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Multi-Objective Constrained Optimization using Discrete Mechanics and NSGA-II Approach

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ABSTRACT

A novel approach to solve multi-objective optimization problems of complex mechanical systems is proposed based on evolutionary algorithm. Discrete mechanics derives structure preserving constraint equations and objective functions. Standard non-linear optimization techniques used to obtain optimal solution to these equations fails to find global optimum solution and also requires system satisfying initial guess. Multi-objective optimization technique like non-dominated sorting genetic algorithm-II (NSGA-II) finds global optimal solution without giving any initial guess for multiple conflicting objectives. This method is numerically illustrated by optimizing an underactuated mechanical system called 2D SpiderCrane system. In SpiderCrane, fast and precise payload positioning is to be achieved while keeping payload swing minimum along the trajectory. Minimizing the time of operation requires greater amount of force which may lead to unacceptable payload sway, while decreasing forces increases the time of operation. Proposed control law to optimize this conflicting multi-objectives is validated with simulation results.

Keywords:

Optimization, Non-dominated sorting genetic algorithm, Discrete mechanics optimal control, Bio-inspired 2D SpiderCrane.ifx

1. INTRODUCTION

In order to solve multi-objective optimization problem of mechanical systems, one is often interested in preserving certain properties of the mechanical system for the approximated solution and steer a mechanical system from an initial to a final state under the influence of control forces such that a given quantity, for example control effort or maneuver time is minimal i.e multiple conflicting objectives are needed to be optimized. The presence of these multiple conflicting objectives formulates the task as a (global) multi-objective optimization problem (MOP), which resorts to a number of trade-off optimal solutions. Classical methods like the objective weighted method, the hierarchical optimization, the constraint method, the goal programming method and many more aggregates the multiple-objective in a single, parametrized objective function. However, for systematically varying the parameters, knowledge of problem is very much necessary which may not be available [1]. Also, there are possibilities of producing biased result by setting priorities to objectives and finding one solution in one simulation run. Because of which several optimization runs are required to obtain approximate Pareto-optimal set. Evolutionary algorithms (EAs), on the other hand, can find multiple optimal solutions in one single simulation run due to their population-approach. EAs are ideal for solving multi-objective optimization problems. Although there exist a number of multi-objective evolutionary algorithms (EMO), non-dominated sorting genetic algorithm II (NSGA-II), have gained tremendous popularity in solving different kinds of engineering problems [1], [2]. NSGA-II implements elitism for multi-objective search which enhances the convergence properties towards the true Pareto-optimal set. The constraint handling method does not make use of penalty parameters. The algorithm implements a modified definition of dominance in order to solve constrained multi-objective efficiently.

Discrete Mechanics and Optimal Control (DMOC) is used to derive structure preserving constraint equations and objective functions. These equations are then used by NSGA-II to obtain global optimum solution. DMOC is introducesd in [4], [5]. In the context of variational integrators [6], the discretization of the Lagrange-d'Alembert principle leads to structure preserving time stepping equations which serve as equality constraints for the resulting finite dimensional non-linear optimization problem. This problem can be solved by standard non-linear optimization techniques such as Sequential Quadratic Programming (SQP) leading to local optimal solutions dependent on the initial guess [7]. Although this method works very successfully in many applications, they fail to find the global optimal solution for MOP without using any initial guess. A remedy for these difficulty is found in the MOEA.

In this paper, 2D SpiderCrane mechanism is considered and the objective is to steer its payload from stable equilibrium point with zero initial velocity to other stable equilibrium point with minimum effort in minimum time, i.e. stabilization of the load along with the time and force minimization by keeping the payload swing minimum along the trajectory. This bio-inspired mechanism is proposed by the Laboratory of Automatic Control at École Polytechnique Fédérale de Lausanne [8], [9], [10].

The main contribution of this work is to provide a methodology for performing multi-objective constrained optimization of complex mechanical systems without providing any initial guess and using equations derived from DMOC as objective function and as constraint equations. The paper is organised as follows: Section II includes brief introduction of DMOC. Section III summarizes basic principles of NSGA-II. Section IV includes dynamics and modelling of 2D SpiderCrane. Section V includes application of NSGA-II to 2D SpiderCrane system and based on the simulation results, the performances of NSGA-II and local optimization method are compared and discussed. Section VI outlines the conclusion and future research.

2. DISCRETE MECHANICS OPTIMAL CONTROL (DMOC)

In order to locally solve optimal control problems, DMOC is used which relies on a direct discretization of the variational formulation of the system dynamics. For convenience, we briefly summarize the basic idea, the more elaborate discussion can be found in [11], [4].
A mechanical system with configuration space M is to be moved on a curve $x(t) \in M, t \in [0, T]$, form an initial state $(q^0, \dot{q}^0)$ to a final state $(q^T, \dot{q}^T)$ under the influence of a force $f : TM \times U \rightarrow T^*M$, where TM and T^*M are the tangent and cotangent space of the configuration space M, respectively. This force depends on a time-dependent control effort $u(t) \in U$. The curves q and u shall minimize a given objective functional $J : TM \times U \rightarrow \mathbb{R}$

$$J(q, \dot{q}, u) = \int_0^T C(q(t), \dot{q}(t), u(t))dt \quad (1)$$

with the cost function $C : TM \times U \rightarrow \mathbb{R}$. If $L : TM \rightarrow \mathbb{R}$ denotes the Lagrangian of the system, its motion $q(t)$ satisfies the *Lagrange-d'Alembert principle,* which requires that

$$\delta \int_0^T L(q(t), \dot{q}(t))dt + \int_0^T f(q(t), \dot{q}(t), u(t))dt = 0 \quad (2)$$

for all variations δq with $\delta q(0) = \delta q(T) = 0$. This principle leads to a system of second order differential equations denoted as the *forced Euler-Lagrange equations*

$$\frac{d}{dt}\left[\frac{\partial}{\partial \dot{q}} L(q, \dot{q})\right] - \frac{\partial}{\partial q} L(q, \dot{q}) = f(q, \dot{q}, u) \quad (3)$$

