^1$ or (ii) $x \in N$, $y \notin N_{f_a}{}^1$ or (iii) $x,y \notin N_{f_a}{}^1$.

Case(i): Let $x,y \in N_{f_a}{}^1$. Suppose $f_a(x-y) < \min\{f_a(x), f_a(y)\}$. Then $f_a(x-y) < 1$ which implies that then $x-y \notin N_{f_a}{}^1$ but since $N_{f_a}{}^1$ is a right ternary subnear-ring of N we have $x - y \in N_{f_a}{}^1$. This contradiction leads us to conclude that $f_a(x-y) \geq \min\{f_a(x), f_a(y)\}$.

Case (ii): Let $x \in N$, $y \notin N_{f_a}{}^1$. Since $N_{f_a}{}^1$ is a right ternary subnear-ring of N , $x - y \notin N_{f_a}{}^1$.This implies that $f_a(x-y) = \alpha = f_a(y) = \min\{f_a(x), f_a(y)\}$ as $f_a(x)$ can be 1 or α and $\alpha < 1$.

Case (iii): Let $x,y \notin N_{f_a}{}^1$.Since $N_{f_a}{}^1$is a right ternary subnear-ring of N, $x - y \notin N_{f_a}{}^1$. This implies that $f_a(x-y) = \alpha = \min\{f_a(x), f_a(y)\}$ as $f_a(x) = \alpha$ and $f_a(y) = \alpha$. A similar argument holds for $f_a([x\ y\ z])$ and it can be shown that $f_a([x\ y\ z]) \geq \min\{f_a(x), f_a(y), f_a(z)\}$.

To prove (ii)

By (i) $f_a(x-y) \geq \min\{f_a(x), f_a(y)\}$ and $f_a([x\ y\ z]) \geq \min\{f_a(x), f_a(y), f_a(z)\}$.To establish conditions (iii) (iv),(v) and (vi) of Definition 3.8, let x , $y \in N$. Then we have the following three cases (i) $x,y \in N_{f_a}{}^1$ or (ii) $y \in N$, $x \notin N_{f_a}{}^1$ or (iii) $x,y \notin N_{f_a}{}^1$.

Case(i): Let x , $y \in N_{f_a}{}^1$. As $N_{f_a}{}^1$ is an ideal, $y+x-y \in N_{f_a}{}^1$. Thus $f_a(y + x - y) = 1 = f_a(y)$, as $y \in N_{f_a}{}^1$.

Case(ii): Again as $N_{f_a}{}^1$ is an ideal, if $y \in N$, $x \notin N_{f_a}{}^1$ then $y + x - y \notin N_{f_a}{}^1$.Therefore, $f_a(y + x - y) = \alpha = f_a(x)$.

Case (iii) : If $x,y \notin N_{f_a}{}^1$, then $y + x - y \notin N_{f_a}{}^1$and therefore $f_a(y + x - y) = \alpha$ and $f_a(x) = \alpha$ and hence $f_a(y + x - y) = f_a(x)$.Now, if $x,y,z \in N$. Then either (i) $x \in N_{f_a}{}^1$and $y,z \in N$ and hence $f_a(x) = 1 = f_a([x\ y\ z])$, as $[N_{f_a}{}^1\ N\ N] \subseteq N_{f_a}{}^1$ or (ii) $x,y \in N_{f_a}{}^1$ and $z \in N$ and hence $f_a(x) = 1 = f_a([x\ y\ z])$, as $[N_{f_a}{}^1\ N\ N] \subseteq N_{f_a}{}^1$ or (iii) x, y, $z \notin N_{f_a}{}^1$ and hence $f_a(x) = \alpha = f_a([x\ y\ z])$, as $[N_{f_a}{}^1\ N\ N]$ can not be a subset of $N_{f_a}{}^1$. Thus in all the three cases $f_a([x\ y\ z]) \geq f_a(x)$. Similarly it can be established that, $f_a([x\ y\ (z + i)] - [xyz]) \geq f_a(i)$ and $f_a([x\ (y+ i)\ z] - [xyz]) \geq f_a(i)$. Thus (f , A) is a fuzzy soft ideal over N.

Proposition 3.15 The intersection of two non-empty fuzzy soft right ternary near-rings and two non-empty fuzzy soft ideals over N are respectively fuzzy soft right ternary near-ring and fuzzy soft ideal over N.

