Chance Constrained Linear Plus Linear Fractional Bi-level Programming Problem

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ABSTRACT
We present fuzzy goal programming approach to solve chance constrained linear plus linear fractional bi-level programming problem. The chance constraints with right hand parameters as random variables of prescribed probability distribution functions are transformed into equivalent deterministic system constraints. We construct nonlinear membership functions based on deterministic system constraints. The nonlinear membership functions are transformed into linear membership functions by using first order Taylor's series approximation. In the bi-level decision making context, decision deadlock may arise due to the dissatisfaction of the lower level decision maker with the decision of upper level decision maker. To overcome this problem, decision maker of each level gives his preference bounds on decision variables under his/her control to provide some relaxation on their decisions. Fuzzy goal programming model is used to achieve highest membership goals by minimizing negative deviational variables. Euclidean distance function is used in order to find out the most satisfactory solution. We solve a chance constrained linear plus linear fractional bi-level programming problem to illustrate the proposed approach.

General Terms
Bi-level programming, linear plus linear fractional programming.

Keywords
Bi-level programming, linear plus linear fractional programming, chance constraints, fuzzy goal programming, Taylor's series.

1. INTRODUCTION
In game theory, inventory problems, production house problems, banking systems, the objective functions may be either linear fractional or the sum of linear and linear fractional functions. In 1962, Charnes and Cooper [1] developed variable transformation method to solve multi-objective linear fractional programming problem (MOLFPP). Linear programming with a fractional objective function was studied by Bitran and Noveas [2] in 1973. Goal programming (GP) approach to linear fractional criteria was introduced by Kornbluth and Steuer [3]. In GP, the goals are stated explicitly. In real situation, the target goals may not be explicitly stated due to uncertainty. To deal with uncertainty, Luhandjula [4] proposed fuzzy approaches for solving MOLFPP. Sakawa and Kato [5] used interactive decision making to solve MOLFPP involving fuzzy numbers.

In 1970, Teterav [6] first studied optimality criteria for solving linear plus linear fractional programming problem. Schaible [7] studied the sum of linear and linear fractional function in 1977. Chadha [8] and Hirche [9] developed different models for solving the sum of linear and linear fractional programming problem. Under fuzzy constraints, linear plus linear fractional programming problem (LPLFPP) was studied by Jain and Lachhwani [10]. In 2010, Jain and Lachhwani [11] developed LPLFPP with homogeneous constraints using fuzzy approach. Jain et al. [12] discussed multi-objective linear plus linear fractional programming problem (MOLPLFPP) containing non-differential term. In 2011, Singh et al. studied fuzzy goal programming (FGP) approach for solving MOLPLFPP. Using Taylor's Series approximation Pramanik et al. [14] developed FGP model to solve MOLPLFPP. Recently, Pramanik and Banerjee [15] studied chance constrained MOLPLFPP based on first order Taylor's series approximation.

Linear plus linear fractional bi-level programming problem (LPLFBLPP) is a special type of non-linear bi-level programming problem. In this paper, we consider the objective function of each level DM as linear plus linear fractional function. We also consider the constraints as linear functions and probabilistically defined. There are many research fields where LPLFBLPP arises such as robust data fitting, traffic assignment problems, portfolio optimizations, banking systems, any management systems, etc.

In this paper, the concept of Pramanik and Banerjee is extended to chance constrained linear plus linear fractional bi-level programming problem (CCLPLFBLPP). In bi-level programming problem (BLPP), two types of decision makers (DMs) i.e. upper level decision maker (ULDM) and lower level decision maker (LLDM) execute their decision in hierarchical way. Each level DM independently controls a set of decision variables. Candler and Townsley [16] as well as Fortuny –Amart and McCarl [17] developed the formal BLPP. After that many researchers [18, 19] studied BLPP in various perspectives. Sakawa and Nishizaki [20, 21] introduced linear fractional BLPP using interactive fuzzy programming. Pramanik and Dey [22] presented bi-level linear fractional programming problem based on FGP using first order Taylor's series approximation.

