Soft Set Theory in Medical Diagnosis using Trapezoidal Fuzzy Number

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ABSTRACT
Soft set theory is a newly emerging mathematical tool to deal with uncertain problems. The Parameterization tools of soft set theory enhance the flexibility of its applications to different problems. In this paper, we apply fuzzy soft set technology through the well known Sanchez's approach for Medical Diagnosis using arithmetic operations on Trapezoidal fuzzy numbers and exhibit the technique with a hypothetical case study.

Keywords
Soft set, fuzzy soft set, fuzzy number, Trapezoidal fuzzy number, defuzzificaztion

1. INTRODUCTION
In real life situation, most of the problems in economics, social science, environment etc., have various uncertainties. However most of the existing mathematical tools for formal modeling, reasoning and computing are crisp deterministic and precise in character. There are theories viz., theory of probability, evidence, fuzzy set, intuitionistic fuzzy set, vague set, interval mathematics, rough set for dealing with uncertainties. These Theories have their own difficulties as pointed out by Molodtsov[1].In 1999, Molodtsov[1] initiated a novel concept of soft set theory, which is completely new approach for modeling vagueness and uncertainties. Soft set theory has a rich potential for application in solving practical problems in economics, social science, medical science etc. Later on Maji et al [2] have studied the theory of fuzzy soft set. In this paper, we study Sanchez's[3] method for medical diagnosis using the notion of fuzzy soft set together with arithmetic operations on trapezoidal fuzzy number and exhibit the technique with a case study.

2. PRELIMINARIES
2.1 Soft set [1]
Suppose that U is an initial universe set and E is a set of parameters, let P(U) denotes the power set of U.A pair(F,E) is called a soft set over U where F is a mapping given by

F: E→P(U) .Clearly, a soft set is a mapping from parameters to P(U),and it is not a set, but a parameterized family of subsets of the Universe.

Example 2.1.Suppose that U={s_1,s_2,s_3,s_4} is a set of students and E={e_1,e_2,e_3} is a set of parameters, which stand for result, conduct and sports performances respectively. Consider the mapping from parameters set E to the set of all subsets of power set U.Then soft set (F,E) describes the character of the students with respect to the given parameters, for finding the best student of an academic year.Then (F,E) = {{result = s_1,s_3,s_4 }, {conduct = s_1,s_2 },{sports performances = s_2,s_3,s_4 }}.We can represent a soft set in the form of Table1

Table1.The Tabular representation of soft set

U	Result(e_1)	Conduct(e_2)	Sports(e_3)
s_1	1	1	0
s_2	0	1	1
s_3	1	0	1
s_4	1	0	1

2.2 Fuzzy soft set [2]
Let U be an initial Universe set and E be the set of parameters. Let $A \subset E$. A pair (F,A) is called fuzzy soft set over U where F is a mapping given by F: A→I^U,where I^U denotes the collection of all fuzzy subsets of U.

Example 2.2. Consider the example2.1,in soft set(F,E),if s_1 is medium in studies, we cannot expressed with only the two numbers 0 and 1,we can characterize it by a membership function instead of the crisp number 0 and 1,which associates with each element a real number in the interval [0,1].

Then fuzzy soft set can describe as

(F,A)={F(e_1) = {(s_1,0.9), (s_2,0.3), (s_3,0.8), (s_4,0.9)}, F(e_2) = {(s_1,0.8), (s_2,0.9), (s_3,0.4), (s_4,0.3)}} where A={ e_1,e_2}

We can represent a fuzzy soft set in the form of Table2.

Table2.The Tabular representation of fuzzy soft set

U	Result(e_1)	Conduct(e_2)
s_1	0.9	0.8
s_2	0.3	0.9
s_3	0.8	0.4
s_4	0.9	0.3

2.3 Fuzzy soft super set[2]
For two fuzzy soft sets (F,A) and (G,B) over a common universe U, we have (F,A) $\subseteq$ (G,B) if A $\subset$ B and $\forall$ e ∈ A, G(e) is a fuzzy superset of F(e).

i.e., (G,B) is a fuzzy soft super set of (F,A).