Final state is given by final time constraint $r(q(t), \dot{q}(t), q^T, \dot{q}^T) = 0$ with $r : TM \times TM \rightarrow \mathbb{R}^{n_r}$, where $(q^T, \dot{q}^T) \in TM$ is fixed for desired final state. The minimization of (1) subject to the equations (3) and initial and final state conditions constitutes the optimal control problem in the continuous setting. Formulation of optimal control problem for a Lagrangian system is as follows:
Minimise cost function J

$$\min_{(q(\cdot),\dot{q}(\cdot),u(\cdot),T)} J(q, \dot{q}, u) = \int_0^T C(q(t), \dot{q}(t), u(t))dt, \quad (4)$$

subjected to

$$\delta \int_0^T L(q(t), \dot{q}(t))dt + \int_0^T f_{LC}(q(t), \dot{q}(t), u(t)) \cdot \delta q(t)dt = 0,$$

$$q(0) = q^0, \dot{q}(0) = \dot{q}^0,$$

$$h(q(t), \dot{q}(t), u(t)) \geq 0,$$

$$r(q(T)), \dot{q}(T), q^T, \dot{q}^T) = 0.$$

In this paper, we pose our optimization problem as moving the payload from an initial position to a given desired position with minimum swing, time and minimum control effort. Here, initial and final positions of the payload are considered as fixed boundary conditions.

Fixed Boundary Conditions. This is the special case of optimisation problem with fixed initial and final velocities and configuration without path constraints,

$$\min_{q(\cdot),\dot{q}(\cdot),u(\cdot)} J(q, \dot{q}, u) = \int_0^T C(q(t), \dot{q}(t), u(t))dt, \quad (5)$$

subjected to

$$\delta \int_0^T L(q(t), \dot{q}(t))dt + \int_0^T f_{LC}(q(t), \dot{q}(t), u(t)) \cdot \delta q(t)dt = 0,$$

$$q(0) = q^0, \dot{q}(0) = \dot{q}^0, q(T) = q^T, \dot{q}(T) = \dot{q}^T.$$

2.1 Discretization

During discretization the state space TM of the continuous system is replaced by R and consider the grid $\Delta t = \{t_k = kh | k = 0, ..., N\}$, $Nh = T$, where N is a positive integer and h the step size. The path $q : [0, 1] \rightarrow Q$ is replaced by discrete path $q_d : [0, h, 2h, ..., (Nh = 1)] \rightarrow M$. Similarly continuous force $f : [0, 1] \rightarrow T^*M$ is approximated by discrete force $f_d : [0, h, 2h, ..., (Nh = 1)] \rightarrow T^*M$.
Notations used $q_k = q_d(kh)$ and $f_k = f_d(kh)$. An approximation of the action integral in (2) over small time interval $[kh, (k + 1)h]$, by a discrete Lagrangian $L_d : Q \times Q \rightarrow \mathbb{R}$ yields

$$L_d(q_k, q_{k+1}) \approx \int_{kh}^{(k+1)h} L(q(t), \dot{q}(t))dt, \quad (6)$$

and *discrete forces* expressed as

$$f_k^- \cdot \delta q_k + f_k^+ \cdot \delta q_{k+1} \approx \int_{kh}^{(k+1)h} f(q(t), \dot{q}(t), u(t)) \cdot \delta q(t)dt, \quad (7)$$

where f_k^- and $f_k^+ \in T^*Q$ are called *left* and *right discrete forces* respectively. Now depending on (q_k, q_{k+1}, u_k), *the discrete Lagrange-d'Alembert principle* is obtained in (8). This requires to find discrete paths $\{q_k\}_{k=0}^N$ such that for all variations $\{\delta q_k\}_{k=0}^N$ with $\delta q_0 = \delta q_N = 0$, one has

$$\delta \sum_{k=0}^{N-1} L_d(q_k, q_{k+1}) + \sum_{k=0}^{N-1} f_k^- \cdot \delta q_k + f_k^+ \cdot \delta q_{k+1} = 0. \quad (8)$$

which is equivalent to the *forced discrete Euler-Lagrange equations*

$$D_2 L_d(q_{k-1}, q_k) + D_1 L_d(q_k, q_{k+1}) + f_{k-1}^+ + f_k^- = 0, \quad (9)$$

where $k = 1, 2, ..., N - 1$, D_i denotes the derivative w.r.t. the ith slot. In the same manner approximation of the objective functional (1) is done and the *discrete objective functional* $J_d(q_d, u_d)$, is obtained such that the *Discrete Constrained Optimization Problem* is

$$min_{q_d,u_d} J_d(q_d, u_d) = \sum_{k=0}^{N-1} C_d(q_k, q_{k+1}, u_k) \quad (10)$$

subject to the discretized boundary constraints and the discrete Euler-Lagrange equations (9).

