A Simulink Modeling to Develop a Control System of Stirred Tank Heater with Multifarious Operating Conditions

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ABSTRACT

This paper represents a novel design and control architecture of a non-linear stirred tank heater based on its mathematical equivalent modeling of the physical system. The method involves both single and two manipulated of four input variables mainly the temperature of the fluid through the jacket and the flow rate of the fluid through the jacket where the others are considered as unmeasured disturbances. A Model Predictive Controller (MPC) is used over PID based controller to maintain the tank temperature at 150^0F using MATLAB simulations. Finally, a comparative tabulated result is presented for both MPC and PID based controllers for single manipulated input variables and offered MPC based control considering the operating decisive factor as a MISO system(single output and two manipulated input variables) for optimum control process.

General Terms

Nonlinear Control, Model Predictive Control, Manipulated Inputs, Stirred Tank Heater.

Keywords

PID controller, Model Predictive Controller, Temperature Control, MISO control

1. INTRODUCTION

MPC is a widely used technology in process control due its ability of dealing with multiple input variables [8][9]. It works in two stages: Identification step and implementation step. A previous identification is necessary to get a linear model of the process plant and then it takes less time for implementation step. In this paper our motto is to keep the tank temperature of a stirred tank heater at a fixed temperature for optimum utilization in different industries by using both MPC and PID controllers [1][3].The performance and quality of responses was judged on the basis of settling time and overshoot of the tank's temperature from the step responses at manipulated inputs (jacket inlet flow rate & temperature) of the simple linearized system which is presented in this paper.

The rest of the paper is organized as follows: In section 2, we started with the physical modeling of the stirred tank heater and developing a mathematical model in section 3. Simulink model of the STH is also shown in this section along with LTI state space equations to be applicable appropriately in MPC and PID controller. Next in section 4 & 5, Simulink model of PID and MPC controllers are developed respectively to manipulate jackets temperature to achieve an optimum control of tank inlet temperature[4][5]. The simulation results associated with each manipulation is also presented in this section. Section 6 deals with the manipulation of two input variables in case of MPC controller as PID's can't work with MISO systems. Our simulation results are discussed in section 7 along with a result table showing all the results we obtained from our simulation work. We concluded in section 8 with references in section 9.

2. PHYSICAL MODELLING OF STH

The parameters for the model are taken as standardized values of a certain chemical process where the temperature of the tank needs to be maintained at a certain optimum temperatures[7][13]. The assumptions taken to develop the mathematical dynamic model of the stirred tank heater are as follows:

1. A constant volume with constant liquid density and heat capacity
2. Perfect Mixing in both Tank and Jacket
3. Inflow and outflow of the fluid in the Jacket and Tank is assumed to be constant.

A pictorial representation of a typical Stirred Tank heater is illustrated as follows:

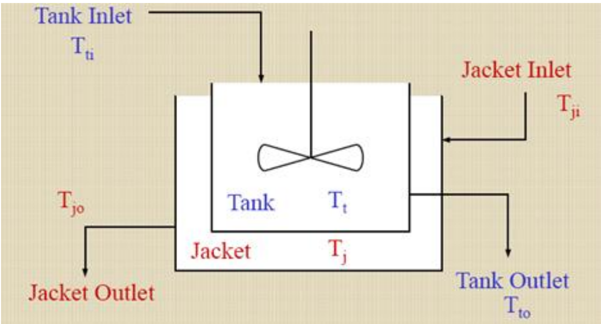

Figure 1:Stirred Tank Heater

3. MATHEMATICAL EQUIVALENT MODEL OF STH

The governing mathematical equation can be represented as follows:

- **Conservation of Mass around the Tank and Jacket gives:**

$$\frac{d}{dt}(\rho_t V_t) = \rho_t \dot{V}_{ti} - \rho_t \dot{V}_{to} (1)$$

$$\frac{d}{dt}(\rho_j V_j) = \rho_j \dot{V}_{ji} - \rho_j \dot{V}_{jo} (2)$$

Since constant volume is assumed hence both the equations will yield zero.

