

Domino Recognizability of Triangular Picture Languages

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ABSTRACT

The notion of local iso-triangular picture languages and recognizable iso-triangular picture languages are introduced. Domino recognizability of iso-triangular picture languages and HRL-domino systems are defined. Also the concept that recognizable iso-triangular picture languages are characterized as projections of hrl-local triangular picture languages is derived. Theorems are proved.

Keywords

Iso triangular domino system, overlapping of iso triangular pictures, hrl domino systems

1. INTRODUCTION

A generalization of formal languages to two dimensions is possible to different ways and several formal models to recognize or generate two dimensional objects have been proposed in the literature. These approaches were initially motivated by problems arising in the framework of pattern recognition and image processing [3], but two dimensional patterns are also appear in studied concerning cellular automata and other models of parallel computing [5].

Already a notion of recognizability of a set of pictures in terms of tiling systems is introduced [4]. The underlying idea is to define recognizability by projection of local properties. Informally recognition in a tiling system is defined in terms of a finite set of square pictures of side two which correspond somehow to automaton transitions and are called ‘tiles’. In a picture to be recognized (over the alphabet Σ) each quadruple of positions form a square to be covered by a tile (with symbols say in the alphabet Γ) such that a coherent assignment of picture positions to labels in Γ is built up, and such that a projection from Γ to Σ reestablishes the considered picture. Then the tiles can be viewed as local ‘automaton transitions’ and tiling a given picture means to construct a run of the automaton on it [2].

The local languages are languages given by a finite set of authorized tiles of size (2, 2). The use of blocks of size (2, 2) implies that in a computational procedure to recognize a given picture, the horizontal and vertical controls are done at the same time. Then it is natural to ask what happens when the two scanning are done separately and in particular what this can imply when apply the projections afterwards.

In [1] the so called hv-local picture languages are defined where the square tiles of side 2 are replaced by “dominoes” that correspond to two kinds of tiles: horizontal dominoes of size (1, 2) and vertical dominoes of size (2, 1).

In this paper the notion of domino systems to recognize iso-triangular picture languages.

2. PRELIMINARIES

In this section some definitions of tiling systems are recollected [6]. Let Σ be a finite alphabet of symbols. A

picture P over Σ is a rectangular array of symbols over Σ . The set of all pictures over Σ is denoted by Σ^{**} . Given $p \in \Sigma^{**}$, $\ell_1(p)$ and $\ell_2(p)$ denote the number of rows and columns respectively of p . The pair $(\ell_1(p), \ell_2(p))$ is the size of p ; $p(i, j)$ denotes the symbol at row i and column j , $1 \leq i \leq \ell_1(p)$ and $1 \leq j \leq \ell_2(p)$. A picture language L is a subset of Σ^{**} . Let p be a picture of size (m, n) . Let \hat{p} be the picture of size $(m+2, n+2)$ obtained by bordering p with a special symbol $\# \notin \Sigma$. $B_{h,k}(p)$ denotes the set of all subpictures of p of size (h, k) . A tile is a picture of size (2, 2). $\Sigma^{m \times n}$ denotes the set of all pictures of size (m, n) over the alphabet Σ . Here some basic concepts of iso-triangular picture languages are given.

Definition 2.1

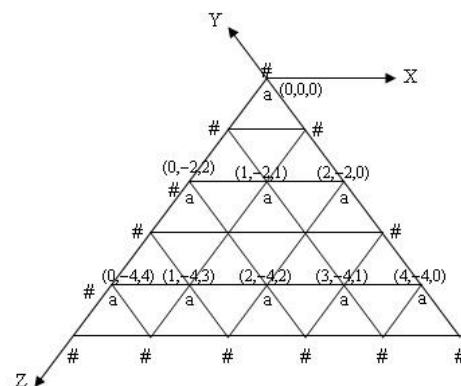
An iso-triangular picture p over the alphabet Σ is an isosceles triangular arrangement of symbols over Σ . The set of all iso-triangular pictures over the alphabet Σ is denoted by Σ_T^{**} . An iso-triangular picture language over Σ is a subset of Σ_T^{**} .

Given an iso-triangular picture p the number of rows (counting from the bottom to top) denoted by $r(p)$ is the size of an iso-triangular picture. The empty picture is denoted by Λ .

Iso-triangular pictures can be classified into four categories.