Discrete Cost Function. Approximation of the cost functional in short interval of time $[kh, (k + 1)h]$ is given as

$$C_d(q_k, q_{k+1}, u_k) \approx \int_{kh}^{(k+1)h} C(q(t), \dot{q}(t), u(t))dt \quad (11)$$

results in discrete cost function

$$J_d(q_d, u_d) = \sum_{k=0}^{N-1} C_d(q_k, q_{k+1}, u_k) \quad (12)$$

In our case we select the following cost function.

$$J_{1d}(u_d) = \sum_{k=0}^{N} u_k^2, J_{2d}(\theta_d) = \sum_{k=0}^{N} \theta_k^2, \quad (13)$$

$$J_{3d} = \textit{Total time required for operation} = T$$

The cost function depends on the forces, Theta (θ) and T.

Boundary Conditions. All the initial and final conditions, configurations and velocities need to be assigned in discrete form in discrete fixed boundary optimisation problem. $q(0) = q^0, \dot{q}(0) = \dot{q}^0$ and $q(1) = q^1, \dot{q}(1) = \dot{q}^1$ as initial condition. $q(N-1) = q^{N-1}, \dot{q}(N-1) = \dot{q}^{N-1}$ and $q(N) = q^N, \dot{q}(N) = \dot{q}^N$ these two points lead to two discrete boundary conditions as discussed in [12] using *standard Legendre transformation*

$$D_2L(q_0, \dot{q}_0) + D_1L_d(q_0, q_1) + f_0^- = 0, \quad (14)$$

$$-D_2L(q_N, \dot{q}_N) + D_2L_d(q_{N-1}, q_N) + f_{N-1}^+ = 0. \quad (15)$$

Equation (14) is applied to initial discrete point and at final point (15) is applied. Equation (14) and (15) describes a non-linear optimization problem with equality constraints, which can be solved by standard optimization methods. With a local solver, for example, the SQP-method, one can find a local optimum. The initial guess of the optimization is chosen in a simple way. SQP is an iterative method for non-linear constrained optimization, used on problems for which the objective function and the constraints are twice continuously differentiable.

2.2 Advantages of The Proposed Approach

The SQP method used alongwith DMOC [13], have some notable drawback:

1. SQP is local optimal solver so the obtained result is not the global optimal solution.
2. It is a single objective problem.
3. This technique requires an excellent initial guess and the rate of convergence of the solution are very sensitive to these guesses. The wrong selection of initial guess misleads the search.

To overcome the above drawbacks, Multi-objective optimization techniques like NSGA II can be used in place of SQP. The advantages of Evolutionary techniques are as follows:

1. They are population based search algorithms, so global optimal solution is possible. Population-based search techniques give multi-directional search. Therefore, they search whole solution space for global optimum solution.
2. Optimizing all the objectives simultaneously and generating a set of alternative solutions offers more flexibility. The simultaneous optimization can fit nicely with population-based approaches such as EAs, because they generate multiple solutions in a single run.
3. In case of population-based techniques, the final solution does not depend on the initial guess.

3. NON-DOMINATED SORTING GENETIC ALGORITHM-II (NSGA-II)

A single objective optimization algorithm mostly terminates upon obtaining an optimal solution. In a typical multi-objective optimization problem, there exists a family of equivalent solutions that are superior to the rest of the solutions and are considered equal from the perspective of simultaneous optimization of multiple conflicting objective functions. Such solutions are called non-inferior, non-dominated or Pareto-optimal solutions, and are such that no objective can be improved without degrading at least one of the others, and, given the constraints of the model, no solution exist beyond the true Pareto front.
The goal of NSGA-II actually consists of two parts as mentioned in [1], namely that the solutions found must be: (i) close to the Pareto-optimal front, and (ii) diverse. This is also illustrated in Fig 1, where it is clear how solutions near the Pareto-optimal front are first obtained followed by a search for diversity along the front. The first requirement is obtained by using the dominance concept and does not have a need for any niching or crowding measures. Therefore, a good algorithm can find a set of solutions as close to the Pareto-optimal front as possible. However, the second requirement can be more difficult to obtain. In order to obtain a diverse set, it must be specified what can be considered as a set of diverse solutions, but it must also be understood how dominance has influenced the diversity of the solutions.