Proof: Let N be a right ternary near-ring and A , B be two subsets of a parameter set E with $A \cap B \neq \emptyset$. Let (f , A) and

(g , B) be any two non-empty fuzzy soft ternary near-rings over N. We prove that $(f, A) \tilde{\cap} (g, B) = (h, C)$ is also a fuzzy soft right ternary near-ring over N.

Consider

$h_c(x-y) = \min\{f_c(x-y), g_c(x-y)\}$

$\geq \min\{\min\{f_c(x), f_c(y)\}, \min\{g_c(x), g_c(y)\}\}$

$= \min\{\min\{f_c(x), g_c(x)\}, \min\{f_c(y), g_c(y)\}\}$

$= \min\{h_c(x), h_c(y)\}$.

Now, $h_c([xyz])$

$= \min\{f_c([xyz]), g_c([xyz])\}$

$\geq \min\{\min\{f_c(x), f_c(y), f_c(z)\}, \min\{g_c(x), g_c(y), g_c(z)\}\}$

$= \min\{\min\{f_c(x), g_c(x)\}, \min\{f_c(y), g_c(y)\}, \min\{f_c(z), g_c(z)\}\}$

$= \min\{h_c(x), h_c(y), h_c(z)\}$,for every $c \in C$ and x, y, $z \in N$.

This implies that (h, C) is also a fuzzy soft right ternary near-ring over N.

Let (f , A) and (g , B) be any two non-empty fuzzy soft ideals over N. Then to prove that (h, C) is a fuzzy soft ideal it is enough to prove conditions (iii),(iv),(v),(vi) given in Definition 3.8.

Consider, $h_c(y+x-y) = \min\{f_c(y+x-y), g_c(y+x-y)\} \geq \min\{f_c(x), g_c(x)\} = h_c(x)$. Now, $h_c([xyz]) = \min\{f_c(([xyz]), g_c(([xyz])\} \geq \min\{f_c(x), g_c(x)\} = h_c(x)$. Also, $h_c([x\ y\ (z+i)] - [x\ y\ z]) = \min\{f_c([x\ y\ (z+i)] - [x\ y\ z]), g_c([x\ y\ (z+i)] - [x\ y\ z])\} \geq \min\{f_c(i), g_c(i)\} = h_c(i)$. Similarly it can be proved that

$h_c([x(y+i)z] - [x\ y\ z]) \geq h_c(i)$. Thus (h,C) is a fuzzy soft ideal over N.

Proposition 3.16 The $\tilde{\wedge}$ intersection of any two non-empty fuzzy soft right ternary near-rings and any two non-empty fuzzy soft ideals over N are respectively fuzzy soft right ternary near-ring and fuzzy soft ideal over N.

Proof : Let N be a right ternary near-ring and A,B be two subsets of a parameter set E . Let (f , A) and (g , B) be any two non-empty fuzzy soft right ternary near-rings over N. We prove that (f , A) $\widetilde{\wedge}$ (g , B) = (h,C), where C = A × B and h: C→ I^N, h(a,b)= $h_{(a,b)} = f_a \wedge g_b$ for every, (a,b) ∈ C is also a fuzzy soft right ternary near-ring over N. Consider

$h_c(x\text{-}y) = \min\{f_a(x\text{-}y), g_b(x\text{-}y)\}$

$\geq \min\{\min\{f_a(x), f_a(y)\}, \min\{g_b(x), g_b(y)\}\}$

$= \min\{\min\{f_a(x), g_b(x)\}, \min\{f_a(y), g_b(y)\}\}$

$= \min\{h_c(x), h_c(y)\}$.

Now,

$h_c([xyz])$

$= \min\{f_a([xyz]), g_b([xyz])\}$

$\geq \min\{\min\{f_a(x), f_a(y), f_a(z)\}, \min\{g_b(x), g_b(y), g_b(z)\}\}$

$= \min\{\min\{f_a(x), g_b(x)\}, \min\{f_a(y), g_b(y)\}, \min\{f_a(z), g_b(z)\}\}$

$= \min\{h_c(x), h_c(y), h_c(z)\}$, for every c ∈C and x,y,z ∈ N.

Hence (h, C) is a fuzzy soft ternary near-ring over N**.**

Let (f , A) and (g , B) be any two non-empty fuzzy soft ideals over N. Then to prove that (h, C) is a fuzzy soft ideal by the above discussion, it is enough to prove conditions (iii),(iv),(v),(vi) given in Definition 3.8 **.**

Consider, $h_c(y+x\text{-}y) = \min\{f_a(y+x\text{-}y), g_b(y+x\text{-}y)$

$\geq \min\{f_a(x), g_b(x)\} = h_c(x)$.