1. Upper iso-triangular picture
2. Lower iso-triangular picture
3. Right iso-triangular picture
4. Left iso-triangular picture

The upper triangular iso-picture P can be represented in the co-ordinate system as follows:



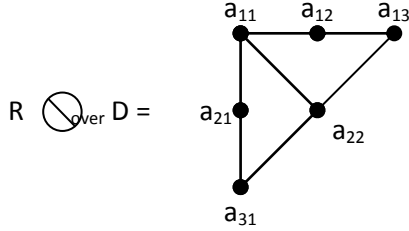
A lower triangular iso-picture P can be represented in the co-ordinate system as follows:

Definition 3.4 Left Overlapping

The left overlapping is defined between any two glueable iso-triangular pictures of same size and it is denoted by the symbol $\overline{\circ}$. This overlapping includes the following.

- (a) $U \overline{\circ} R$ (b) $U \overline{\circ} L$
(c) $L \overline{\circ} R$ (d) $R \overline{\circ} D$.

Example 3.4

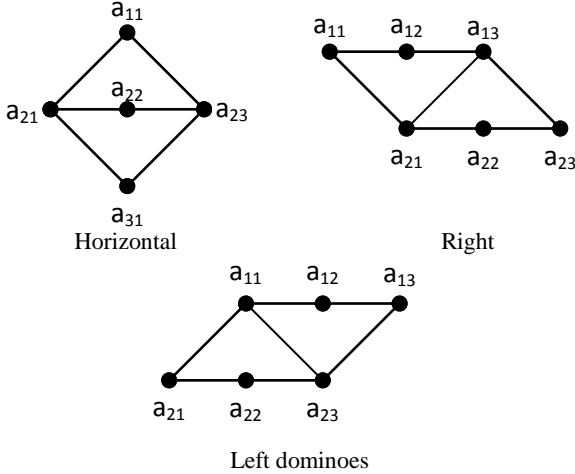


Definition 3.5

The set of all pictures obtained by overlapping an iso-triangular pictures of same size is denoted by $O(p)$.

Here dominoes of the following types are considered.

- (i) Horizontal dominoes (ii) Right and left dominoes.



Definition 3.6

Let L be an iso-triangular picture language. The language L is hrℓ-local if there exists a set Δ of dominoes over the alphabet $\Sigma \cup \{\#\}$ such that

$$L = \{p \in \Sigma_T^{**} / O(B_2(\hat{p})) \subseteq \Delta\}.$$

In this case we write $L = L(\Delta)$.

Definition 3.7

An iso-triangular domino system (ITDS) is a 4-tuple $(\Sigma, \Gamma, \Delta, \pi)$ where Σ and Γ are two finite alphabets, Δ is a finite set of dominoes over the alphabet $\Gamma \cup \{\#\}$ such that $\pi : \Gamma \rightarrow \Sigma$ is a projection.

The iso-triangular domino system recognized by an iso-triangular picture language L over the alphabet Σ and is defined as $L = \pi(L')$ where $L' = L(\Delta)$ is the hrℓ-local iso-triangular picture language over Γ . The family of iso-triangular picture languages recognized by iso-triangular domino system is denoted by $\mathcal{L}(\text{ITDS})$.

Proposition 3.1

If $L \subseteq \Gamma_T^{**}$ is a hrℓ-local iso triangular picture language then L is local iso-triangular picture language. That is $\mathcal{L}(\text{ITDS}) \subseteq \mathcal{L}(\text{ITTS})$.

Proof

Let $L \subseteq \Gamma_T^{**}$ be a hrℓ-local iso triangular picture language. Then $L = L(\Delta)$ where Δ is a finite set of dominoes. Here construct a finite set Θ of iso-triangular tiles of size 2 and show that $L = L(\Theta)$.

Define Θ as follows

$$\Theta = \left\{ \theta \in \Gamma \cup \{\#\} / \theta \neq \begin{matrix} \# & \# & \# \\ \# & \# & \# \end{matrix}, O(\theta) \subseteq \Delta \right\}$$

Where the symbol 'O' denotes overlapping.

Let $L' = L(\Theta)$. Now show that $L' = L$.

Let $p \in L'$ then by definition $B_2(\hat{p}) \in \Theta$. This implies that

$$O(B_2(\hat{p})) \subseteq O(\Theta) \subseteq \Delta.$$

Hence $p \in L$.