In the present paper, we convert the chance constraints into equivalent deterministic constraints with known distribution functions and confidence levels. We form non-linear membership functions by using individual best solutions subject to the deterministic constraints. Using first order Taylor's series, the non-linear membership functions are transformed into linear membership functions by expanding about the respective individual best solution point. In decision making process, each level decision maker provides preference bounds on the decision variables controlled by him/her for avoiding decision deadlock. Two FGP models are formulated and Euclidean distance function is used to determine the most compromise solution. A numerical example is solved to demonstrate the proposed approach.

The rest of the paper is developed in the following way. In Section 2, we formulate CCLPLFBLPP. In Section 3, chance constraints are reduced into equivalent deterministic constraints. Non-linear membership functions are constructed in Section 4. In Section 5, technique of linearization of non-linear membership function is discussed by using first order Taylor's series. In Section 6, preference bounds on the decision variables are defined. Section 7 is devoted to develop two FGP models for solving CCLPLFBLPP. The Euclidean distance function is described in the next Section 8. The step wise description of the process for solving CCLPLFBLPP is presented in the Section 9. Section 10 presents illustrative numerical example of CCLPLFBLPP. Section 11 presents conclusion and future work.

2. FORMULAON OF CCLPLFBLPP

CCLPLFBLPP can be presented as:

$$\text{[ULDM]} \ \underset{\overline{X}_1}{\text{Max}}\ Z_1(\overline{X}) = (\overline{p}_1^T \overline{x} + \gamma_1) + \frac{\overline{c}_1^T \overline{x} + \alpha_1}{\overline{d}_1^T \overline{x} + \beta_1} \tag{1}$$

$$\text{[LLDM]} \ \underset{\overline{X}_2}{\text{Max}}\ Z_2(\overline{X}) = (\overline{p}_2^T \overline{x} + \gamma_2) + \frac{\overline{c}_2^T \overline{x} + \alpha_2}{\overline{d}_2^T \overline{x} + \beta_2} \tag{2}$$

subject to

$\overline{X} \in X' =$

$$\{\overline{X} \in \overline{R}^n : \Pr(\overline{C}\overline{X} \ {\textstyle\leq \atop \geq}\ \overline{d}) > \overline{I} - \overline{m},\ \overline{X} \geq \overline{0}\} \tag{3}$$

Here, $\overline{p}_1^T$, $\overline{p}_2^T$, $\overline{c}_1^T$, $\overline{c}_2^T$, d_1^T, $d_2^T \in \overline{R}^n$ and α_1, α_2, β_1, β_2, γ_1, γ_2 are constants. The decision vector $\overline{X}_1 = (x_{11}, x_{12}, x_{13}, \ldots, x_{1n1})$ is controlled by ULDM and $\overline{X}_2 = (x_{21}, x_{22}, x_{23}, \ldots, x_{2n2})$ is controlled by LLDM. $\overline{X}_1 \cup \overline{X}_2 = \overline{X} \in \overline{R}^n$, $n_1 + n_2 = n$, 'T' means transposition of vector. $\overline{I}$, $\overline{d}$, $\overline{m}$ are vectors of order $p \times 1$, every elements of $\overline{I}$ is unity. $\overline{C}$ is the given matrix of order $p \times n$. The polyhedron X' is assumed to be non-empty and bounded.

3. REDUCTION OF STOCHASTIC CONSTRAINTS INTO EQUIVALENT DETERMINISTIC CONSTRAINTS

We consider the chance constraints of the form:

$$\Pr\left(\sum_{j=1}^{n} c_{ij} x_j \leq d_i\right) \geq 1 - m_i, \quad i = 1, 2, \ldots, p_1. \tag{4}$$

$$\Rightarrow \Pr\left(\frac{\sum_{j=1}^{n} c_{ij}x_j - E(d_i)}{\sqrt{\text{var}(d_i)}} \leq \frac{d_i - E(d_i)}{\sqrt{\text{var}(d_i)}}\right) \geq 1 - m_i,\ i = 1, 2, \ldots, p_1$$

$$\Rightarrow m_i \geq 1 - \Pr\left(\frac{\sum_{j=1}^{n} c_{ij}x_j - E(d_i)}{\sqrt{\text{var}(d_i)}} \leq \frac{d_i - E(d_i)}{\sqrt{\text{var}(d_i)}}\right)$$