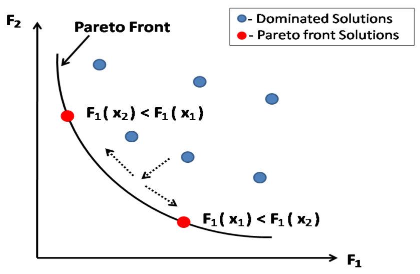

Fig. 1. Pareto-front and Non-domination illustration

The NSGA proposed by Srinivas and Deb (1994) has been successfully applied to solve many problems, the main criticisms of this approach has been its high computational complexity of non-dominated sorting, lack of elitism, and need for specifying a tunable parameter called sharing parameter [14]. Recently, Deb et al. reported an improved version of NSGA, which they called NSGA-II, to address all the above issues [2].
NSGA-II only differs from a simple genetic algorithm in the selection process. The population is initialized as usual. Once the population in initialized NSGA-II sorts the population based on non-domination into each front. The first front is completely non-dominant set in the current population and the second front is dominated only by the individuals in the first front and the front goes so on. Each individual in the each front are assigned rank (fitness) values based on front in which they belong to. Individuals in first front are given a fitness value of 1 and individuals in second are assigned fitness value as 2 and so on. In addition to fitness value a new parameter called crowding distance is calculated for each individual. The crowding distance is a measure of how close an individual is to its neighbours. Large average crowding distance means better diversity in the population. Parents are selected from the population by using tournament selection based on the rank and crowding distance. When comparing two individual, an individual belonging to lesser rank is given priority but if both individual have same rank, extreme individuals prevails over not extreme ones. If both individuals are not extreme, the one with the bigger crowding distance wins. The selected population then generates off-springs from crossover and mutation operators.
The population with the current population and current off-springs is sorted again based on non-domination and only the best N individuals are selected, where N is the population size. The selection is based on rank and the on crowding distance on the last front.

4. 2D SPIDERCRANE

The proposed method is applied to 2D SpiderCrane system. SpiderCrane is a pulley-cable system and cable systems are generally underactuated which works on tensile forces. This system is similar to pendulum on a cart system where pendulum swing is non actuated. SpiderCrane is a crane designed to reduce the time required, for carrying loads, consider the 2D SpiderCrane mechanism as illustrated in Fig 2 [8].
Crane operators moves the load in such a way that cable by which load is attached remains vertical for safety reasons, this strategy induces large economical loss due to additional time involved in process. To improve work rate one should anticipate swing of the load. The problem is to achieve fast and precise payload positioning while minimizing the swing. This problem has been approached by various control strategies [15]-[19].

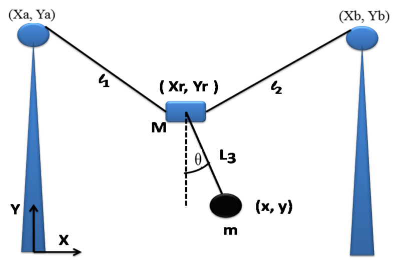

Fig. 2. 2D SpiderCrane mechanism

In this model, the problem is to find the optimum effort path of the payload of mass *m* suspended by the cable from the ring of mass M on which the actuating forces F_x and F_y are applied. This type of analysis of the 2D SpiderCrane is termed as decoupled SpiderCrane. The configuration variables for gantry crane mechanism are

$$q = (\mathrm{X}_r, \mathrm{Y}_r, \theta)^T \tag{16}$$

and Lagrangian may be defined as

$$\mathcal{L}(q, \dot{q}) = \frac{1}{2}\dot{q}^T M(q)\dot{q} - V(q) \tag{17}$$

where,

$$M(q) = \begin{bmatrix} (M+m) & 0 & mL_3\cos\theta \\ 0 & (M+m) & mL_3\sin\theta \\ ml_3\cos\theta & mL_3\sin\theta & mL_3^2 \end{bmatrix} \tag{18}$$

and

$$V(q) = (M+m)gY_r - mgL_3\cos\theta \tag{19}$$

The resulting Euler-Lagrange equations are:

$$F_x = (M+m)\ddot{X}_r + (mL_3\cos\theta)\ddot{\theta} - (mL_3\sin\theta)\dot{\theta}^2 \tag{20}$$

$$F_y = (M+m)\ddot{Y}_r + (mL_3\sin\theta)\ddot{\theta} + (mL_3\cos\theta)\dot{\theta}^2 + (M+m)g \tag{21}$$

$$0 = (mL_3\cos\theta)\ddot{X}_r + (mL_3\sin\theta)\ddot{Y}_r + (mL_3^2)\ddot{\theta} + mgL_3\sin\theta \tag{22}$$

As from Fig 3, F_x and F_y are control forces applied on the ring of mass M on which payload of mass m is suspended with non elastic cable. This subsystem is referred as *gantry mechanism* or *decoupled SpiderCrane*, where pulleys, cables and pylons are neglected for simplicity.

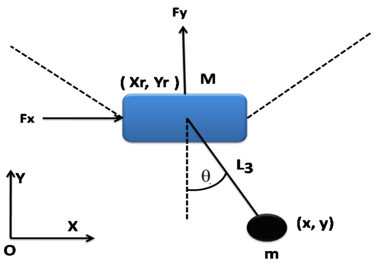

Fig. 3. Decoupled SpiderCrane model

4.1 Discretization

In this paper, a midpoint rule for integral approximation and derivative approximation over small intervals h is considered as follows

$$\int_{kh}^{(k+1)h} f(x)dx \approx hf(\frac{a+b}{2}).$$

We obtain velocity vector according to midpoint rule,

$$\frac{(q_{k+1} - q_k)}{h} \approx \dot{q}$$

and position vector approximation according to midpoint rule,

$$\frac{q_{k+1} + q_k}{2} \approx q_k$$

as well as in case for discrete forces, we obtain

$$\int_{kh}^{(k+1)h} f(t).\delta q(t)dt \approx h\frac{f_{k+1} + f_k}{2}.\frac{\delta q_{k+1} + \delta q_k}{2}$$

$$= \frac{h}{4}(f_{k+1} + f_k).\delta q_k + \frac{h}{4}(f_{k+1} + f_k).\delta q_{k+1}, \tag{23}$$

i.e.$f_k^- = f_k^+ = \frac{h}{4}(f_{k+1}^k)$ were used as the left and right discrete forces. Discrete Lagrange is equivalent to continuous Lagrange

$$L_d(q_k, q_{k+1}) = hL\left(\frac{q_{k+1} + q_k}{2}, \frac{q_{k+1} - q_k}{h}\right), \tag{24}$$