Conversely let $p \in L$ and $q \in B_2(\hat{p})$. Then

$$O(q) \subseteq O(B_2(\hat{p})) \subseteq \Delta. \text{ Therefore } q \in \Theta \text{ and } p \in L'.$$

Hence $L = L'$.

Remark 3.1

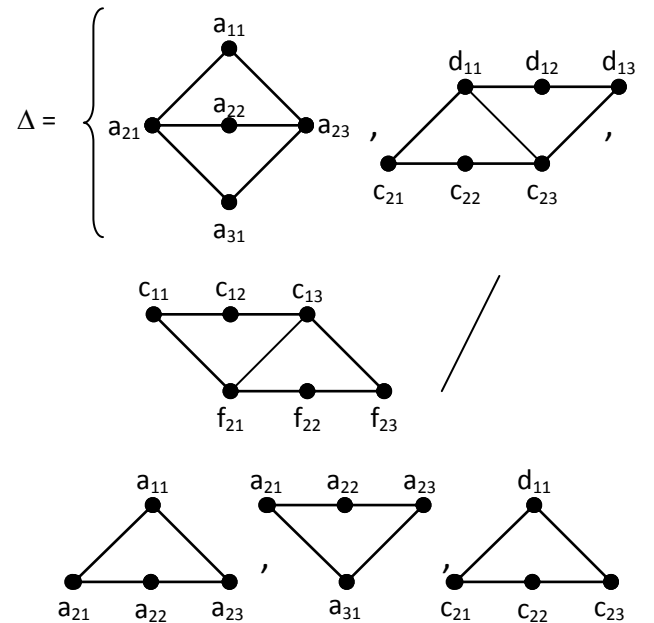
The converse of the Proposition 1.1 is not true. That is there are languages that are in ITLOC but not in hrℓ-local.

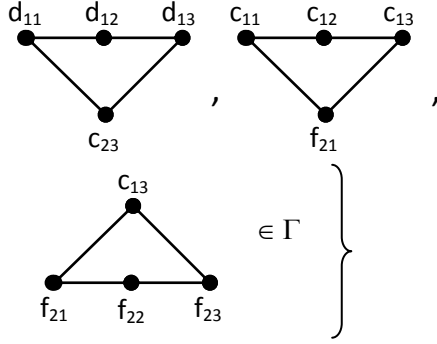
Lemma 3.1

Let L be a local iso-triangular picture language over an alphabet Σ . Then there exists an HRL-local language L' over the alphabet Γ and a mapping $\pi : \Gamma \rightarrow \Sigma$ such that $L = \pi(L')$.

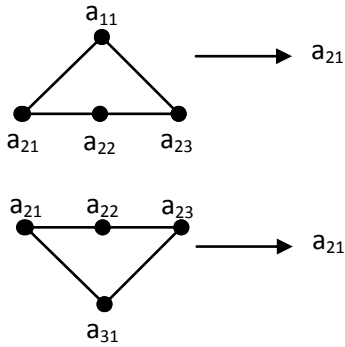
Proof

Let $L = L(\theta)$ where θ is a finite set of iso-triangular tiles of size 2 over $\Sigma \cup \{\#\}$. By definition θ contains all allowed subpictures of size 2 of pictures in L . The idea of the proof is to show that the property of being an allowed subpicture of size 2 of a picture in L by means of dominoes over an larger alphabet Γ can be expressed. This is accomplished by choosing Γ as the set θ itself and defining the set Δ of dominoes. Let $\Gamma = \theta \subseteq (\Sigma \cup \{\#\})^2$ and let





Let $L' = L(\Delta)$. Then define a mapping π between the two alphabets Γ and Σ such that



To complete the proof, first $\pi(L') = L$ to be proved. Before proving it formally, give an example to clarify how a picture $p \in L$ and a picture $p' \in L'$ such that $\pi(p') = p$ are related.