- **Conservation of Energy around the Tank and Jacket gives:**

$$\frac{dT_t}{dt} = \frac{\dot{V}_t}{V_t}(T_{ti} - T_{to}) + \frac{\dot{Q}}{\rho_t V_t c_{pt}} \quad (3)$$

$$\frac{dT_j}{dt} = \frac{\dot{V}_j}{V_j}(T_{ji} - T_{jo}) - \frac{\dot{Q}}{\rho_j V_j c_{pj}} \quad (4)$$

$$\dot{Q} = hA(T_j - T_t) \quad (5)$$

3.1 EQUIVQLENT SIMULINK MODEL OF STH

From the above equations a dynamic model of a stirred tank heater can be obtained. The Simulink model of the stirred water tank heater is shown below:

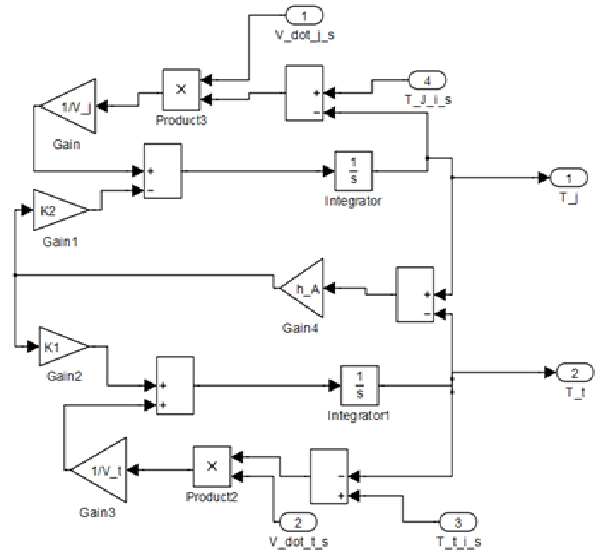

Figure 2: Simulink Model of Stirred Tank Heater

The input-output Simulink model of the stirred water tank heater is shown below:

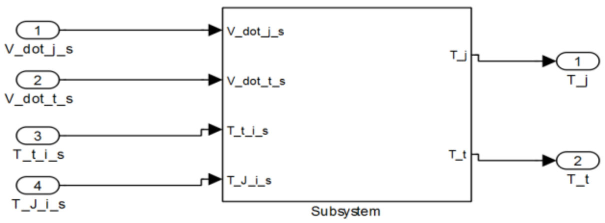

Figure3: Input-Output Model of Stirred Tank Heater

3.2 LTI MODELING OF NONLINIEAR TANK HEATER

Since it is convenient and necessary for the model predictive controller to be able to process a linear model, the linearized model is obtained via the "linmod" command in Matlab with the operating point set such that the initial temperature of the tank was set to 0^0F and the temperature of the Jacket at 150 ^{0}F. The obtained LTI State Space equation for the nonlinear Stirred Water Tank Heater model is as follows:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-4.5 & 3\\ 0.3 & -0.4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}50 & 0 & 0 & 1.5\\ 0 & -7.5 & 0.1 & 0\end{bmatrix}\begin{bmatrix}\dot{V}_j\\ \dot{V}_t\\ T_{ti}\\ T_{ji}\end{bmatrix} \quad (6)$$

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\dot{V}_t\\ \dot{V}_j\\ T_{ti}\\ T_{ji}\end{bmatrix} \quad (7)$$

Where, x_2 is the temperature of the tank to be controlled. The system is stable because the eigenvalues are -3.5793 and -0.1169. They are negative and they lie on the left half plane.