$$\Rightarrow m_i \geq \Pr\left(\frac{\sum_{j=1}^{n} c_{ij}x_j - E(d_i)}{\sqrt{\text{var}(d_i)}} > \frac{d_i - E(d_i)}{\sqrt{\text{var}(d_i)}}\right)$$

$$\Rightarrow \Psi^{-1}(m_i) \geq \frac{\sum_{j=1}^{n} c_{ij}x_j - E(d_i)}{\sqrt{\text{var}(d_i)}}$$

$$\Rightarrow \Psi^{-1}(m_i)\sqrt{\text{var}(d_i)} \geq \sum_{j=1}^{n} c_{ij}x_j - E(d_i)$$

$$\Rightarrow \sum_{j=1}^{n} c_{ij}x_j \leq E(d_i) + \Psi^{-1}(m_i)\sqrt{\text{var}(d_i)},$$

$$i = 1, 2, \ldots, p_1 \tag{5}$$

Here, Ψ (.) and Ψ^{-1} (.) represent the distribution function and inverse of the distribution function of standard normal variable respectively.

We consider the case when

$$\Pr\left(\sum_{j=1}^{n} c_{ij} x_j \geq d_i\right) \geq 1 - m_i,\ i = p_1+1, p_1+2, \ldots, p. \tag{6}$$

The constraints can be rewritten as:

$$\Pr\left(\frac{\sum_{j=1}^{n} c_{ij}x_j - E(d_i)}{\sqrt{\text{var}(d_i)}} \geq \frac{d_i - E(d_i)}{\sqrt{\text{var}(d_i)}}\right) \geq 1 - m_i,\ i = p_1+1, p_1+2, \ldots, p.$$

$$\Rightarrow \Psi\left(\frac{\sum_{j=1}^{n} c_{ij}x_j - E(d_i)}{\sqrt{\text{var}(d_i)}}\right) \geq 1 - m_i$$

$$\Rightarrow 1 - \Psi\left(-\frac{\sum_{j=1}^{n} c_{ij}x_j - E(d_i)}{\sqrt{\text{var}(d_i)}}\right) \geq 1 - m_i$$

$$\Rightarrow \Psi^{-1}(m_i) \geq \frac{\sum_{j=1}^{n} c_{ij}x_j - E(d_i)}{\sqrt{\text{var}(d_i)}}$$

$$\Rightarrow \sum_{j=1}^{n} c_{ij}x_j \geq E(d_i) - \Psi^{-1}(m_i)\sqrt{\text{var}(d_i)}, \tag{7}$$

$i = p_1 + 1, p_1 + 2, \ldots, p.$

$$\overline{X} \geq \overline{0} \tag{8}$$

Let us denote the equivalent deterministic system constraints (5), (7) and (8) by X. Here, X` and X are equivalent set of constraints.

4. CONSTRUCTION OF MEMBERSHIP FUNCTIONS

In order to construct non-linear membership function subject to the equivalent deterministic system constraints, the objective functions are maximized separately.

Let the individual best solution for the objective function $Z_i(\overline{X})$, i = 1, 2 be

$$\overline{X}_i^B = (x_{i1}^B, x_{i2}^B, x_{i3}^B, \ldots, x_{in_i}^B, x_{in_i+1}^B, \ldots, x_{in}^B),\ i = 1, 2.$$

Let $\max_{\overline{X}\in X} Z_1(\overline{X}) = Z_1^B = Z_1(\overline{X}_1^B)$ and
$\max_{\overline{X}\in X} Z_2(\overline{X}) = Z_2^B = Z_2(\overline{X}_2^B)$.

If we consider the individual best solution as the aspiration level, the fuzzy goal assumes the form:

$$Z_i(\overline{X}) \underset{\sim}{\geq} Z_i^B, \; i = 1, 2 \tag{9}$$

$Z_1^B = Z_1(\overline{X}_1^B)$ and $Z_2^B = Z_2(\overline{X}_2^B)$ are the upper tolerance limits of the fuzzy objective goals of ULDM and LLDM respectively. Similarly, $Z_1^W = Z_1(\overline{X}_2^B)$ and $Z_2^W = Z_2(\overline{X}_1^B)$ are the lower tolerance limits of the fuzzy objective goals of ULDM and LLDM.