2.4 Fuzzy soft sub set[2]
For two fuzzy soft sets (F,A) and (G,B) over a common universe U, we have (F,A) $\subseteq$ (G,B) if A $\subset$ B and $\forall$ e ∈ A, F(e) is a fuzzy subset of G(e).

i.e., (F,A) is a fuzzy soft sub set of (G,B).

2.5 Fuzzy soft complement set[2]

The complement of fuzzy soft set (F,A) denoted by $(F,A)^c$ is defined by $(F,A)^c = (F^C, \sim A)$, where $F^c: \sim A \to I^U$ is a mapping given by $F^c(e)$ = fuzzy complement of $F(\sim e)$

$$= [F(e)]^c \quad \forall e \in A.$$

2.6 Fuzzy soft Absolute set[2]

A fuzzy soft set (F,A) over U is said to be absolute fuzzy soft set with respect to the parameter set A, denoted by $\tilde{A}$

if F(e)=U, $\forall e \in A$.

2.7 Fuzzy soft Null set[2]

A fuzzy soft set (F,A) over U is said to be null fuzzy soft set with respect to the parameter set A, denoted by $\tilde{\Phi}$

if F(e)=Φ, $\forall e \in A$.

2.8 Fuzzy soft AND operation and OR operation[2]

For two fuzzy soft sets (F,A) and (G,B) over a common universe U, we have

i) (AND ^) operation as (F,A)^(G,B)=(H,AxB) where

$$H(\alpha,\beta)=F(\alpha) \cap G(\beta), \ \forall \alpha \in A, \ \forall \beta \in B.$$

ii) (OR $\vee$) operation as (F,A) $\vee$ (G,B)=(H,AxB) where

$$H(\alpha,\beta)=F(\alpha) \cup G(\beta), \ \forall \alpha \in A, \ \forall \beta \in B.$$

Example 2.3 Let U={ h_1,h_2,h_3} be the set of three houses and E ={ cheap(e_1),beautiful(e_2),big(e_3),good location(e_4)} be the set of parameters. Consider two fuzzy soft sets (F,A) and (G,B) where A={ e1,e2} and B={e_1,e_2,e_3} $\subset$ E given by

(F,A)={ $F(e_1)$ ={(h_1,0.5),(h_2,0.2),(h_3,0.8)},

$F(e_2)$ ={(h_1,0.9),(h_2,0.8),(h_3,0.2)} }

(G,B) ={ $G(e_1)$ ={(h_1,0.5),(h_2,0.2),(h_3,0.8)},

$G(e_2)$ ={(h_1,0.9),(h_2,0.8),(h_3,0.2)},

$G(e_3)$ ={(h_1,0.6),(h_2,0.7),(h_3,0.5)}}

Then i) $(F,A)^c$ ={ $F(\sim e_1)$ ={(h_1,0.5),(h_2,0.8),(h_3,0.2)}

$F(\sim e_2)$ ={(h_1,0.1),(h_2,0.2),(h_3,0.8)} }

ii)(F,A) $\subseteq$ (G,B) .

3.ARITHMETIC OPERATIONS ON TRAPEZOIDAL FUZZY NUMBERS

3.1 Convex fuzzy set

A fuzzy set A on the Universe of discourse R (the set of real numbers)is convex iff for a_1,a_2 in U

$$f_A(\lambda a_1 + (1-\lambda) a_2) \geq \min(f_A(a_1), f_A(a_2)) \quad \text{where } \lambda \in [0,1].$$

3.2 Normal fuzzy set

A fuzzy set A on the Universe of discourse U is called a normal fuzzy set if $\exists a_i \in U \ni f_A(a_i)=1$

3.3 Fuzzy number

A fuzzy number is a fuzzy set defined on the universe of discourse R which is both convex and normal.