4. JACKET'S TEMPERATURE MANIPULATION for the Model

4.1 PID BASED SIMULINK MODEL OF STH

Our objective is to keep the temperature of the tank at 150 ^{0}F under different operating conditions. The simulation is carried out with Jacket inlet temperature(T_{ji}) as the manipulated variables with the PID controller [6] while keeping the input disturbance values to 1 ft^3/min (Tank inlet flow rate $\dot{V}_t$), 50 ^{0}F (Tank inlet temperatureT_{ti}) and 2 ft^3/min (Jacket inlet flow rate$\dot{V}_j$). The Simulink block diagram of the system with such configuration is shown below:

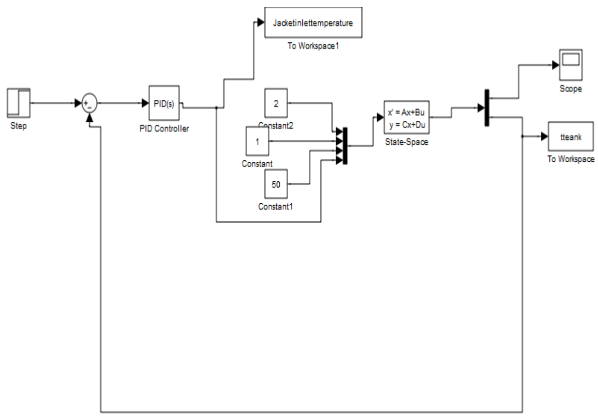

Figure 4

We finally simulate the model with all the specifications using first of all using the PID controller.

4.2 MPC BASED MODEL OF STH

The Model Predictive Controller Block Diagram [6] used to carry out the simulations under different operating conditions with the Jacket Inlet temperature T_j as the manipulated variable is shown below:

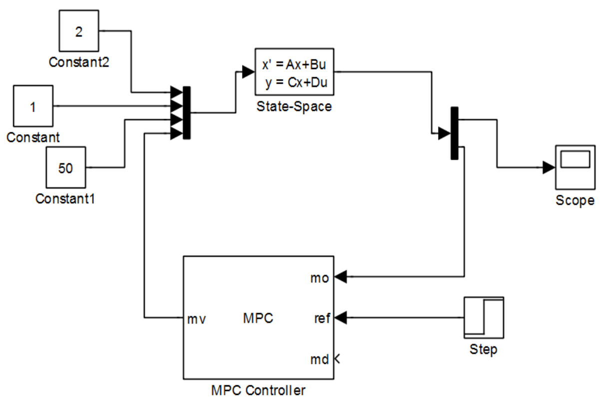

Figure 5

5. SIMULATIONS AND RESULTS

5.1 Simulation Results for PID controller

The results obtained after simulation with the PID controller keeping the input disturbance values when the Tank inlet flow rate $\dot{V}_t$ is 1 ft^3/min, Tank inlet temperature T_{ti} is 50 ^{0}F and Jacket inlet flow rate $\dot{V}_j$ is 2 ft^3/min are shown below:

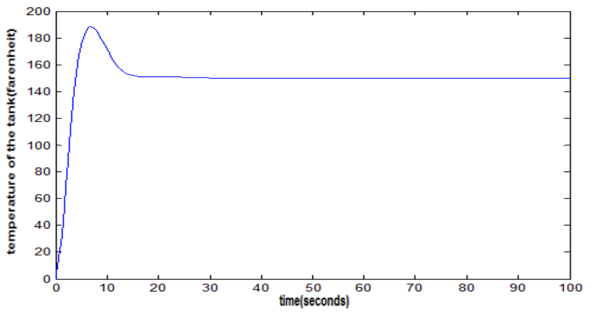

Figure 6

The settling time is 20 seconds and overshoot percentage is 38%.

The corresponding input (Jacket inlet temperature T_j) is shown below:

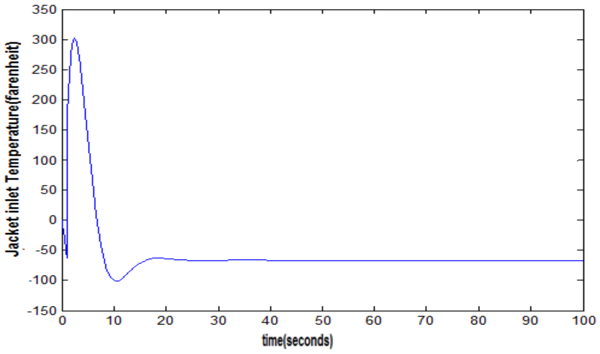

Figure 7

The Jacket inlet temperature (T_j) has become negative at certain time intervals which is not desirable at all.