Now,

$h_c([xyz]) = \min\{f_a(([xyz]), g_b(([xyz])\}$

$\geq \min\{f_a(x), g_b(x)\} = h_c(x)$.

Also,

h_c ([x y (z +i)] - [x y z]) = $\min\{f_a($ [x y (z +i)] - [x y z]),$g_b(($[x y (z +i)] - [x y z])}≥ $\min\{f_a(i), g_b(i)\} = h_c(i)$. Similarly it can be proved that h_c ([x (y + i) z] - [x y z]) ≥ h_c (i) . Thus (h,C) is a fuzzy soft ideal over N.

Remark 3.17 (a) (f , A) $\widetilde{\cup}$ (g , B) = (h,C) can be a fuzzy soft right ternary near-ring over N if A∩B = ∅. For, from Example 3.9 (ii) and (iii) we have A∩B={0} and $h_c(x\text{-}y) = f_c(x\text{-}y) \vee g_c(x\text{-}y) = f_c(z) \vee g_c(z) = 0$ and $h_c(x) = f_c(x) \vee g_c(x) = 0.7$ and $h_c(y) = f_c(y) \vee g_c(y) = 0.9$ and $\min\{h_c(x), h_c(y)\} = 0.7$. Thus, $h_c(x\text{-}y) < \min\{h_c(x), h_c(y)\}$ and hence (h,C) is not a fuzzy soft right ternary near-ring over N.

(b) (f , A) $\widetilde{\vee}$ (g , B) = (h,C) need not be a fuzzy soft right ternary near-ring . For, from Example 3.9 (ii) and (iii) we have A×B = {(0,0), 0,y), (x,0), (x,y)} and $h_{(0,0)}(x\text{-}y) = h_{(0,0)}(z) = f_0(z) \vee g_0(z) = 0$ but $h_{(0,0)}(x) = 0.7$ and $h_{(0,0)}(y) = 0.9$ which implies that $\min\{h_{(0,0)}(x), h_{(0,0)}(y)\} = 0.7 > h_{(0,0)}(x\text{-}y)$ and hence (h,C) is not a fuzzy soft right ternary near-ring over N.

(c) The propositions 3.12 and 3.13. and the above remarks are true for finite and arbitrary sets of fuzzy soft right ternary near-rings and ideals.

Proposition 3.18 The cartesian product of any two non-empty fuzzy soft right ternary near-rings over N and M and any two non-empty fuzzy soft ideals over N and M are respectively fuzzy soft right ternary near-ring and fuzzy soft ideal over N×M.

Proof : Let N and M be any two non-empty right ternary near-rings. Let E_1, E_2 be parameter set of N and M respectively. Let A ⊆ E_1, B ⊆ E_2 and (f , A) and (g , B) be any two non-empty fuzzy soft right ternary near-rings over N and M respectively. Then (f, A) × (g, B) = (h, C), where C = A × B is a fuzzy soft right ternary near-ring over N×M where N×M is a right ternary near ring under the operations (x,y)+(u,v)= (x+u,y+v) and [(x,u) (y,v) (z,w)] =([xyz],[uvw]). Now,

$h_c((x,y)\text{-}(u,v))$

$= h_c((x-u),(y-v))$

$= \min\{f_a(x-u), g_b(y-v)\}$

$\geq \min\{\min\{f_a(x), f_a(u)\}, \min\{g_b(y), g_b(v)\}$

$= \min\{\min\{f_a(x), g_b(y)\}, \min\{f_a(u), g_b(v)\}\}$

$= \min\{h_c(x,y)), h_c(u,v)\}$,

for x,u ∈ N and y,v ∈ M. Now

$h_{(a,b)}([(x,u)(y,v)(z,w)]) = h_{(a,b)}([xyz],[uvw])$

$= \min\{f_a([xyz]), g_b([uvw])\}$

$\geq \min\{\min\{f_a(x), f_a(y), f_a(z)\}, \min\{g_b(u), g_b(v), g_b(w)\}$

$= \min\{\min\{f_a(x), g_b(u)\}, \min\{f_a(y), g_b(v)\}, \min\{f_a(z), g_b(w)\}$

$= \min\{h_c(x,u)), h_c(y,v), h_c(z,w)\}$,

for x, y, z ∈ N and u, v, w ∈ M.

Hence (h, C) is a fuzzy soft right ternary near-ring over N×M.