Using (17), (18), (19) and (24) Lagrangian is defined as

$$L = \frac{1}{2}[(M+m)(\dot{X_r}^2 + \dot{Y_r}^2) + 2\dot{X_r}^2\dot{\theta}(mL_3cos\theta)$$

$$+2\dot{Y_r}^2\dot{\theta}(mL_3sin\theta) + \dot{\theta}^2(mL_3^2)] - (M+m)gY_r + mgL_3cos\theta$$

discrete Lagrangian is,

$$\begin{aligned} &L_d(q_k, q_{k+1}) \\ &= \frac{h}{2}(((M+m)[((X_{r_{k+1}} - X_{r_k})/h)^2 + ((Y_{r_{k+1}} - Y_{r_k})/h)^2]) \\ &+2((X_{r_{k+1}} - X_{r_k})/h)(\theta_{k+1} - \theta_k/h)(mL_3cos((\theta_{k+1} + \theta_k)/2)) \\ &+2((Y_{r_{k+1}} - Y_{r_k})/h)((\theta_{k+1} - \theta_k)/h)(mL_3sin((\theta_{k+1} + \theta_k)/2)) \\ &+((\theta_{k+1} - \theta_k)/h)^2(mL_3^2)) - (M+m)g((Y_{r_{k+1}} + Y_{r_k})/2) \\ &+mgL_3cos((\theta_{k+1} + \theta_k)/2) \end{aligned} \tag{25}$$

$$L_d(q_{k-1}, q_k)$$
$$= \frac{h}{2}(((M+m)[((X_{r_k} - X_{r_{k-1}})/h)^2 + ((Y_{r_k} - Y_{r_{k-1}})/h)^2])$$
$$+2((X_{r_k} - X_{r_{k-1}})/h)(\theta_k - \theta_{k-1}/h)(mL_3cos((\theta_k + \theta_{k-1})/2))$$
$$+2((Y_{r_k} - Y_{r_{k-1}})/h)((\theta_k - \theta_{k-1})/h)(mL_3sin((\theta_k + \theta_{k-1})/2))$$
$$+((\theta_k - \theta_{k-1})/h)^2(mL_3^2)) - (M+m)g((Y_{r_k} + Y_{r_{k-1}})/2)$$
$$+mgL_3cos((\theta_k + \theta_{k-1})/2) \quad (26)$$

and discrete cost function as

$$C_d(f_k, f_{k+1}) = hC(\frac{f_{k+1} + f_k}{2}). \quad (27)$$

In this 2D SpiderCrane problem as seen from (20) and (21) actuating discrete force f_k can be represented as

$$u_k = f_k = \begin{pmatrix} F_{x_k} \\ F_{y_k} \end{pmatrix} \quad (28)$$

Now from (13) and (27) discrete cost function for 2D SpiderCrane can be formulated. Since we formulated our problem as constrained optimization problem, Forced discrete Euler Lagrange equation (9) and discrete boundary condition (14) and (15) serves as constraints and are as follows.
Constraint for initial θ,

$$(M+m)((\theta(2) - \theta(1))/h) + ((mL)cos((\theta(2) + \theta(1))/2)$$
$$((Y_r(2) - Y_r(1))/h)) + h(((M+m)((\theta(2)$$
$$-\theta(1))/h)(-1/h)) + (((mL)((Y_r(2)$$
$$-Y_r(1))/h))((cos((\theta(2) + \theta(1))/2)(-1/h))$$
$$+(((\theta(2) - \theta(1))/h)sin((\theta(2) + \theta(1))/2)(-1/2))))$$
$$+((mL)((X_r(2) - X_r(1))/h)((Y_r(2)$$
$$-Y_r(1))/h)(cos((\theta(2) + \theta(1))/2))0.5)$$
$$+(mgL(-0.5)sin((\theta(2) + \theta(1))/2))) = 0 \quad (29)$$

Constraint for final θ,

$$(-1)((M+m)((\theta(11) - \theta(10))/h)$$
$$+((mL)cos((\theta(11) + \theta(10))/2)((Y_r(11) - Y_r(10))/h)))$$
$$+h(((M+m)((\theta(11) - \theta(10))/h)(1/h))$$
$$+((mL)((Y_r(11) - Y_r(10))/h))((cos((\theta(11) + \theta(10))/2)(1/h))$$
$$+(((\theta(11) - \theta(10))/h)(-1/2)sin((\theta(11) + \theta(10))/2)))$$
$$+((mL)((X_r(11) - X_r(10))/h)((Y_r(11)$$
$$-Y_r(10))/h)(0.5)(cos((\theta(11) + \theta(10))/2)))$$
$$+((-mgL(0.5))(sin((\theta(11) + \theta(10))/2)))) = 0 \quad (30)$$

