

# Fuzzy Metagraph and Vague Metagraph based Techniques and their Applications

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## ABSTRACT

Metagraphs are graphical hierarchical structure in which every node is a set having one or more elements. Fuzzy Metagraph and Vague Metagraph are an emerging technique used in the design of many information processing systems like transaction processing systems, Decision Support Systems (DSS), and workflow Systems. In this paper, distinct matrixes have been proposed for Fuzzy Metagraph and Vague Metagraph respectively. This method has reduced time complexity and space complexity. In complex situations, our Fuzzy Expert System integrated with the metagraphs will yield goods decision as quickly as possible. The main purpose of this DSS is to help a user make effective and quick decisions that the user can concentrate only on solving the problem.

## General Terms

Metagraph, Fuzzy set, Fuzzy Expert System.

## Keywords

Fuzzy Metagraph, Vague Metagraph, adjacency matrix.

## 1. INTRODUCTION

A graph is defined by a pair  $G=\{X, E\}$ , where  $X =\{x_1, x_2, x_3, \dots, x_n\}$  is a finite set of vertices (i.e. Nodes) and  $E$  a collection of edges (i.e. Links) that happen to connect vertices, and are typically used to model data from complex applications such as bioinformatics, social networks analysis, text retrieval, chemical compounds, protein structures and XML documents. Metagraph is a graphical hierarchical structure in which every node is a set having one or more elements. It has all the properties of graphs. In a metagraph, there is set-to-set mapping in place of node to node as in a conventional graph structure [1, 2].

Fuzzy logic is based on fuzzy set theory. In contrast to standard set theory in which each element is either completely in or not in a set. Fuzzy set theory allows partial member in sets. This provides a powerful mechanism for representing vague concepts. The world of information is surrounded by uncertainty and imprecision. The human reasoning process can handle inexact, uncertain, and vague concepts in an appropriate manner. Usually, the human thinking, reasoning, and perception process cannot be expressed precisely. These types of experiences can rarely be expressed or measured using statistical or probability theory. Fuzzy logic provides a framework to model uncertainty, the human way of thinking, reasoning, and the perception process. Fuzzy systems were first introduced by Zadeh (1965) [3].

Vague sets, defined by Gau and Buehrer have an extra potential edge over fuzzy sets of Zadesh. Vague sets are higher order fuzzy sets. Application of higher order fuzzy set make the solution procedure more complex, but if the complexity on computation time, computation volume or memory space are not the matter of concern then a better result could be achieved [9].

The rest of the paper is organized as follows. Section 2 points out various representations of a Metagraph. Fuzzy Expert System architecture is presented in section 3. Section 4 concludes the paper.

## 2. BACKGROUND

This section briefly reviews metagraph, fuzzy-metagraph and vague-metagraph techniques. Basu and Blanning introduced the concept of metagraphs [1]. Metagraph is graphical hierarchical structure in which every node is a set having one or more elements. It has all the properties of graphs. In a metagraph, there is set-to-set mapping in place of node-to-node as in a conventional graph structure.

### 2.1 Metagraph

A metagraph  $S = \{X, E\}$  is a graphical representation comprising two tuples  $X$  and  $E$ . Here  $X$  is its generating set and  $E$  is the set of edges defined on generating sets and metagraph is defined by  $S$ . The set of elements  $X = \{x_1, x_2, x_3, \dots, x_n\}$  represents variables and occurs in the edges of the metagraph.

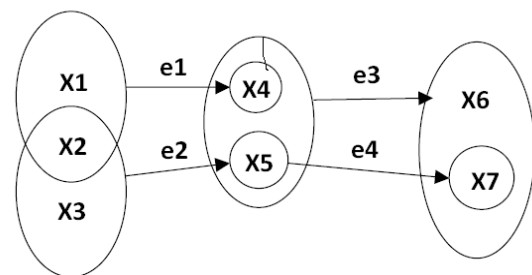


Fig 1: Example of Metagraph

Figure 1, shows an example of a metagraph.  $X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$  is the generating set, and  $E = \{e_1, e_2, e_3, e_4\}$  is the set of edges. The edge set can be specified as  $E = \{\langle \{x_1, x_2\}, \{x_4\} \rangle, \langle \{x_2, x_3\}, \{x_5\} \rangle, \langle \{x_4, x_5\}, \{x_6, x_7\} \rangle, \langle \{x_5\}, \{x_7\} \rangle\}$ . In-vertex is a function having one argument which can find out the internal vertices from a given set. In-vertex  $(\langle \{x_4, x_5\}, \{x_6, x_7\} \rangle) = \{x_4, x_5\}$ . Out-vertex is another function having one argument which can