Now consider,

$h_c((y,u)+(x,v)\text{-}(y,u)) = h_c((y+x\text{-}y, u+v\text{-}u))$

$= \min\{f_a(y+x\text{-}y), g_b(u+v\text{-}u)\}$

$\geq \min\{f_a(x), g_b(v)\} = h_c((x,v))$.

Also,

$h_c([(x,u)(y,v)(z,w)] = h_c([xyz],[uvw])$

$= \min\{f_a([xyz]), g_b([uvw])\}$

$\geq \min\{f_a(x), g_b(u)\} = h_c((x,u))$.

Now,

h_c ([(x,u) (y,v) (z + i, w + j)]- [(x,u) (y,v) (z,w)])

$= h_c$(([x y (z + i)],[u v (w + j)]) - ([x y z],[u v w]))

$= h_c$ ([x y (z +i)] - [x y z], [u v(w +j)] - [u v w])

$= \min\{f_a($ [x y (z +i)] – [x y z]),$g_b(($[u v (w +j)] - [u v w])}

$\geq \min\{f_a(i), g_b(j)\} = h_c((i,j))$.

Similarly it can be proved that h_c([(x,u) (z + i, w + j) (y,v)]- [(x,u) (y,v)(z,w)]) ≥ h_c((i,j). Thus (h,C) is a fuzzy soft ideal over N×M.

Definition 3.19 Let N and M be two non empty right ternary near-rings with parameters from E_1 and E_2 respectively. Let φ:N→M be a right ternary near-ring homomorphism.That is φ(x + y) = φ(x) + φ(y) and φ([xyz]) = [φ(x) φ(y) φ (z)] for every x,y,z ∈N.

Let ψ: A → B and f:A→$[0,1]^N$ where A ⊆ E_1 is such that (f , A) is a fuzzy soft right ternary near-ring over N and $g:B\to[0,1]^M$ where $B \subseteq E_2$ is such that (g , B) is a fuzzy soft right ternary near-ring over M. Then (φ, ψ):(f , A) → (g , B) is a *fuzzy soft right ternary near-ring homomorphism* where(φ,ψ) (f , A) = (φ(f),ψ(A)) .

Proposition 3.20 Homomorphic image of a fuzzy soft right ternary near-ring (fuzzy soft ideal) over a right ternary near-ring is a fuzzy soft right ternary near-ring(fuzzy soft ideal)**.**

Proof: Let φ:N→M be an onto right ternary near-ring homomorphism. Let E_1 and E_2 be parameter sets for N, M respectively. Let $\psi: A \to B$ where $A \subseteq E_1$, $B \subseteq E_2$ be a one-to-one mapping such that $\psi(a) = b$, where $a \in A, b \in B$. To prove that the image of (f , A) under (φ,ψ) is a fuzzy soft right ternary near-ring over M.

Since φ is onto for u, v, w ∈ M , there exists x, y, z respectively in N such that $\varphi(x) = u, \varphi(y) = v$, $\varphi(z) = w$. Also $\varphi(x - y) = \varphi(x) - \varphi(y) = u - v$ and $\varphi([xy z]) = [\varphi(x)\varphi(y)\varphi(z)] = [uvw]$.

Now,

$$(\varphi(f))_b(u) = \vee_{x\in\varphi^{-1}(u)}\left(\vee_{e\in\psi^{-1}(b)\cap A} f_e(x)\right)$$

$$=\vee_{x\in\varphi^{-1}(u)} f_a(x),$$

as ψ is one-one .Similarly, $(\varphi(f))_b(v) = \vee_{y\in\varphi^{-1}(v)} f_a(y)$ and $(\varphi(f))_b(w) = \vee_{z\in\varphi^{-1}(w)} f_a(z)$.

Now, $(\varphi(f))_b((u - v)) = \vee_{t\in\varphi^{-1}(u-v)} f_a(t) \geq f_a(x-y) \geq \min\{f_a (x), f_a(y)\}$. Hence $(\varphi(f))_b((u - v)) \geq \min\{ (\varphi(f))_b(u), (\varphi(f))_b(v)\}$.Also, $(\varphi(f))_b([uvw])= \vee_{t\in\varphi^{-1}([uvw])} f_a(t) \geq f_a([xyz]) \geq \min\{ f_a(x),f_a(y),f_a(z)\}$, x,y,z ∈ N. Hence $(\varphi(f))_b([uvw]) \geq \min\{(\varphi(f))_b(u), (\varphi(f))_b(v),(\varphi(f))_b(w)\}$

Thus the homomorphic image of (f , A) is also a fuzzy soft right ternary near-ring over M. The case for fuzzy soft ideal can similarly be proved.