Suppose

$$\hat{p} = \begin{array}{ccccccc} & & \# & & & & \\ & & \# & P_{11} & \# & & \\ \# & & P_{21} & P_{22} & P_{23} & \# & \\ \# & \# & \# & \# & \# & \# & \# \end{array}$$

Then the corresponding picture \hat{p}' will be the following

$$\hat{p}' = \begin{array}{ccccccccccccccc} & & & & \# & & & & & & & & & \\ & & & & \# & \# & \# & & & & & & & \\ & & & \# & P_{11} & \# & \# & \# & & & & & & \\ \# & & P_{21} & P_{22} & P_{11} & P_{22} & P_{23} & \# & \# & \# & & & & \\ \# & P_{21} & P_{21} & P_{21} & P_{22} & P_{23} & P_{23} & P_{23} & \# & \# & \# & & & \\ \# & \# & \# & \# & \# & \# & \# & \# & \# & \# & \# & \# & \# & \# \\ \# & \# & \# & \# & \# & \# & \# & \# & \# & \# & \# & \# & \# & \# \end{array}$$

In the definition of \hat{p}' several different border symbols are used. More precisely the border symbols for Γ are all iso-triangular tiles of size 2 containing $\begin{array}{ccc} \# & \# & \# \\ \# & \# & \# \end{array}$.

Now $L = \pi(L')$ will be proved. Let $p \in L$ be of size m . Consider a picture \hat{p} over Γ as follows.

$$P'(i,j) = \begin{array}{ccc} \# & & \\ \# & \# & \# \\ P_{ij} & \# & \# \end{array}, i = 1, 2, 3, \dots, m, j = 2i-1$$

$$P'(i,j) = \begin{array}{ccc} P_{1i-1} & & \\ P_{ij} & P_{ij+1} & P_{ij+2} \end{array}, i = 2, 3, \dots, m, j = 1, 3, 5, \dots, 2i-3$$

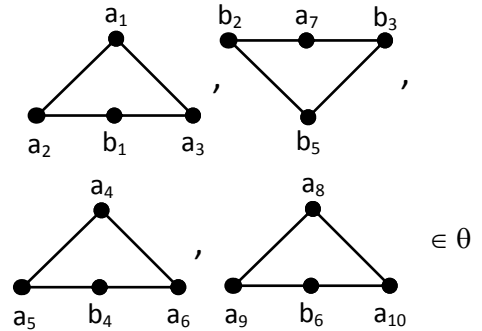
$$P'(i,j) = \begin{array}{ccc} P_{ij} & P_{ij+1} & \# \\ & P_{ij+2} & \end{array}, i = 2, 3, \dots, m-1, j = 2i-2$$

$$P'(i,j) = \begin{array}{ccc} P_{ij} & P_{ij+1} & P_{ij+2} \\ & P_{i+1j+2} & \end{array}, i = 3, \dots, m-1, j = 2, 4, \dots, 2i-4$$

It is easy to verify that $p' \in L'$ and $\pi(p') = p$. Conversely let $p' \in L'$ and let $q \in B_2(\hat{p}')$ be a subpicture of \hat{p}' of size 2. To prove that $\pi(p') \in L$. It suffices to show that $\pi(q) \in \theta$. Suppose the iso-triangular picture q is the following

$$q = \begin{array}{cccc} & a_1 & & \\ & a_2 & b_1 & a_3 \\ a_4 & b_2 & a_7 & b_3 & a_8 \\ a_5 & b_4 & a_6 & b_5 & a_9 & b_6 & a_{10} \end{array}$$

where all the tiles $a_1, a_2, \dots, a_{10}, b_1, b_2, \dots, b_5 \in \Sigma \cup \left\{ \begin{array}{ccc} \# & \# & \# \\ \# & \# & \# \end{array}, \begin{array}{ccc} \# & \# & \# \end{array} \right\}$ and the four iso-triangular tiles of size 2.



$$\text{Then } \pi(q) = \begin{array}{ccc} a_4 & & \\ a_5 & b_4 & a_6 \end{array} \in \theta.$$

Similarly q can also be any block (D-iso triangular tile) and in this case $\pi(q) \in \theta$.

Theorem 3.1
 $\mathcal{L}(\text{ITTS}) = \mathcal{L}(\text{ITDS})$.

Proof

The inclusion $\mathcal{L}(\text{ITDS}) \subseteq \mathcal{L}(\text{ITTS})$ is an immediate consequence of Proposition 3.1.

The inverse inclusion follows from Lemma 3.1.

Before concluding an example as an application of the theorem as given.