find out what are the out vertices from the given set. Out-vertex  $\langle \{x_4, x_5\}, \{x_6, x_7\} \rangle = \{x_6, x_7\}$ . Two more functions of metagraph are the co-input and Co output functions each have two arguments. Co input function gives the co-input from a given set. Co-input  $\langle \{x_4, \{x_4, x_5\}, \{x_6, x_7\} \rangle = \{x_5\}$ , Co-output  $\langle \{x_6, \{x_4, x_5\}, \{x_6, x_7\} \rangle = \{x_7\}$ . Generally the edges of the metagraph are labeled as  $e_1 = \langle \{x_1, x_2\}, \{x_4\} \rangle$ ,  $e_2 = \langle \{x_2, x_3\}, \{x_5\} \rangle$ ,  $e_3 = \langle \{x_4, x_5\}, \{x_6, x_7\} \rangle$ ,  $e_4 = \langle \{x_5\}, \{x_7\} \rangle$ . A simple meta path  $h(x, y)$  from an element  $x$  to an element  $y$  is a sequence of edges  $\langle e_1, e_2, e_3, \dots, e_n \rangle$  such that  $x \in \text{in vertex}(e_1)$ ,  $y \in \text{out vertex}(e_n)$  and for all  $e_i$ ,  $i=1,2,3,\dots,n-1$ . Out vertex  $(e_i) \cap \text{in vertex}(e_{i+1}) = \emptyset$ .

### 2.1.1 Adjacency Matrix of Metagraph

Table I. Adjacency matrix of metagraph

	$X_4$	$X_5$	$X_6$	$X_7$
$X_1$	$\langle x_2, \emptyset, e_1 \rangle$	$\emptyset$	$\emptyset$	$\emptyset$
$X_2$	$\langle x_1, \emptyset, e_1 \rangle$	$\langle x_3, \emptyset, e_2 \rangle$	$\emptyset$	$\emptyset$
$X_3$	$\emptyset$	$\langle x_2, \emptyset, e_2 \rangle$	$\emptyset$	$\emptyset$
$X_4$	$\emptyset$	$\emptyset$	$\langle x_5, x_7, e_3 \rangle$	$\langle x_5, x_6, e_3 \rangle$
$X_5$	$\emptyset$	$\emptyset$	$\langle x_4, x_7, e_3 \rangle$	$\langle x_4, x_6, e_3 \rangle$ $\langle \emptyset, \emptyset, e_4 \rangle$

The adjacency matrix of Figure-1 metagraph will be as follows the adjacency matrix  $A$  of a metagraph is a square matrix with one row and one column for each element in the generating set  $X$  [6]. The  $ij^{\text{th}}$  element of  $A$ , denoted as  $a_{ij}$  is a set of triples one for each edge  $e$  connecting  $x_i$  to  $x_j$ . Each triple is of the form of  $\langle CI_e, CO_e, e \rangle$  in which  $CI_e$  is the co-input of  $x_i$  in  $e$  and  $CO_e$  is the co-output of  $x_j$  in  $e$ . Table I represents adjacency matrix of metagraph from figure 1.

### 2.1.2 Adjacency List

An adjacency list basically has  $V$  linked lists, with each corresponding linked list containing the elements that are adjacent to a particular vertex [1].

- $x_1 \rightarrow \langle x_2, \emptyset, e_1 \rangle$
- $x_2 \rightarrow \langle x_1, \emptyset, e_1 \rangle \rightarrow \langle x_3, \emptyset, e_2 \rangle$
- $x_3 \rightarrow \langle x_2, \emptyset, e_2 \rangle$
- $x_4 \rightarrow \langle x_5, x_7, e_3 \rangle \rightarrow \langle x_5, x_6, e_3 \rangle$
- $x_5 \rightarrow \langle x_4, x_7, e_3 \rangle \rightarrow \langle x_4, x_6, e_3 \rangle$
- $x_5 \rightarrow \langle x_4, x_7, e_3 \rangle \rightarrow \langle \emptyset, \emptyset, e_4 \rangle$
- $x_6 \rightarrow \langle \emptyset, \emptyset, \emptyset \rangle$
- $x_7 \rightarrow \langle \emptyset, \emptyset, \emptyset \rangle$

### 2.2 Fuzzy Set Declaration and Definition

Fuzzy logic allows intermediate categories between notations such as true/false, hot/cold, black/white etc. as used in Boolean logic. In fuzzy system, values are indicated by a number in the range of 0 to 1. Where 0 represents absolute falseness and 1 represents absolute truthfulness.