Proposition 3.21 The inverse homomorphic image of (g , B) is a fuzzy soft right ternary near-ring (fuzzy soft ideal) over M if (g , B) is a fuzzy soft right ternary near-ring (fuzzy soft ideal) over N.

Proof: Let $\varphi: N \to M$ be an onto right ternary near-ring homomorphism. Let E_1 and E_2 be parameter sets for N and M respectively.Let ψ: A→ B where $A \subseteq E_1$, $B\subseteq E_2$. Let (g , B) be a fuzzy soft set over M. To prove that $(\varphi, \psi)^{-1}$ (g, B) = $(\varphi^{-1}(g), \psi^{-1}(B))$ is a fuzzy soft right ternary near-ring over N . Let $\varphi^{-1}(g) = h$, $\psi^{-1}(B) = C$. Then $h : C \to I^N$ and $h_c: N \to I$. Now, $h_c(x) = g_{\psi(c)}(\varphi(x))$.

Hence $h_c(x-y) = g_{\psi(c)}(\varphi(x - y))$

$$= g_{\psi(c)}(\varphi(x) - \varphi(y))$$

$$\geq \min \{g_{\psi(c)}(\varphi(x)), g_{\psi(c)}(\varphi(y))\}$$

$$= \min\{ h_c(x), h_c(y)\}.$$

Also,

$$h_c([xyz]) = g_{\psi(c)}(\varphi([xyz]))$$

$$= g_{\psi(c)}([\varphi(x)\varphi(y)\varphi(y)])$$

$$\geq \min \{g_{\psi(c)}(\varphi(x)), g_{\psi(c)}(\varphi(y)), g_{\psi(c)}(\varphi(z))\}$$

$$= \min\{ h_c(x), h_c(y), h_c(z)\}.$$

Hence $(\varphi, \psi)^{-1}$ (g, B) is a fuzzy soft fuzzy soft right ternary near-ring over N .The case for fuzzy soft ideal can similarly be proved.

Proposition 3.22 Let N be a right ternary near-ring and A be a subset of a parameter set E. Let (F, A) be a soft set over N. i.e. F : A → ℘(N) where ℘(N) is the set of subsets of N for all a ∈ A . Then F(a) is a soft right ternary subnear-ring (soft ideal) over N iff (f , A) is a fuzzy soft right ternary near-ring (fuzzy soft ideal) over N.

Proof: Let N be a right ternary near-ring and A be a subset of a parameter set E. Let (f , A) be a soft set over N i.e., F(a) ∈ ℘(N) for all a ∈ A . Let (f, A) be a fuzzy soft set over N. Let F(a) be a right ternary near-ring over N. To prove that (f, A) is a fuzzy soft right ternary near-ring over N. That is to prove that $f_a([x y z]) \geq \min\{ f_a (x), f_a (y), f_a(z)\}$. Let x, y, z ∈ N. Since F(a) is a soft set over N, either (i) x, y, z ∈ F(a) or (ii) x ∉ F(a) but y, z ∈ F(a) or (iii) x ∉ F(a), y ∉ F(a), z ∉ F(a).

Case(i) If x, y, z ∈ F(a) then x-y ∈ F(a) , [x y z] ∈ F(a). Now, $f_a(x-y) \geq \min\{ f_a (x), f_a (y)\}$. Similarly, $f_a([x y z]) \geq \min\{ f_a (x), f_a (y), f_a(z)\}$.Hence (f, A) is a fuzzy soft right ternary near-ring over N.

Case (ii) Let x ∉ F(a) and y, z ∈ F(a) .Then x-y ∉ F(a),

[x y z] ∉ F(a) which implies $f_a(x - y) = 0$ and $f_a[x y z] = 0$. Also, as x ∉ F(a) , $f_a(x) = 0$ .Therefore, $\min\{ f_a (x), f_a (y)\}= 0$ and $\min\{ f_a (x), f_a (y) , f_a(z)\} = 0$. Hence $f_a(x-y) \geq \min\{ f_a (x), f_a (y)\}$ and $f_a([x y z]) = \min \{ f_a(x), f_a(y),f_a(z)\}$. Thus, (f, A) is a fuzzy soft right ternary near-ring over N.