Constraint for initial X_r,

$$(((M+m)((X_r(2) - X_r(1))/h))$$
$$+((mL)((Y_r(2) - Y_r(1))/h)(sin((\theta(2) + \theta(1))/2))))$$
$$+h(((M+m)((X_r(2) - X_r(1))/h)(-1/h))$$
$$+(((-mL)/h)(sin((\theta(2) + \theta(1))/2))((Y_r(2) - Y_r(1))/h)))$$
$$+((h/2)(F_x(1))) = 0 \quad (31)$$

Constraint for final X_r,

$$(-1)(((M+m)((X_r(11) - X_r(10))/h))$$
$$+((mL)((Y_r(11) - Y_r(10))/h)(sin((\theta(11) + \theta(10))/2))))$$
$$+((M+m)((X_r(11) - X_r(10))/h))$$
$$+((mL)((Y_r(11) - Y_r(10))/h)(sin((\theta(11) + \theta(10))/2)))$$
$$+((h/2)(F_x(11))) = 0 \quad (32)$$

Constraint for initial Y_r,

$$(mL)(cos((\theta(2) + \theta(1))/2))((\theta(2) - \theta(1))/h)$$
$$+(mL)(sin((\theta(2) + \theta(1))/2))((X_r(2) - X_r(1))/h)$$
$$+(mL^2)((Y_r(2) - Y_r(1))/h)$$
$$+h(((mL)(cos((\theta(2) + \theta(1))/2))((\theta(2) - \theta(1))/h)(-1/h))$$
$$+((mL)(sin((\theta(2) + \theta(1))/2))((X_r(2) - X_r(1))/h)(-1/h))$$
$$+((mL)((Y_r(2) - Y_r(1))/h)(-1/h))$$
$$-((M+m)g0.5)) + ((h/2)(F_y(1))) = 0 \quad (33)$$

Constraint for final Y_r,

$$(-1)(((mL)(cos((\theta(11) + \theta(10))/2))((\theta(11) - \theta(10))/h))$$
$$+((mL)(sin((\theta(11) + \theta(10))/2))((X_r(11) - X_r(10))/h))$$
$$+((mL^2)((Y_r(11) - Y_r(10))/h)))$$
$$+h(((mL)(cos((\theta(11) + \theta(10))/2))((\theta(11) - \theta(10))/h)(1/h))$$
$$+((mL)(sin((\theta(11) + \theta(10))/2))((X_r(11) - X_r(10))/h)(1/h))$$
$$+((mL)((Y_r(11) - Y_r(10))/h)(1/h))$$
$$-((M+m)g0.5)) + ((h/2)F_y(11)) = 0 \quad (34)$$

Constraint for mid values,
for $i = 1:9$
Constraint for mid θ,

$$h(((M+m)((\theta(i+1) - \theta(i))/h)(1/h))$$
$$+((mL)((Y_r(i+1) - Y_r(i))/h))((cos((\theta(i+1)$$
$$+\theta(i))/2)(1/h)) + (((\theta(i+1)$$
$$-\theta(i))/h)(-1/2)sin((\theta(i+1) + \theta(i))/2)))$$
$$+((mL)((X_r(i+1) - X_r(i))/h)((Y_r(i+1)$$
$$-Y_r(i))/h)0.5(cos((\theta(i+1) + \theta(i))/2)))$$
$$+((-mgL0.5)(sin((\theta(i+1) + \theta(i))/2))))$$
$$+h(((M+m)((\theta(i+2) - \theta(i+1))/h)(-1/h))$$
$$+(((mL)((Y_r(i+2) - Y_r(i+1))/h))((cos((\theta(i+2)$$
$$+\theta(i+1))/2)(-1/h)) + (((\theta(i+2)$$
$$-\theta(i+1))/h)sin((\theta(i+1) + \theta(i))/2)(-1/2))))$$
$$+((mL)((X_r(i+2) - X_r(i+1))/h)((Y_r(i+2)$$
$$-Y_r(i+1))/h)(cos((\theta(i+1) + \theta(i+2))/2))0.5)$$
$$+(mgL(-0.5)sin((\theta(i+2) + \theta(i+1))/2))) = 0 \quad (35)$$

Constraint for mid X_r,

$$(((M+m)((X_r(i+1) - X_r(i))/h))$$
$$+((mL)((Y_r(i+1) - Y_r(i))/h)(sin((\theta(i+1) + \theta(i))/2))))$$
$$+h(((M+m)((X_r(i+2) - X_r(i+1))/h)(-1/h))$$
$$+(((-mL)/h)(sin((\theta(i+2) + \theta(i+1))/2))((Y_r(i+2)$$
$$-Y_r(i+1))/h))) + (((h)(F_x(i+1)))) = 0 \quad (36)$$