Let  $X$  be a space of objects and  $x$  be a generic element of  $X$ . A classical set  $A$ ,  $A \subseteq X$ , is defined as a collection of elements or objects  $x \in X$ , such that  $x$  can either belong or not belong to the set  $A$ . A fuzzy set  $A$  in  $X$  is defined as a set of ordered pairs [3]. In fuzzy set there is no repeated elements. All elements must be unique [3, 11].

$A = \{(x, \mu_A(x)) \mid x \in X\}$ , Where  $\mu_A(x)$  is called as the membership function (MF) for the fuzzy set  $A$ . The MF maps each element of  $X$  to a membership grade between zero and one. The two fuzzy sets  $A$  and  $B$  is specified in general by a function  $T : [0,1] \times [0,1] \rightarrow [0,1]$ , which aggregates two membership grades as follows: This class of fuzzy set operators is usually referred to as  $T$ -norm (Triangular Norm). The most frequently used  $T$ -norm operators are

- Commutative:  $T(p, q) = T(q, p)$
- Associative:  $T(p, T(q, r)) = T(T(p, q), r)$
- Monotonicity:  $p \leq q, r \leq s \rightarrow T(p, r) \leq T(q, s)$
- Boundary conditions:  $T(p, 0) = 0, T(p, 1) = p$
- Logical product:  $T_L(p, q) = p \wedge q$
- Algebraic product:  $T_A(p, q) = pq$
- Minimum:  $T_{\min}(a, b) = \min(a, b) = a \wedge b$
- Maximum:  $T_{\max}(a, b) = \max(a, b) = a \vee b$
- Intersection:  $\mu_{A \cap B}(x) = T(\mu_A(x), \mu_B(x)) = \min(\mu_A, \mu_B)$
- Union:  $\mu_{A \cup B}(x) = T(\mu_A(x), \mu_B(x)) = \max(\mu_A, \mu_B)$

Over the past few years, a number of fuzzy graphs have been proposed to represent the uncertain relationship between fuzzy elements and sets of fuzzy elements. However, as mentioned before, existing fuzzy graphs are not capable of effectively modeling the directed relationships between sets of fuzzy elements. It is critical that all the graphs lack powerful algebraic analytic methods for manipulating the directed relationships between sets of elements. This motivates the development of Fuzzy Metagraphs.

### 2.3 Fuzzy Metagraph

The concept of a fuzzy graph is the “fuzzification” of the crisp graphs using fuzzy sets. A fuzzy graph  $\tilde{G}$  can be defined as a triple  $\{X, \tilde{X}, \tilde{E}\}$ , where  $\tilde{X}$  is a fuzzy set on  $X$  and  $\tilde{E}$  is a fuzzy relation on  $X \times X$ .

A fuzzy set  $\tilde{X}$  on  $X$  is completely characterized by its membership function  $\mu: X \rightarrow [0, 1]$  for each  $x \in X$ ,  $\mu(x)$  illustrates the truth value of the statement of  $x \in \tilde{X}$ . The fuzzy metagraph is the concept of Fuzzification of the crisp Metagraph using fuzzy generating set. Fuzzy generating set is the node set of all the elements of fuzzy metagraph [8, 20]. Consider a finite set  $X = \{x_1, x_2, x_3, \dots, x_n\}$ . A fuzzy metagraph is a triple  $\tilde{S} = \{X, \tilde{X}, \tilde{E}\}$  in which  $\tilde{X}$  is a fuzzy set on  $X$  and  $\tilde{E}$  is a fuzzy edge set  $\{\tilde{e}_m, m=1, 2, 3, \dots, m\}$ .