3.4 Trapezoidal fuzzy number

A trapezoidal fuzzy number $\tilde{n}$ can be defined as (n_1,n_2,n_3,n_4) shown in Figure1 which has the membership function as follows

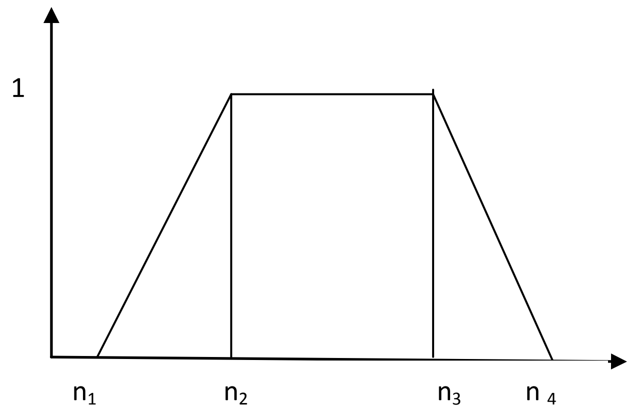

Fig 1:Trapezoidal fuzzy number $\tilde{n}$

$$\mu_{\tilde{n}}(x) = \begin{cases} 0 & x < n_1 \\ \dfrac{x-n_1}{n_2-n_1} & n_1 \leq x \leq n_2 \\ 1 & n_2 \leq x \leq n_3 \\ \dfrac{x-n_4}{n_3-n_4} & n_3 \leq x \leq n_4 \\ 0 & x > n_4 \end{cases}$$

Let $\tilde{m}=(m_1,m_2,m_3,m_4)$ and $\tilde{n}=(n_1,n_2,n_3,n_4)$ be two trapezoidal fuzzy numbers, then we have the following operations

i) $\tilde{m} \leq \tilde{n}$ iff $m_1 \leq n_1$, $m_2 \leq n_2$,$m_3 \leq n_3$ and $m_4 \leq n_4$

ii) The addition of $\tilde{m}$ and $\tilde{n}$ can be defined as

$\tilde{m} \oplus \tilde{n}$ =($m_1+n_1, m_2+n_2, m_3+n_3, m_4+n_4$)

iii) The multiplication of $\tilde{m}$ and $\tilde{n}$ can be defined as $\tilde{m} \otimes \tilde{n}$ =($m_1 x n_1, m_2 x n_2, m_3 x n_3, m_4 x n_4$)

iv) The union of $\tilde{m}$ and $\tilde{n}$ can be defined as

$\tilde{m} \cup \tilde{n}$ =(max(m_1, n_1),max(m_2 ,n_2),max(m_3, n_3),max(m_4 n_4))

v) The intersection of $\tilde{m}$ and $\tilde{n}$ can be defined as $\tilde{m} \cap \tilde{n}$ =(min(m_1, n_1),min(m_2 ,n_2),min(m_3, n_3), min(m_4 , n_4))

vi) The difference between $\tilde{m}$ and $\tilde{n}$ can be defined as

$$d_v(\tilde{m},\tilde{n}) = \sqrt{\frac{1}{4}\left(m_1 - n_1\right)^2 + \left(m_2 - n_2\right)^2 + \left(m_3 - n_3\right)^2 + \left(m_4 - n_4\right)^2}$$

vii)The complement of $\tilde{n}$ can be defined as

$\tilde{n}^c$ =(1- n_4,1-n_3,1- n_2,1- n_1)

4. FUZZY SOFT SETS IN MEDICAL DIAGNOSIS USING TRAPEZOIDAL FUZZY NUMBERS

Let us assume that there is a set of m patients

P ={$p_1,p_2,p_3,\ldots\ldots p_m$} with a set of n symptoms

S={$s_1,s_2,s_3,s_4\ldots\ldots s_n$} related to a set of k diseases D={$d_1,d_2,d_3,\ldots d_k$}.We apply fuzzy soft set technology to diagnose which patient is suffering from what disease. we construct a fuzzy soft set (F,P) over S where F is a mapping F:P→I^s.This fuzzy soft set gives a relation matrix R_1 called patient symptom matrix, where the entries are trapezoidal fuzzy numbers $\tilde{m}$ parameterized by a quadruplet(m-2,m-1,m+1,m+2).Then construct another fuzzy soft set (G,S) over D where G is a mapping G:S→I^D.This fuzzy soft set gives a relation matrix R_2 called symptom disease matrix, where each element denote the weight of the symptoms for a certain disease. These elements are also taken as trapezoidal fuzzy numbers. Thus the general form of R_1,R_2 are