Constraint for mid Y_r,

$$h(((mL)(cos((\theta(i+1) + \theta(i))/2))((\theta(i+1) - \theta(i))/h)(1/h))$$
$$+((mL)(sin((\theta(i+1) + \theta(i))/2))((X_r(i+1)$$
$$-X_r(i))/h)(1/h)) + ((mL)((Y_r(i+1) - Y_r(i))/h)(1/h))$$
$$-((M+m)g0.5)) + h(((mL)(cos((\theta(i+2)$$
$$+\theta(i+1))/2))((\theta(i+2) - \theta(i+1))/h)(-1/h))$$
$$+((mL)(sin((\theta(i+2) + \theta(i+1))/2))((X_r(i+2)$$
$$-X_r(i+1))/h)(-1/h)) + ((mL)((Y_r(i+2)$$
$$-Y_r(i+1))/h)(-1/h)) - ((M+m)g0.5))$$
$$+(((h)(F_y(2)))) = 0 \quad (37)$$

end

5. APPLICATION OF NSGA-II TO 2D SPIDERCRANE SYSTEM

In 2D-space, a path is constructed by a series of points. The coordinates of these points are taken as codes and arranged in terms of its location in trajectory. If the initial point of path is (x_i, y_i) and final point is (x_f, y_f),the path can be expressed as follows : $(x_s, x_{j1}, ...x_{j(L-2)}, x_e, y_s, y_{j1}, ...y_{j(L-2)}, y_e)$ where, L is the length of code and x co-ordinates of all path points are placed at front followed by y co-ordinates [20].

5.1 Generation of Initial Population

The population is initialized based on the problem range and constraints if any. In 2D SpiderCrane problem considered in this paper, optimization of forces, θ and time is to be achieved while finding optimized path. Consider the population size of N, then based on the above coding rules chromosomes are generated as follows: First gene of chromosome is set as initial point of x co-ordinate, and as number of discrete points considered are 11, 11^{th} gene is set as final point of x co-ordinate, remaining 9 in between points are set randomly. Similarly, 12^{th} gene is set as first point of y co-ordinate and so on. In chromosome representation values of F_x and F_y are also represented based on coding rules. Forces in x and y direction are generated by randomly choosing values between starting force and ending force while total time is randomly generated by choosing value between 1 sec and maximum time required, where maximum time is considered as 50 sec [13]. Then I_j can be expressed as

$$I_j = [X_s, X_1, ..., X_9, X_e, Y_s, Y_1, ..., Y_9, Y_e, \theta_s, \theta_1, ...,$$
$$\theta_9, \theta_e, F_{x_s}, F_{x_1}, ..., F_{x_9}, F_{x_e}, F_{y_s}, F_{y_1}, ..., F_{y_9}, F_{y_e}, T] \quad (38)$$

Non-Dominated Sort. The initialized population is sorted based on non- domination. The fast sort algorithm is used for sorting the population [2]. This algorithm is better than the original NSGA since it works on the information about the set that an individual dominate (S_p) and number of individuals that dominate the individual (n_p) [14].

Crowding Distance. Once the non-dominated sort is complete the crowding distance is assigned. Since the individuals are selected based on rank and crowding distance all the individuals in the population are assigned a crowding distance value. Crowding distance is assigned front wise and comparing the crowding distance between two individuals in different front is meaning less. The basic idea behind the crowing distance is finding the euclidean distance between each individual in a front based on their objectives. The individuals in the boundary are always selected since they have infinite distance assignment.

5.2 Fitness Function

Fitness function is an important factor to convergence and stability of a NSGA-II. In 2D SpiderCrane problem our objective is to minimize forces, time, swing of load (θ) and generate smooth path.
The fitness function is defined as:

$$F_1 = \sum_{k=0}^{N} f_k^2, \quad F_2 = \sum_{k=0}^{N} \theta_k^2, \quad (39)$$

F_3 = Total time required for operation = T

subjected to constraint equations (29)-(37).

5.3 NSGA-II Operator Design

In this paper, dominance-based selection scheme is used to incorporate constraints into the fitness function. The approach does not require the use of penalty function to generate feasible solution. Tournament selection is used. While comparing first preference is given to feasible candidates irrespective of their front and crowding distance.
Crossover operator used is two-point crossover. Crossover is performed with probability of 0.85. Two individuals selected by Tournament selection are further used as parent individual for crossover. Two points are chosen randomly, then two new individuals of next generation are obtained by changing the parts of the parent individuals between the two points. Crossover operator can find some good individuals from a global view. However, the searching space can't be searched in details by using crossover operator only. If mutation operator is used to adjust some genes of each individual, the optimal solution is approximated from the local view. In addition, mutation operator can maintain the diversity of population and avoid premature phenomenon effectively. In mutation operator, which can improve the local search ability, the points chosen in individual are set randomly. Mutation is performed with probability of 0.2.

5.4 Recombination

The offspring population is combined with the current generation population and selection is performed to set the individuals of the next generation. Since all the previous and current best individuals are added in the population, elitism is ensured. Population is now sorted based on non-domination. The new generation is filled by each front subsequently until the population size exceeds the current population size. If by adding all the individuals in front F_j the population exceeds N then individuals in front F_j are selected based on their crowding distance in the descending order until the population size is N. And hence the process repeats to generate the subsequent generations.