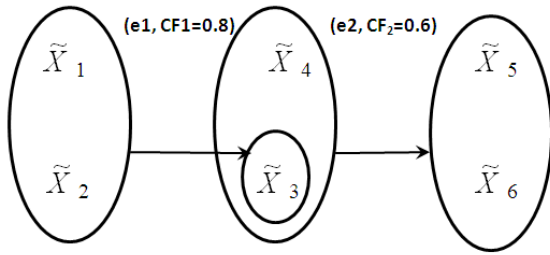


Fig 2: Fuzzy Metagraph

Often, the membership value of an edge is also called certainty factor (CF) of the edge. For simplicity, assign  $\tilde{X}_i$  denoting  $(X_i, \mu(\tilde{X}_i))$  and  $\tilde{e}_k$  denoting  $(e_k, CF_k)$ . Figure 2 shows fuzzy metagraph whose element set is  $X = \{ \tilde{X}_1, \tilde{X}_2, \dots, \tilde{X}_6 \}$  is known as fuzzy Meta Node and whose edge set consists of:  $\tilde{e}_1 = \langle \{ \tilde{X}_1, \tilde{X}_2 \}, \{ \tilde{X}_3 \} \rangle$  and  $\tilde{e}_2 = \langle \{ \tilde{X}_3, \tilde{X}_4 \}, \{ \tilde{X}_5, \tilde{X}_6 \} \rangle$ .

### 2.3.1 Adjacency Matrix of Fuzzy Metagraph

Each entry  $a_{ij}$  in the matrix is a null set or a few ordered triples according to whether  $\tilde{X}_i$  and  $\tilde{X}_j$  are adjacent or not and how many edges connect  $\tilde{X}_i$  to  $\tilde{X}_j$ . In each triple, the first component is the co-input of  $\tilde{X}_i$ ; the second is the co output of  $\tilde{X}_j$ ; the third is a simple path from  $\tilde{X}_i$  to  $\tilde{X}_j$  with a length of one [20].

Table II. Adjacency matrix of Fuzzy Metagraph

	$\tilde{X}_3$	$\tilde{X}_5$	$\tilde{X}_6$
$\tilde{X}_1$	$\langle \tilde{X}_2, \emptyset, \tilde{e}_1 \rangle$	$\emptyset$	$\emptyset$
$\tilde{X}_2$	$\langle \tilde{X}_1, \emptyset, \tilde{e}_1 \rangle$	$\emptyset$	$\emptyset$
$\tilde{X}_3$	$\emptyset$	$\langle \tilde{X}_4, \tilde{X}_6, \tilde{e}_2 \rangle$	$\langle \tilde{X}_4, \tilde{X}_5, \tilde{e}_2 \rangle$
$\tilde{X}_4$	$\emptyset$	$\langle \tilde{X}_3, \tilde{X}_6, \tilde{e}_2 \rangle$	$\langle \tilde{X}_3, \tilde{X}_5, \tilde{e}_2 \rangle$

### 3.3.2 The closure of Fuzzy Metagraph Adjacency matrix

Given an Fuzzy Metagraph  $\tilde{S} = \{X, \tilde{X}, \tilde{E}\}$  and its Fuzzy Metagraph adjacency matrix A is defined as an infinite sum, namely,

$$A^* = A + A^2 + A^3 + A^4 + \dots + A^n, n \rightarrow \infty$$

$$A^* = \sum_{n=1}^{\infty} A^n$$

The Fuzzy Metagraph closure matrix  $A^*$  is formed by adding successive powers of the fuzzy metagraph adjacency matrix. The Fuzzy Metagraph adjacency matrix of Fuzzy Metagraph in Figure 2 can be illustrated as Table III. The square,  $A_2$ , of the adjacency matrix of Fuzzy Metagraph in Figure 2 is given in Table IV.

Table III. The Closure of Adjacency matrix of Fuzzy Metagraph

	$\tilde{X}_3$	$\tilde{X}_5$	$\tilde{X}_6$
$\tilde{X}_1$	$\langle \tilde{X}_2, \emptyset, \tilde{e}_1 \rangle$	$\langle \{ \tilde{X}_2, \tilde{X}_4 \}, \{ \tilde{X}_3, \tilde{X}_6 \}, \langle \tilde{e}_1, \tilde{e}_2 \rangle \rangle$	$\langle \{ \tilde{X}_2, \tilde{X}_4 \}, \{ \tilde{X}_3, \tilde{X}_5 \}, \langle \tilde{e}_1, \tilde{e}_2 \rangle \rangle$
$\tilde{X}_2$	$\langle \tilde{X}_1, \emptyset, \tilde{e}_1 \rangle$	$\langle \{ \tilde{X}_1, \tilde{X}_4 \}, \{ \tilde{X}_3, \tilde{X}_6 \}, \langle \tilde{e}_1, \tilde{e}_2 \rangle \rangle$	$\langle \{ \tilde{X}_1, \tilde{X}_4 \}, \{ \tilde{X}_3, \tilde{X}_6 \}, \langle \tilde{e}_1, \tilde{e}_2 \rangle \rangle$
$\tilde{X}_3$	$\emptyset$	$\langle \tilde{X}_4, \tilde{X}_6, \tilde{e}_2 \rangle$	$\langle \tilde{X}_4, \tilde{X}_5, \tilde{e}_2 \rangle$
$\tilde{X}_4$	$\emptyset$	$\langle \tilde{X}_3, \tilde{X}_6, \tilde{e}_2 \rangle$	$\langle \tilde{X}_3, \tilde{X}_5, \tilde{e}_2 \rangle$