Case(iii) Let x, y, z ∈ F(a).Then $f_a([x y z])= 0$ and $f_a([x y z]) = 0$. Also, $\min\{f_a (x), f_a (y)\} = 0$ and $\min\{ f_a (x), f_a (y), f_a(z)\} = 0$. As in case (ii), (f, A) is a fuzzy soft right ternary near-ring over N.

Conversely, let x, y, z ∈ F(a). Since (f, A) is a fuzzy soft right ternary near-ring over N, $f_a(x-y) \geq \min\{ f_a (x), f_a (y)\}$ and $f_a([x y z]) \geq \min \{ f_a (x), f_a (y), f_a (z)\}$. This implies that $f_a(x-y) \geq \min\{1,1\}$ and $f_a ([x y z]) \geq \min (1, 1, 1)$. Hence $f_a(x-y) =1$ and $f_a([x y z])= 1$ which implies that x-y ∈ F(a) and [x y z] ∈ F(a). Hence F(a) is a right ternary sub near-ring of N. The case for fuzzy soft ideal can similarly be proved.

Theorem 3.23 (i) A non-empty subset L of N is a right ternary subnear-ring (ideal) of N iff (f, A) is a fuzzy soft right ternary nearing (fuzzy soft ideal) over N, where $f : A \to I^N$ is defined by

$$(f_a) (x) = \begin{cases} r & \text{if } x \in L \\ t & \text{if } x \in N - L \end{cases} \quad \text{where } r > t, \text{ for every } a \in A.$$

(ii) In particular, a non-empty subset L of N is a right ternary subnear-ring (ideal) of N iff the characteristic function (Ψ_L , E) is a fuzzy soft right ternary near-ring (fuzzy soft ideal) over N, where $\Psi_L : E \to I^N$ is defined by

$$(\Psi_L)_e(x)=\begin{cases} 1 \text{ if } x \in L \\ 0 \text{ if } x \notin L \end{cases} \text{, for every } e \in E.$$

Proof: To prove (i)

We first show that if L is a right ternary subnear-ring of N then (f , A) is a fuzzy soft right ternary near-ring over N.Let x, y ∈ N. Then we have the following three cases. (i) x,y ∈ L or (ii) y ∈ N, x ∉ L or (iii) x,y ∉ L.

Case (i): Since L is a right ternary subnear-ring, x–y ∈ L.Thus $f_a(x-y) = \min\{ f_a (x), f_a (y)\}$.

Case (ii): Again as L is a right ternary subnear-ring, if y∈ N, x ∉ L then x-y ∉ L.Therefore, f_a(x-y) = t and min{ f_a (x), f_a (y)}= t and hence f_a(x-y) = min{ f_a (x), f_a (y)}.

Case (iii): If x,y ∉ L, then x-y ∉ L and therefore f_a(x-y) = t and min{f_a(x),f_a(y)}= t and hence f_a(x-y) = min{ f_a (x), f_a (y)}. A similar argument holds for f_a([x y z]) and it can be shown that f_a([x y z]) ≥ min { f_a (x), f_a (y), f_a (z)}.Thus (f , A) is a fuzzy soft right ternary near-ring over N.

Now by assuming L is an ideal, we prove (f , A) to be a fuzzy soft ideal over N . Let x, y ∈ N. Then (i) x, y ∈ L or (ii) y ∈ N, x ∉ L or (iii) x,y ∉ L.

Case (i): Since L is an ideal, y + x – y ∈ L. Therefore, f_a(y + x – y) = r = f_a(x), as x ∈ L.

Case (ii): Again as L is an ideal, if y ∈ N, x ∉ L then y + x – y ∉ L. Therefore f_a(y + x – y) = t = f_a(x).

Case (iii) : If x,y ∉ L, then y + x – y ∉ L and therefore f_a(y + x – y) = t and f_a (x) = t and hence f_a(y + x – y) = f_a (x). Again arguing in the same manner it can be established that f_a([xyz]) ≥ f_a(x), f_a ([x y (z + i)] – [xyz]) ≥ f_a(i) and f_a ([x (y+ i) z] – [xyz]) ≥ f_a (i). Thus (f , A) is a fuzzy soft ideal over N.

Conversely, if x , y ∈ L then f_a(x) = r, f_a(y) = r and hence min {f_a(x), f_a(y)}= r which in turn implies that f_a(x-y) ≥ r as (f , A) is a fuzzy soft ideal over N. Thus, x-y ∈ L. Otherwise, f_a(x-y) = t and we get t ≥ r but $r > t$. Also if x, y, z ∈ L, since f_a([xyz]) ≥ min{ f_a(x), f_a(y), f_a(z)}, [xyz] ∈ L. Otherwise, f_a([xyz]) = t and we get t ≥ r but $r > t$.