Changing the Tank inlet temperature T_{ti} to 60 ^{0}F the simulation results with PID are as follows:

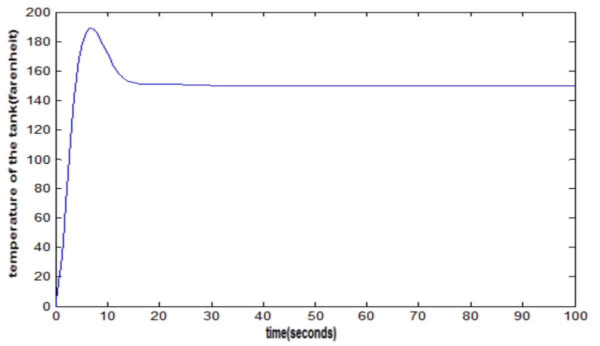

Figure 8

The settling time is 22 seconds and the overshoot percentage is 38%.

The corresponding input (Jacket inlet temperature T_j) is shown below:

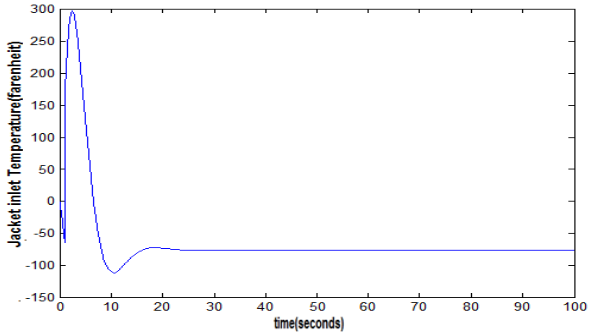

Figure 9

The Jacket inlet temperature (T_j) has become negative at certain time intervals which is not desirable at all.

The PID controller with the input variable as the (Jacket inlet temperature T_j) was adaptively tuned [5] to:

Table 1: PID Controller Parameters

Parameters	Values
Proportional Constant(K_p)	0.08
Integral Constant (K_i)	0.013
Derivative Constant (K_d)	-0.06
Filter Coefficient (N)	0.85

5.2 Simulation Results for MPC controller

The results obtained after simulation with the MPC controller keeping the input disturbance values to 1 ft^3/min (Tank inlet flow rate $\dot{V}_t$), 50 ^{0}F (Tank inlet temperature T_{ti}) and 2 ft^3/min (Jacket inlet flow rate $\dot{V}_j$) is shown below:

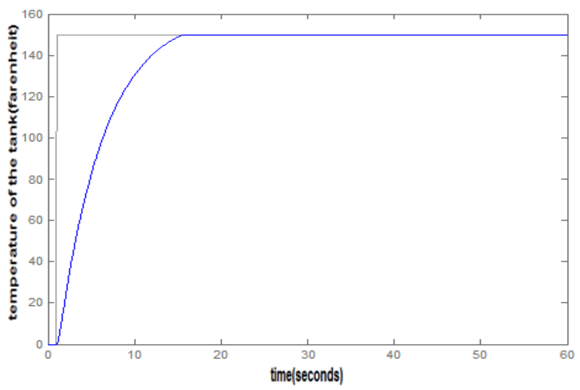

Figure 10

The settling time is 17 seconds and the overshoot percentage is zero.

The corresponding input (Jacket inlet temperature T_j) is shown below:

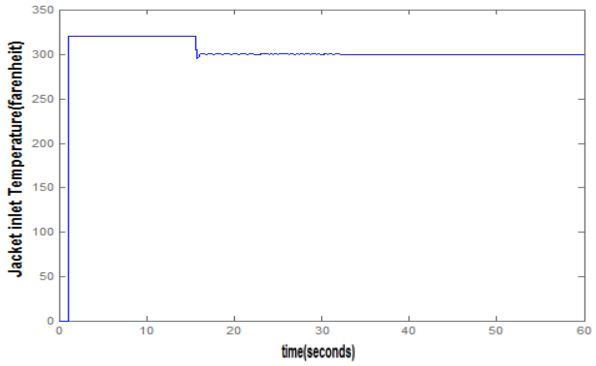

Figure 11

The Jacket inlet temperature (T_j) has never reached negative which is a constraint that has to be maintained.