Table IV. The Square of Adjacency matrix of Fuzzy Metagraph

	$\tilde{X}_5$	$\tilde{X}_6$
$\tilde{X}_1$	$\langle \{ \tilde{X}_2, \tilde{X}_4 \}, \{ \tilde{X}_3, \tilde{X}_6 \}, \langle \tilde{e}_1, \tilde{e}_2 \rangle \rangle$	$\langle \{ \tilde{X}_2, \tilde{X}_4 \}, \{ \tilde{X}_3, \tilde{X}_5 \}, \langle \tilde{e}_1, \tilde{e}_2 \rangle \rangle$
$\tilde{X}_2$	$\langle \{ \tilde{X}_1, \tilde{X}_4 \}, \{ \tilde{X}_3, \tilde{X}_6 \}, \langle \tilde{e}_1, \tilde{e}_2 \rangle \rangle$	$\langle \{ \tilde{X}_1, \tilde{X}_4 \}, \{ \tilde{X}_3, \tilde{X}_6 \}, \langle \tilde{e}_1, \tilde{e}_2 \rangle \rangle$

Fuzzy matrix as well as the Symmetric matrix of Fig. 2 is shown in Fig. 3 and Fig.4.

	1	2	3	4	5	6
1	1	1	0.8	-----	-----	-----
2	1	1	0.8	-----	-----	-----
3	-----	-----	1	1	0.6	0.6
4	-----	-----	1	1	0.6	0.6
5	-----	-----	-----	-----	1	1
6	-----	-----	-----	-----	1	1

Fig 3: Fuzzy Matrix of Fuzzy Metagraph

	1	2	3	4	5	6
1	1	1	-----	-----	-----	-----
2	1	1	-----	-----	-----	-----
3	-----	-----	1	1	-----	-----
4	-----	-----	1	1	-----	-----
5	-----	-----	-----	-----	1	1
6	-----	-----	-----	-----	1	1

Fig 4: Symmetric Matrix of Fuzzy Metagraph

In order to analyze the similarity structure of nodes for a fuzzy Metagraph, we use the symmetric relation matrix  $S = (s_{ij})$ . Before that we have to construct the fuzzy matrix of Fuzzy Metagraph.

	1	2	3	4	5	6
1	1	1	1	1	1	1
2	1	1	1	1	1	1
3	-----	-----	1	1	1	1
4	-----	-----	1	1	1	1
5	-----	-----	-----	-----	1	1
6	-----	-----	-----	-----	1	1

Fig 5: Transitive closure of Fuzzy Metagraph

Fuzzy matrix of above fuzzy Metagraph will be shown in Fig. 3. The Symmetric matrix will be shown in Fig. 4. Transitive closure matrix of above fuzzy matrix in Fig. 2 will be shown in Fig. 5. By viewing this transitive closure matrix we can analyze that some nodes are equivalent we will make a cluster of those nodes. In spite of putting them separately. In the above Fig. 2 node {1, 2} will be clustered as a single node [9, 20].