Thus L is right ternary subnear-ring of N.

Now, let x ∈ L and y,z ∈ N. Then f_a(y + x – y) ≥ r .This in turn implies that y + x – y ∈ L for if y + x – y ∉L , then f_a(y + x – y) = t and we get t ≥ r but $r > t$.

A similar argument holds to establish that [xyz]∈ L if x ∈ L and y,z ∈ N, ([x (y+ i) z] – [xyz]) ∈ L and ([x y (z + i)] – [xyz])∈ L whenever i∈ L and x,y,z ∈ N. Thus L is an ideal of N.

To Prove (ii) . Let$(\Psi_L)_e$ (x - y) ≥ min{$(\Psi_L)_e$ (x), $(\Psi_L)_e$ (y)} and $(\Psi_L)_e$ ([x y z]) ≥ min{$(\Psi_L)_e$ (x), $(\Psi_L)_e$ (y), $(\Psi_L)_e$(z)}.Let x,y ∈ L. Then $(\Psi_L)_e$ (x) = 1 , $(\Psi_L)_e$ (y) = 1 and hence $(\Psi_L)_e$ (x - y) =1 which implies that x - y ∈ L. Similarly if x,y,z ∈ L then [x y z] ∈ L. Thus L is a ternary subnear-ring of N. Now, let x∈L and y, z ∈ N. Then as $(\Psi_L)_e$ ([x y z]) ≥ $(\Psi_L)_e$ (x) we have $(\Psi_L)_e$ ([x y z]) = 1 , hence[LTT] ⊆ L and again as $(\Psi_L)_e$ ([x y (z+i)]-[x y z]) ≥ $(\Psi_L)_e$ (i) if i∈L then $(\Psi_L)_e$ ([x y (z+i)] - [x y z]) = 1, hence [x y (z+i)]-[x y z]∈ L. Similarly it can be proved that [x y + i z] - [x y z] ∈ L. Hence L is an ideal of N.

Conversely, if L is a right ternary subnear-ring (an ideal) of N, then for x, y ∈ N we have the following three cases (i) x , y ∈ L or (ii) x ∈ L,y∉ L or (iii) x ∉ L, y ∉ L.

Case (i): Since $(\Psi_L)_e$ (x) = 1 , $(\Psi_L)_e$ (y) = 1 and $(\Psi_L)_e$ (x-y) =1 we have $(\Psi_L)_e$ (x-y) = min{$(\Psi_L)_e$ (x), $(\Psi_L)_e$ (y)}.

Case(ii): Since $(\Psi_L)_e$ (x) = 1, $(\Psi_L)_e$ (y) = 0, min{$(\Psi_L)_e$ (x), $(\Psi_L)_e$ (y)}=0. Also, $(\Psi_L)_e$ (x-y) = 0, hence equality holds.

Case (iii): Similar to the above cases. Similarly it can be proved that $(\Psi_L)_e$ ([xyz]) ≥ min{$(\Psi_L)_e$ (x), $(\Psi_L)_e$ (y), $(\Psi_L)_e$(z)}. Thus (Ψ_L,E) is a fuzzy soft right ternary near-ring over N.

Also $(\Psi_L)_e$ (y + x – y) ≥$(\Psi_L)_e$(x) and $(\Psi_L)_e$ ([x y z]) ≥ $(\Psi_L)_e$ (x), $(\Psi_L)_e$ ([x y (z+i)]-[x y z]) ≥ $(\Psi_L)_e$(i) and $(\Psi_L)_e$([x y+i z]-[x y z]) ≥ $(\Psi_L)_e$ (i). Hence (Ψ_L, E) is a fuzzy soft ideal over N.

Proposition 3.24 Let N be a right ternary near-ring. Let A be a subset of a parameter set E. Let (f, A) be a fuzzy soft set over N. Then (f, A) is a fuzzy soft right ternary near-ring (fuzzy soft ideal) over N iff for each f_a , each non-empty level subset $(f_a)_t$, t ∈ Im f_a is a right ternary subnear-ring (ideal) of N.

Proof: Let (f , A) be a fuzzy soft right ternary near-ring over N . Let t ∈ Im f_a be such that $(f_a)_t \neq \emptyset$. To prove that $(f_a)_t$ is a right ternary sub near-ring of N.