Changing the Tank inlet temperature T_{ti} to 60 ^{0}F the simulation results with MPC are as follows:

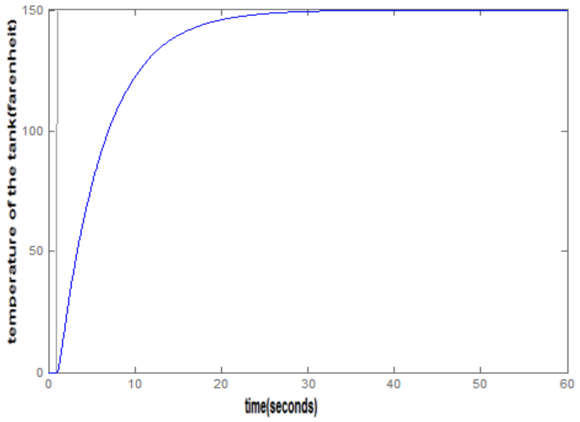

Figure 12

The settling time is 34 seconds and the overshoot percentage is zero.

The corresponding input (Jacket inlet temperatureT_j) is shown below:

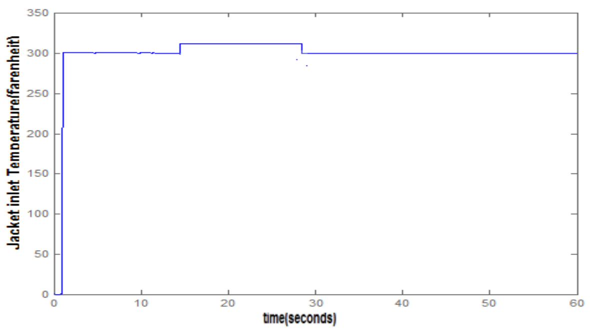

Figure 13

The Jacket inlet temperature (T_j) has never reached negative which is a constraint that has to be maintained.

6. MPC BASED CONTROLLER MANIPULATING TWO INPUT VARIABLES

6.1 Simulink Model of STH

Finally two manipulated variables which are the Jacket inlet flow rate $\dot{V}_j$ and Jacket inlet temperature (T_j) are considered for maintaining the temperature of the tank at 150 ^{0}F while keeping the other variables constant at 1 ft^3/min (Tank inlet flow rate $\dot{V}_t$) and 50 ^{0}F (Tank inlet temperatureT_{ti}) respectively. MPC controller is designed to carry out this operation while the PID controller is rendered quite unworthy because it is generally applicable for Single-Input Single-Output (SISO) systems but not for a Multiple-input Single-Output (MISO) systems [10].

The block diagram of the system utilizing the MPC controller is shown below:

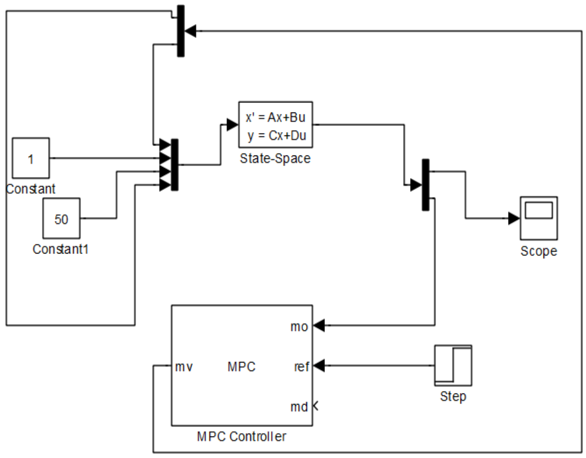

Figure 14: Simulink model of MPC based STH

6.2 SIMULATION RESULTS

The simulation results obtained for this MISO system are shown below.