### 2.4. Vague Metagraph

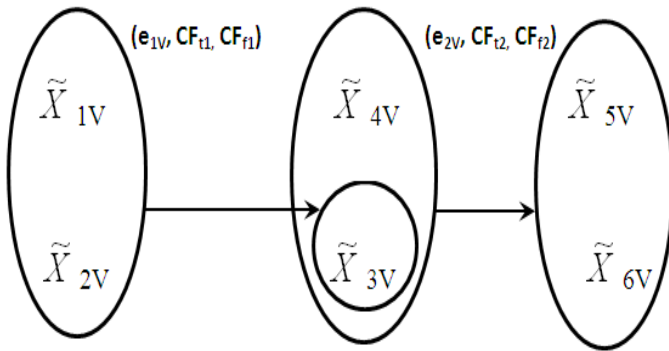


Fig 6: vague metagraph

Vague Metagraph is a new concept of data structure in computer science. Therefore, for any kind of hard and soft computation with vague metagraph, we need to know how to store a vague metagraph in computer memory, how to access it or its components from computer memory. Consider a finite set  $X = \{x_1, x_2, x_3, \dots, x_n\}$ . A Vague metagraph is a triple  $\tilde{S} = \{X, \tilde{X}_v, \tilde{E}_v\}$  in which  $\tilde{X}_v$  is a vague set on  $X$  and  $\tilde{E}_v$  is a vague edge set  $\{\tilde{e}_{mv}, m=1, 2, 3, \dots, m\}$ . Each component of  $\tilde{e}_{mv}$  in  $\tilde{E}_v$  is characterized by an ordered pair  $\langle \tilde{V}_{mv}, \tilde{W}_{mv} \rangle$ . In the pair  $\tilde{V}_{mv} \subseteq \tilde{X}_v$  is the in-vertex of  $\tilde{e}_{mv}$  and  $\tilde{W}_{mv} \subseteq \tilde{X}_v$  is the out-vertex [9]. Often, the membership value of an edge is also called certainty factor (CF) of the edge. For simplicity, assign  $\tilde{X}_{iv}$  denoting  $(X_{iv}, \mu(\tilde{X}_{iv}))$  and  $\tilde{e}_{mv}$  denoting  $(e_{mv}, CF_{tm}, CF_{rm})$ . Figure 6 shows vague metagraph whose element set is  $X_v = \{\tilde{X}_{1v}, \tilde{X}_{2v}, \dots, \tilde{X}_{6v}\}$  is known as vague Meta Node and whose edge set consists of:  $\tilde{e}_{1v} = \langle \{\tilde{X}_{1v}, \tilde{X}_{2v}\}, \{\tilde{X}_{3v}\} \rangle$  and  $\tilde{e}_{2v} = \langle \{\tilde{X}_{3v}, \tilde{X}_{4v}$

$\}, \{\tilde{X}_{5v}, \tilde{X}_{6v}\} \rangle$ . The in-vertex and out-vertex of  $\tilde{e}_{1v}$  are  $\{\tilde{X}_{1v}, \tilde{X}_{2v}\}$  and  $\{\tilde{X}_{3v}\}$ . Vague Metagraph can be represented in computer memory in the form of an Incidence matrix and The Adjacency matrix.

### 2.4.1 Adjacency Matrix of vague Metagraph

Each entry  $a_{ij}$  in the matrix is a null set or a few ordered triples according to whether  $\tilde{X}_{iv}$  and  $\tilde{X}_{jv}$  are adjacent or not and how many edges connect  $\tilde{X}_{iv}$  to  $\tilde{X}_{jv}$ .

Table V. Adjacency matrix of Vague Metagraph

	$\tilde{X}_{3v}$	$\tilde{X}_{5v}$	$\tilde{X}_{6v}$
$\tilde{X}_{1v}$	$\langle \tilde{X}_{2v}, \emptyset, \tilde{e}_{1v} \rangle$	$\emptyset$	$\emptyset$
$\tilde{X}_{2v}$	$\langle \tilde{X}_{3v}, \emptyset, \tilde{e}_{1v} \rangle$	$\emptyset$	$\emptyset$
$\tilde{X}_{3v}$	$\emptyset$	$\langle \tilde{X}_{4v}, \tilde{X}_{5v}, \tilde{e}_{2v} \rangle$	$\langle \tilde{X}_{4v}, \tilde{X}_{5v}, \tilde{e}_{2v} \rangle$
$\tilde{X}_{4v}$	$\emptyset$	$\langle \tilde{X}_{3v}, \tilde{X}_{5v}, \tilde{e}_{2v} \rangle$	$\langle \tilde{X}_{3v}, \tilde{X}_{5v}, \tilde{e}_{2v} \rangle$

In each triple, the first component is the co-input of  $\tilde{X}_{iv}$ , the second is the co output of  $\tilde{X}_{jv}$ ; the third is a simple path from  $\tilde{X}_{iv}$  to  $\tilde{X}_{jv}$  with a length of one [9].

### 2.4.2 The Closure of Adjacency Matrix and Square Matrix of a Vague Metagraph

The adjacency matrix only shows the adjacent linkages in the graph. Many other paths may be existing, but not visible from the adjacency matrix of Vague Metagraph.