$$R_1 = \begin{array}{c} \\ p_1 \\ p_2 \\ \vdots \\ \vdots \\ p_m \end{array} \begin{array}{c} \begin{array}{cccc} s_1 & s_2 & \ldots\ldots\ldots & s_n \end{array} \\ \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \ldots\ldots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \ldots\ldots\ldots & \tilde{a}_{23} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & \ldots\ldots\ldots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \ldots\ldots\ldots & \tilde{a}_{mn} \end{bmatrix} \end{array}$$

$$R_2 = \begin{array}{c} \\ s_1 \\ s_2 \\ \vdots \\ \vdots \\ s_n \end{array} \begin{array}{c} \begin{array}{cccc} d_1 & d_2 & \ldots\ldots\ldots & d_k \end{array} \\ \begin{bmatrix} \tilde{b}_{11} & \tilde{b}_{12} & \ldots\ldots & \tilde{b}_{1k} \\ \tilde{b}_{21} & \tilde{b}_{22} & \ldots\ldots\ldots & \tilde{b}_{2k} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & \ldots\ldots\ldots & \vdots \\ \tilde{b}_{n1} & \tilde{b}_{n2} & \ldots\ldots\ldots & \tilde{b}_{nk} \end{bmatrix} \end{array}$$

Now performing the transformation operation $R_1 \otimes R_2$ we get the patient diagnosis matrix D_1 as follows

$$D_1 = \begin{array}{c} \\ p_1 \\ p_2 \\ \vdots \\ \vdots \\ p_m \end{array} \begin{array}{c} \begin{array}{cccc} d_1 & d_2 & \ldots\ldots\ldots & d_k \end{array} \\ \begin{bmatrix} \tilde{c}_{11} & \tilde{c}_{12} & \ldots\ldots & \tilde{c}_{1k} \\ \tilde{c}_{21} & \tilde{c}_{22} & \ldots\ldots\ldots & \tilde{c}_{2k} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & \ldots\ldots\ldots & \vdots \\ \tilde{c}_{m1} & \tilde{c}_{m2} & \ldots\ldots\ldots & \tilde{c}_{mk} \end{bmatrix} \end{array} \quad \text{where } c_{il} = \sum_{j=1}^{n} a_{ij} b_{jl} \quad \text{i=1 to m, l=1to k}$$

Then defuzzifying each element of the above matrix by t=(m_1+m_2+m_3+m_4)/4. we get the crisp diagnosis matrix as

$$D_2 = \begin{array}{c} \\ p_1 \\ p_2 \\ \vdots \\ \vdots \\ p_m \end{array} \begin{array}{c} \begin{array}{cccc} d_1 & d_2 & \ldots\ldots\ldots & d_k \end{array} \\ \begin{bmatrix} v_{11} & v_{12} & \ldots\ldots & v_{1k} \\ v_{21} & v_{22} & \ldots\ldots\ldots & v_{2k} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & \ldots\ldots\ldots & \vdots \\ v_{m1} & v_{m2} & \ldots\ldots\ldots & v_{mk} \end{bmatrix} \end{array}$$

Now if $\max_{1 \le l \le k} v_{il} = v_{is}$ then we conclude that the patient P_i is suffering from disease d_s.

Incase $\max_{1 \le l \le k} v_{il}$ occurs for more than one value reassess the symptoms to break the tie.

5. ALGORITHM

Step1:Input the fuzzy soft set (F,P) to obtain the patient - disease matrix R_1.

Step2: Input the fuzzy soft set (G,S) to obtain the symptom-disease matrix R_2.

Step3:Perform the transformation operation $R_1 \otimes R_2$ to get the patient diagnosis matrix D_1.

Step4:Defuzzify all the elements of the matrix D_1 to obtain the matrix D_2.

Step5:Find s for which v_{is}=max v_{il}.

Then we conclude that the patient p_i is suffering from disease d_s. In case max v_{il} occurs for more than one value of l, then we can reassess the symptoms to break the tie.

6. CASE STUDY

Suppose there are three patients Ramu, Mary, Somu in a hospital with symptoms temperature, headache, cough and stomach problem. Let the possible diseases relating to the above symptoms be viral fever, typhoid and malaria. Now take P={p_1,p_2,p_3}as the universal set where p_1,p_2 and p_3 represents patients Ramu, Mary, Somu respectively.