5.5 Implementation in MATLAB

To verify the correctness and validity of method, the simulation is carried out in MATLAB® with the following system parameters: *Ring mass* M = 0.5 kg, *Payload mass* m = 1 kg, *Time Steps* = 10 i.e 11 discrete points, *Length of the cable* L = 0.5 m, *Initial point* = (0.7, 0.7), *Final point* = (0.5, 1), *Max time required* = 50 seconds, *Initial* θ = 0.1745 rad, *Final* θ = 0 rad.
In this paper, NSGA-II is used as an alternative to local solver used for finding optimal solution in DMOC. Parameters for application of NSGA-II are shown in Table 1. The terminal condition is that the population has evolved to 2000th generation. The aim of the NSGA-II was to minimize (i) forces required for positioning payload, (ii) operational time and (iii) swing of payload. Fig 4, Fig 5 and Fig 6 shows the simulation result using NSGA-II and the results obtained indicates that our approach is a viable alternative. The NSGA-II algorithm was able to find minimum time without drastic increment in the forces required for positioning. The result obtained for minimum time and for maximum time required is presented in Table 2. For comparison

Table 1. Parameters for NSGA-II

Population	Generations	Pool Size	Tour Size
200	2000	10	2

Table 2. Results Obtained By NSGA-II

	Time	Theta	Force in X direction	Force in Y direction	No. of constraints violated
NSGA-II	4.245	0.0546	0.737	13.482	0
NSGA-II	50	0.0526	0.5125	11.8101	0
SQP	50	0.0526	0.5131	14.4398	2

purpose result obtained by SQP solver is given. SQP solver finds local optimal solution violating 2 strict constraints.

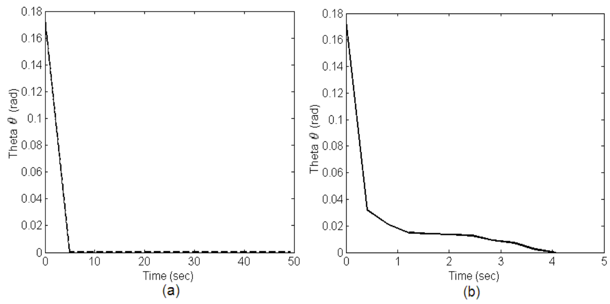

Fig. 4. (a) Load angle as function of time for maximum time (b) Load angle as function of time for minimum time

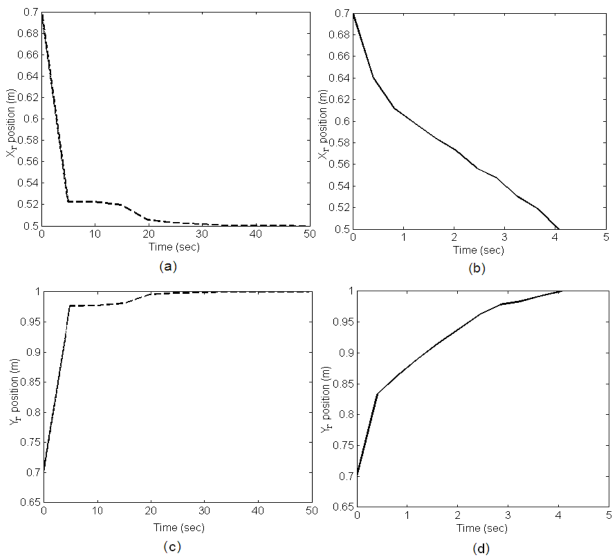

Fig. 5. (a) The X_r position as a function of time for maximum time (b) The X_r position as a function of time for minimum time (c) The Y_r position as a function of time for maximum time (d) The Y_r position as a function of time for minimum time

6. CONCLUSION

This paper proposes a new approach to solve optimal control problems for mechanical systems. Discrete mechanics provides constraint equations and objective function while preserving the sympletic structure and the momentum maps corresponding to symmetry groups for the discrete solution. SQP (non-linear optimization) method used to find local optimum to these equations performs single objective optimization using initial guess. SQP method is applicable to smooth equations. This method can be replaced by an efficient global multi-objective optimization method, NSGA-II. The proposed approach is validated by finding optimal operating conditions of 2D SpiderCrane, in carrying a load over a distance. Using the simulation results obtained by a single-objective SQP solver and multi-objective NSGA-II, we have shown that NSGA-II is a viable alternative to SQP solver.

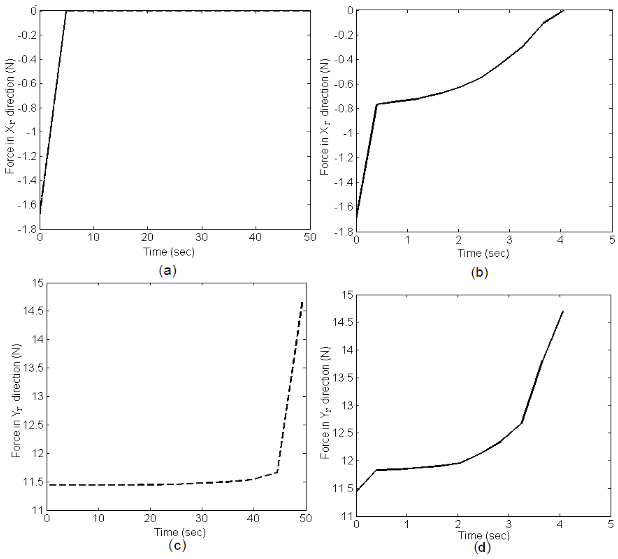

Fig. 6. (a) Control force in X_r direction for maximum time (b) Control force in X_r direction for minimum time (c) Control force in Y_r direction for maximum time (d) Control force in Y_r direction for minimum time

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