**Table VI. The Square of Adjacency matrix of Vague Metagraph**

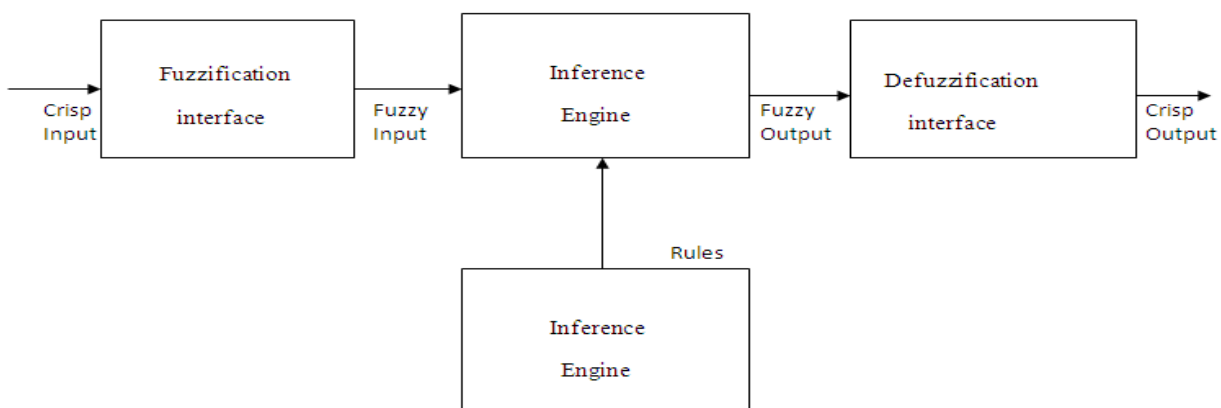
	$\tilde{X}_{5v}$	$\tilde{X}_{6v}$
$\tilde{X}_{1v}$	$\langle \{ \tilde{X}_{2v}, \tilde{X}_{4v} \}, \{ \tilde{X}_{3v}, \tilde{X}_{6v} \}, \langle \tilde{e}_{1v}, \tilde{e}_{2v} \rangle \rangle$	$\langle \{ \tilde{X}_{2v}, \tilde{X}_{4v} \}, \{ \tilde{X}_{3v}, \tilde{X}_{5v} \}, \langle \tilde{e}_{1v}, \tilde{e}_{2v} \rangle \rangle$
$\tilde{X}_{2v}$	$\langle \{ \tilde{X}_{1v}, \tilde{X}_{4v} \}, \{ \tilde{X}_{3v}, \tilde{X}_{6v} \}, \langle \tilde{e}_{1v}, \tilde{e}_{2v} \rangle \rangle$	$\langle \{ \tilde{X}_{1v}, \tilde{X}_{4v} \}, \{ \tilde{X}_{3v}, \tilde{X}_{6v} \}, \langle \tilde{e}_{1v}, \tilde{e}_{2v} \rangle \rangle$

If  $\tilde{X}_{iv}$  is adjacent to  $\tilde{X}_{jv}$  and  $\tilde{X}_{jv}$  is adjacent to  $\tilde{X}_{kv}$ , then it can be inferred that there is a path with the length two from  $\tilde{X}_{iv}$  to  $\tilde{X}_{kv}$ . The closure of adjacency matrix can be developed by all paths of any length connecting two arbitrary vertices  $\tilde{X}_{iv}$  to  $\tilde{X}_{jv}$  if exists. The Vague Metagraph closure matrix  $A^*$  is formed by adding the successive power to Adjacency matrix, namely, the multiplication by itself. Square of Adjacency matrix  $A^2$  of vague Metagraph shown in Table VI and Closure matrix  $A^*$  of vague Metagraph of Figure-6 is represented in Table VII.