Next consider the set S={s_1,s_2,s_3,s_4} as universal set where s_1,s_2,s_3,s_4 represents symptoms temperature, headache, cough and stomach problem respectively and the set D={d_1,d_2,d_3} where d_1,d_2 and d_3 represent the diseases viral fever, typhoid and malaria respectively.

Suppose $F(s_1)=\{p_1/\tilde{6}, p_2/\tilde{8}, p_3/\tilde{4}\}$

$$F(s_2)=\{p_1/\tilde{8}, p_2/\tilde{6}, p_3/\tilde{5}\}$$
$$F(s_3)=\{p_1/\tilde{6}, p_2/\tilde{7}, p_3/\tilde{7}\}$$
$$F(s_4)=\{p_1/\tilde{4}, p_2/\tilde{8}, p_3/\tilde{7}\}$$

Then the fuzzy soft set (F,S) is a parameterized family of all fuzzy set over S and gives a collection of approximate description of the patient-symptoms in the hospital. This fuzzy soft set (F,P) represents the patient-symptom relation matrix R_1 and is given by

$$R_1 = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{c} \begin{matrix} s_1 & s_2 & s_3 & s_4 \end{matrix} \\ \begin{pmatrix} \tilde{6} & \tilde{8} & \tilde{6} & \tilde{4} \\ \tilde{8} & \tilde{6} & \tilde{7} & \tilde{8} \\ \tilde{4} & \tilde{5} & \tilde{7} & \tilde{7} \end{pmatrix} \end{array}$$

Suppose that $G(s_1)=\{d_1/\tilde{9}, d_2/\tilde{8}, d_3/\tilde{3}\}$

$$G(s_2)=\{d_1/\tilde{8}, p_2/\tilde{5}, p_3/\tilde{6}\}$$
$$G(s_3)=\{d_1/\tilde{5}, d_2/\tilde{7}, d_3/\tilde{4}\}$$
$$G(s_4)=\{d_1/\tilde{4}, d_2/\tilde{7}, d_3/\tilde{8}\}$$

Then the fuzzy soft set (G,S) is a parameterized family of all fuzzy sets over the set S where G: S→I^D and is determined from expert medical documentation. Thus the fuzzy soft set (G,S) gives an approximate description of the three diseases and their symptoms. This soft set is represented by a symptom-disease relation matrix R_2 and is given by

$$R_2 = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{array} \begin{array}{c} \begin{matrix} d_1 & d_2 & d_3 \end{matrix} \\ \begin{pmatrix} \tilde{9} & \tilde{8} & \tilde{3} \\ \tilde{8} & \tilde{5} & \tilde{6} \\ \tilde{5} & \tilde{7} & \tilde{4} \\ \tilde{4} & \tilde{7} & \tilde{8} \end{pmatrix} \end{array}$$

Then performing the transformation operation $R_1 \otimes R_2$ we get the patient diagnosis matrix D_1 as

$$D_1 = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{c} \begin{matrix} d_1 & d_2 & d_3 \end{matrix} \\ \begin{pmatrix} \tilde{218} & \tilde{213} & \tilde{170} \\ \tilde{246} & \tilde{259} & \tilde{206} \\ \tilde{192} & \tilde{209} & \tilde{174} \end{pmatrix} \end{array}$$

Where

$$\tilde{218}=(84,118,218,280), \tilde{259}=(103,147,259,341),$$
$$\tilde{174}=(54,86,174,230)$$

Now defuzzifying the above matrix, we get

$$D_2 = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{c} \begin{matrix} d_1 & d_2 & d_3 \end{matrix} \\ \begin{pmatrix} 175 & 167 & 131.75 \\ 202 & 212.5 & 166 \\ 144 & 165 & 136 \end{pmatrix} \end{array}$$

It is clear from the above matrix that patient Ramu(p_1) is suffering from disease viral fever(d_1) and Mary(p_2) and Somu(p_3) are suffering from typhoid(d_2).

7. CONCLUSION

In this paper, we have applied the fuzzy arithmetic operations on trapezoidal fuzzy number to diagnose the disease in fuzzy soft set theory. A case study has been taken to exhibit the simplicity of the technique. As far as future direction are concerned it is hoped that our findings will help enhancing this study on fuzzy soft sets for the researchers.

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