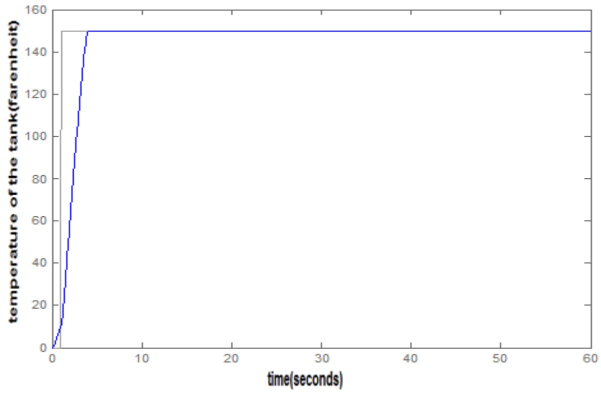

Figure 15

The settling time is 3 seconds and the overshoot percentage is zero.

The corresponding changes in the inputs, Jacket inlet temperatureT_j and Jacket inlet flow rate $\dot{V}_j$ are shown below:

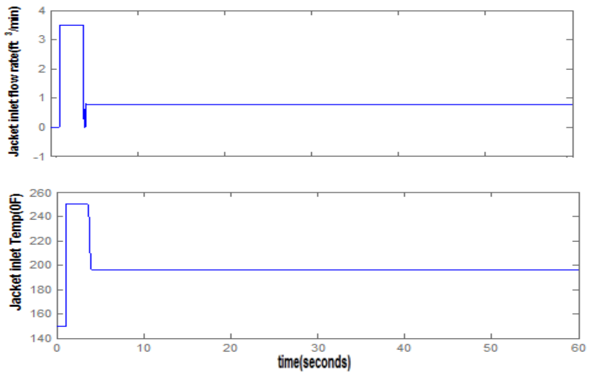

Figure 16

The inputs have never reached negative which is constrained to a desired value.

Changing the Tank inlet temperature T_{ti} to 60 ^{0}F the simulation results with MISO-MPC system are as follows:

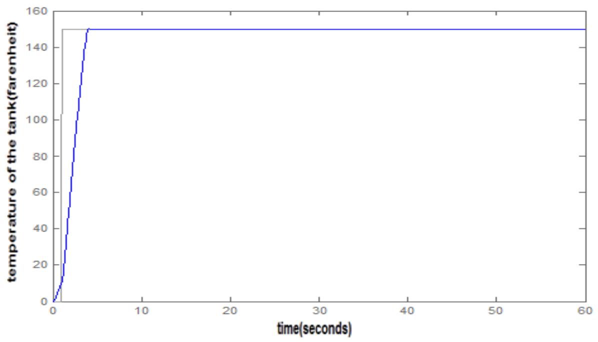

Figure 17

The settling time is 3.1 seconds and the overshoot percentage is zero.

The corresponding changes in the inputs, Jacket inlet temperatureT_j and Jacket inlet flow rate $\dot{V}_j$ are shown below:

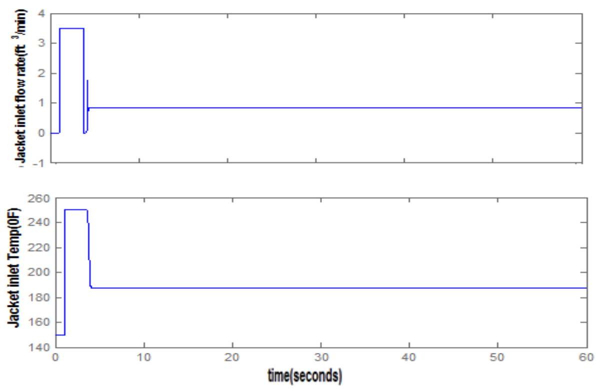

Figure 18

The inputs have never reached negative which is constrained to a desired value.