**Table VII. The Closure of Adjacency matrix of Fuzzy Metagraph**

	$\tilde{X}_{3v}$	$\tilde{X}_{5v}$	$\tilde{X}_{6v}$
$\tilde{X}_{1v}$	$\langle \tilde{X}_{2v}, \emptyset, \tilde{e}_{1v} \rangle$	$\langle \{ \tilde{X}_{2v}, \tilde{X}_{4v} \}, \{ \tilde{X}_{3v}, \tilde{X}_{6v} \}, \langle \tilde{e}_{1v}, \tilde{e}_{2v} \rangle \rangle$	$\langle \{ \tilde{X}_{2v}, \tilde{X}_{4v} \}, \{ \tilde{X}_{3v}, \tilde{X}_{5v} \}, \langle \tilde{e}_{1v}, \tilde{e}_{2v} \rangle \rangle$
$\tilde{X}_{2v}$	$\langle \tilde{X}_{1v}, \emptyset, \tilde{e}_{1v} \rangle$	$\langle \{ \tilde{X}_{1v}, \tilde{X}_{4v} \}, \{ \tilde{X}_{3v}, \tilde{X}_{6v} \}, \langle \tilde{e}_{1v}, \tilde{e}_{2v} \rangle \rangle$	$\langle \{ \tilde{X}_{1v}, \tilde{X}_{4v} \}, \{ \tilde{X}_{3v}, \tilde{X}_{6v} \}, \langle \tilde{e}_{1v}, \tilde{e}_{2v} \rangle \rangle$

### 3. FUZZY EXPERT SYSTEM



**Fig 7: Basic architecture of Fuzzy Expert System**

A fuzzy expert system is simply an expert system that uses a collection of fuzzy membership functions and rules, instead of

	$\tilde{e}_{1v}, \tilde{e}_{2v} \gg \gg$	$\tilde{e}_{1v}, \tilde{e}_{2v} \gg$
$\tilde{X}_{3v}$	$\emptyset$	$\langle \tilde{X}_{4v}, \tilde{X}_{6v}, \tilde{e}_{2v} \rangle$
$\tilde{X}_{4v}$	$\emptyset$	$\langle \tilde{X}_{3v}, \tilde{X}_{6v}, \tilde{e}_{2v} \rangle$

#### 2.4.3 Incidence Matrix of a Vague Metagraph

The incidence matrix  $I'$  of a vague metagraph has one row for each of the element in the vague generating set  $\tilde{X}_v$  and one column for each edge. The  $ij$ th component of  $I'$ ,  $I'_{ij}$ , is -1 if  $\tilde{X}_{iv}$  is an invertex of  $e_{jv}$ , it is +1 if  $\tilde{X}_{iv}$  is in the outvertex of  $e_{jv}$ , and it is 0 otherwise. The incidence matrix of Figure 3 will be as follows [9].

**Table VIII. Incidence matrix of Vague Metagraph**

	$e_{1v}$	$e_{2v}$
$\tilde{X}_{1v}$	-1	$\emptyset$
$\tilde{X}_{2v}$	-1	$\emptyset$
$\tilde{X}_{3v}$	1	-1
$\tilde{X}_{4v}$	$\emptyset$	-1
$\tilde{X}_{5v}$	$\emptyset$	1
$\tilde{X}_{6v}$	$\emptyset$	1

Boolean logic, to reason about data. Fuzzy expert system consists of Fuzzification, inference system, rule base, Defuzzification units. It has the capability to solve decision

making problems for which no exact algorithm exists. Fuzzy expert systems are well to problems that exhibit uncertainty resulting from inexactness, vagueness or subjectivity. Fuzzy expert system architecture used in this system is shown in the figure 7.

Fuzzification is the process of converting crisp input to fuzzy value. Membership Functions (MFs) are used to convert crisp inputs into fuzzy value. The MF maps each element of input to a membership grade (or membership value) between zero and one [3]. A rule-based system consists of if-then rules, a set of facts, and an interpreter controlling the application of the rules, given the facts. Fuzzy rule based expert system could be applied in every important field like business, robotics, manufacturing, online services etc.

#### **4. CONCLUSION**

Metagraphs have a lot of applications in the field of information processing systems, decision support systems, transaction processing systems and workflow systems. Distinct matrixes have been developed for the Fuzzy Metagraph and the Vague Metagraph. The graphical model not only has visualized the process of any system but also does the formal analysis by means of an algebraic representation of the graphical structure. Data can be stored inside the computer memory either in the form of an Adjacency matrix or as an Adjacency list so that it can be used efficiently. Fuzzy Expert System (FES) integrated with the Fuzzy Metagraph or Vague Metagraph can yield excellent state-of-the-art decisions under complex circumstances which other graph structures find it very difficult. Thus a user with this effective decision making system can make effective and quick decisions to resolve the problem. Future works may be concentrated on the optimization techniques in Fuzzy-Metagraphs and Vague- Metagraph to further enhance the performance of the system.

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