Let x,y,z ∈$(f_a)_t$. Then f_a (x) ≥ t, f_a (y) ≥ t, f_a (z) ≥ t. Since f_a(x-y) ≥ min{ f_a (x), f_a (y)} and f_a([x y z]) ≥ min { f_a (x), f_a (y), f_a (z)} we have f_a(x-y) ≥ t and f_a([xyz]) ≥ t which implies that x –y ∈$(f_a)_t$ and [xyz] ∈$(f_a)_t$.

Conversely, let x,y,z ∈ N. Suppose f_a(x-y) < min{ f_a (x), f_a (y)}. Let s = min{ f_a (x), f_a (y)}. Then x ∈$(f_a)_s$, y ∈$(f_a)_s$. Therefore x-y ∈ $(f_a)_s$ which implies that f_a(x-y) ≥ min{ f_a (x), f_a(y)},a contradiction to the assumption. Now Suppose f_a([xyz]) < min{ f_a (x), f_a (y), f_a (z)}. Let r = min { f_a (x), f_a (y), f_a (z)}.Then x,y,z ∈$(f_a)_r$ and therefore, [xyz] ∈$(f_a)_r$ and hence f_a([xyz]) ≥ min { { f_a (x), f_a (y), f_a (z)}, a contradiction to the assumption . Hence for each a ∈A and t ∈ Imf_a . f_a(x-y) ≥ min{ f_a(x), f_a(y)} and f_a([x y z]) ≥ min { f_a (x), f_a (y), f_a (z)}. Hence (f , A) is a fuzzy soft right ternary near-ring over N. The case for fuzzy soft ideal can be similarly proved.

Lemma 3.25 Let N be a right ternary near-ring and A be a subset of a parameter set E. Let (f , A) be a fuzzy soft set over N. Define $(f , A)^m$ = { f_a^m | a∈A},m is a positive integer and $f_a^m(x) = (f_a(x))^m$, for every x ∈ N. Then $(f , A)^m$ is a fuzzy soft right ternary near-ring (fuzzy soft ideal) over N whenever (f , A) is a fuzzy soft right ternary near-ring (fuzzy soft ideal) over N.

Proof: Let N be a right ternary near-ring and (f , A) be a fuzzy soft right ternary near-ring over N. Now,

$$f_a^m(x – y) = (f_a(x – y))^m$$
$$\geq \min\{ f_a(x), f_a(y)\})^m$$
$$= \min\{ (f_a(x))^m, (f_a(y))^m\}$$
$$= \min\{ f_a^m(x), f_a^m(y)\}$$

and

$$f_a^m([xyz]) = (f_a([xyz]))^m$$
$$\geq \min\{ f_a(x), f_a(y), f_a(z)\})^m$$
$$= \min\{ (f_a(x))^m, (f_a(y))^m, (f_a(z))^m \}$$
$$= \min\{ f_a^m(x), f_a^m(y), f_a^m(z)\}.$$

Hence $(f , A)^m$ is a fuzzy soft right ternary near-ring over N. The case for fuzzy soft ideal can be similarly proved.

Corollary 3.26 If m and n are positive integers such that m ≤ n and if (f , A) is a fuzzy soft ternary semigroup (fuzzy soft ideal) over N then $(f , A)^n$ is a fuzzy soft right ternary subnear-ring (fuzzy soft ideal) of $(f , A)^m$ over N.

Proof: Since $(f_a(x))^m \geq (f_a(x))^n$, as $f_a(x) \in [0,1]$ whenever $m \leq n$, $(f, A)^n$ is a fuzzy soft ternary subnear-ring of $(f, A)^m$ over N. The case for fuzzy soft ideal can be similarly proved.

The proof of the following Theorem is straight forward from the above Corollary and Definition 2.11(v).

Theorem 3.27 Let N be a right ternary near-ring and A be a subset of parameter set E. If (f , A) is a fuzzy soft right ternary near-ring (fuzzy soft ideal) over N. Then $(f, A) = (f, A) \widetilde{\cup} (f, A)^2 \widetilde{\cup} (f, A)^3 \widetilde{\cup} \ldots$ if $(f, A)^m$ for $m = 1,2,3, \ldots$ are mutually disjoint.

4. CONCLUSION

In this paper, we considered right ternary near-rings and their ideals and applied fuzzy soft set technology initiated by Maji et al to introduce fuzzy soft right ternary near-rings, fuzzy soft ideals and studied their basic algebraic properties. This theory may be applied to many algebraic structures with problems that contain uncertainty and it would be beneficial to extend the proposed method to subsequent studies.

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