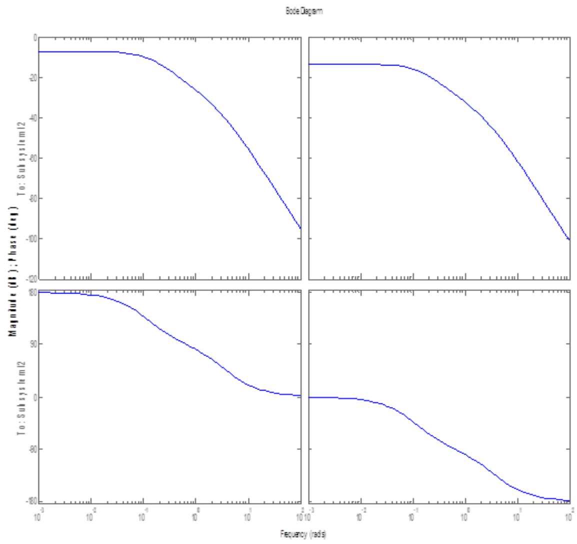

Figure 19

The bode plot showing the system stability is given below:

All the simulations done with the MPC controller was carried out with the parameters of the MPC controller fixed to:

Table 2: MPC Controller Parameters

Parameters	Values
Control Interval	0.05
Prediction Horizon	20
Control Horizon	5

7. Discussion:

While undergoing the control of temperature of the tank it was necessary to maintain the inputs to non-negative value because under no circumstances would the fluid through the jacket be drawn out of the system and the temperature of the fluid in the Jacket cannot be decreased to a temperature below 0 ^{0}F because it would require a complicated actuator to undergo a negative change [14][15]. The temperature of the tank was required to be maintained at 150 ^{0}F which is the optimum temperature for wet process chemical plants such as Ware washes and food processing operations.

The simulated results obtained can be tabulated to delineate the comparison of the performance using both MPC and PID controllers.

Table 3: PID & MPC Controller simulation comparison

Controllers	Input Variables				Settling Time	Overshoot %	Manipulated variable	
	$(\dot{V}_T)$	$(\dot{V}_j)$	(T_{ji})	(T_{ti})			Min	Max
PID	1	Manipulated	200	50	36	26	-0.97	5.4
	1	Manipulated	200	60	33	29	-1.1	5.2
	1	Manipulated	220	50	31	29	5.15	-1.0
	1	Manipulated	220	60	29	30	-1.2	5.1
	1.3	Manipulated	200	50	40	29.5	-1.03	5.24
	1	2	Manipulated	50	20	38	-128	300
	1	2	Manipulated	60	22	38	-124	300
MPC	Input Variables				Settling Time	Overshoot %	Manipulated variable (1/2)	
	$(\dot{V}_T)$	$(\dot{V}_j)$	(T_{ji})	(T_{ti})			Min	Max
	1	Manipulated (1)	200	50	9	0	0 (1)	3 (1)
	1	Manipulated (1)	200	60	7	0	0 (1)	4 (1)
	1	Manipulated (1)	220	50	8	0	0 (1)	4 (1)
	1	Manipulated (1)	220	60	5	0	0 (1)	2.9(1)
	1.3	Manipulated	200	50	13	0	0 (1)	10 (1)
	1	2	Manipulated (2)	50	17	0	0 (1)	319 (2)
	1	2	Manipulated (2)	60	34	0	0 (1)	311 (2)
	1	Manipulated (1)	Manipulated (2)	50	3	0	0 (1) 150 (2)	3.3 (1) 250 (2)
	1	Manipulated (1)	Manipulated (2)	60	3.1	0	0 (1) 150 (2)	3.5 (1) 250 (2)

From the tabulated data showing all the data's obtained from the simulation results, it is quite apparent that the performance of MPC completely overhauls the performance of the adaptive PID controller.

8. CONCLUSION

In the case of controlling the Stirred Tank Heater with a single input, it was observed that the input values had gone negative at certain time intervals while using PID controller. This is derogatory for maintaining STH because it will lead to a complicated and a costly actuator for controlling the temperature of the tank [12]. It is also necessary to eliminate any presence of overshoot responses while controlling STH. This was not possible with the PID controller. All of these negative impacts while using the PID controller were totally eliminated when MPC was used. It was also possible to control the temperature of STH by manipulating both the Jacket Inlet flow rate and Jacket Inlet Temperature with the MPC controller. Moreover the input controlling signals were also constrained for optimum performance of STH. Hence the temperature control of STH has quite remarkably improved by using the Model Predictive Controller [10].

9. REFERENCES

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