Example 3.1

Consider the language L of iso-triangular picture over $\Sigma = \{a, b\}$. In the example given below we observe that $L \in \mathcal{L}(\text{ITTS})$.

```

#
# a #
# b b b #
# a a a a a #
# b b b b b b b #
# # # # # # # # # #

```

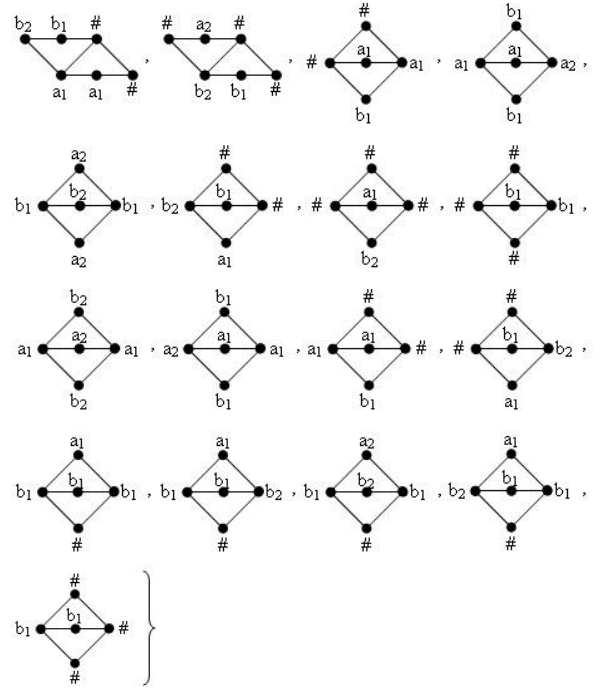
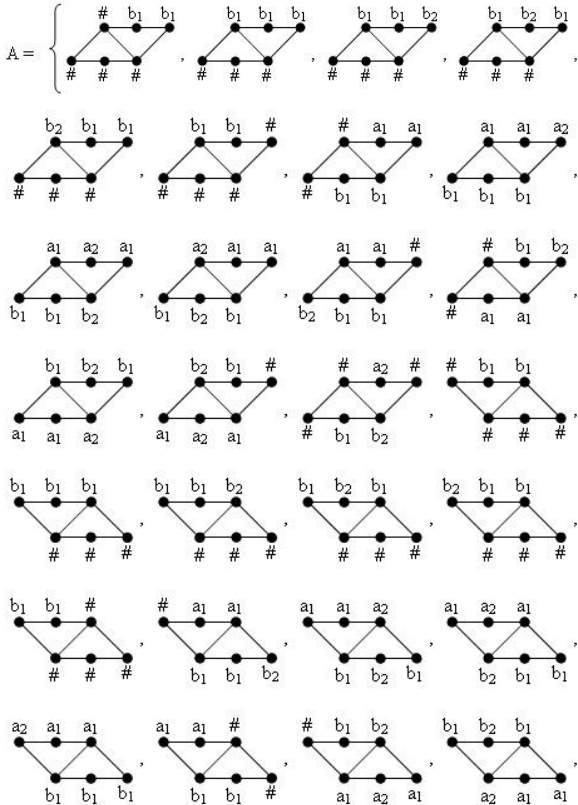
In order to show that $L \in \mathcal{L}(\text{ITDS})$ it suffices to verify that it can be obtained as a projection of the language L'' over $\Gamma = \{a_1, b_1, a_2, b_2\}$ of triangles in which the median of the triangle carry the symbols a_2 and b_2 and the other symbol carry the symbol a_1 and b_1 .

```

#
# a_2 #
# b_1 b_2 b_1 #
# a_1 a_1 a_2 a_1 a_1 #
# b_1 b_1 b_1 b_2 b_1 b_1 b_1 #
# # # # # # # # # #

```

It is clear that L'' is hrl-local. In fact it is represented by the following set of dominoes.



Now $L = \pi(L'')$ where $\pi : \Gamma \rightarrow \Sigma$ is such that $\pi(a_1) = \pi(a_2) = a$ and $\pi(b_1) = \pi(b_2) = b$. Hence L is recognizable by iso-triangular domino system. That is $L \in \mathcal{L}(\text{ITDS})$.

4. CONCLUSION

In this paper the overlapping of iso-triangular pictures have been introduced and the notion of recognizability of iso-triangular pictures by a new formalism called domino system have been investigated. The theorem $\mathcal{L}(\text{ITDS}) = \mathcal{L}(\text{ITTS})$ is proved. Triangular picture languages can generate all pictures in picture languages. The learning of iso-triangular pictures and unary iso-triangular picture languages and their complexity deserve to be studied further.

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