Now, the membership function for the objective function $Z_1(\overline{X})$ of ULDM can be written as:

$$\mu_1(\overline{X}) = \left\langle \begin{array}{ll} 1, & \text{if } Z_1(\overline{X}) \geq Z_1^B, \\ \dfrac{Z_1(\overline{X}) - Z_1^W}{Z_1^B - Z_1^W}, & \text{if } Z_1^W \leq Z_1(\overline{X}) \leq Z_1^B, \\ 0, & \text{if } Z_1(\overline{X}) \leq Z_1^W \end{array} \right\rangle \tag{10}$$

and the membership function for the objective function $Z_2(\overline{X})$ of LLDM can be formulated as:

$$\mu_2(\overline{X}) = \left\langle \begin{array}{ll} 1, & \text{if } Z_2(\overline{X}) \geq Z_2^B, \\ \dfrac{Z_2(\overline{X}) - Z_2^W}{Z_2^B - Z_2^W}, & \text{if } Z_2^W \leq Z_2(\overline{X}) \leq Z_2^B, \\ 0, & \text{if } Z_2(\overline{X}) \leq Z_2^W \end{array} \right\rangle \tag{11}$$

Now, the CCLPLFBLPP reduces to

max $\mu_1(\overline{X})$,

max $\mu_2(\overline{X})$,

subject to

$$\overline{X} \in X. \tag{12}$$

5. CONVERSION OF NON-LINEAR MEMBERSHIP FUNCTION INTO LINEAR MEMBERSHIP FUNCTION BY USING TAYLOR'S SERIES APPROXIMATION

Let $\overline{X}_i^* = (x_{i1}^*, x_{i2}^*, x_{i3}^*, ..., x_{in_i}^*, x_{in_i+1}^*, ..., x_{in}^*)$, $i = 1, 2$ be the individual best solution of $\mu_i(\overline{X})$ subject to the equivalent deterministic system constraints. Then we transform the non-linear membership function $\mu_i(\overline{X})$ into an equivalent linear membership function $\mu_i^*(\overline{X})$ at the point $\overline{X}_i^*$ by using first order Taylor's series as follows:

$$\mu_1(\overline{X}) \cong \mu_1\left(\overline{X}_1^*\right) + (x_1 - x_{11}^*)\frac{\partial}{\partial x_1}\mu_1\left(\overline{X}_1^*\right) + (x_2 - x_{12}^*)\frac{\partial}{\partial x_2}\mu_1\left(\overline{X}_1^*\right) + \ldots + (x_{n_1} - x_{1n_1}^*)\frac{\partial}{\partial x_{n_1}}\mu_1\left(\overline{X}_1^*\right) + (x_{n_1+1} - x_{1n_1+1}^*)\frac{\partial}{\partial x_{n_1+1}}\mu_1\left(\overline{X}_1^*\right) + \ldots + (x_n - x_{1n}^*)\frac{\partial}{\partial x_n}\mu_1\left(\overline{X}_1^*\right)$$

$$= \mu_1^*(\overline{X}) \tag{13}$$

$$\mu_2(\overline{X}) \cong \mu_2\left(\overline{X}_2^*\right) + (x_1 - x_{21}^*)\frac{\partial}{\partial x_1}\mu_2\left(\overline{X}_2^*\right) + (x_2 - x_{22}^*)\frac{\partial}{\partial x_2}\mu_2\left(\overline{X}_2^*\right) + \ldots + (x_{n_2} - x_{2n_2}^*)\frac{\partial}{\partial x_{n_2}}\mu_2\left(\overline{X}_2^*\right) + (x_{n_2+1} - x_{2n_2+1}^*)\frac{\partial}{\partial x_{n_2+1}}\mu_2\left(\overline{X}_2^*\right) + \ldots + (x_n - x_{2n}^*)\frac{\partial}{\partial x_n}\mu_2\left(\overline{X}_2^*\right) = \mu_2^*(\overline{X}) \tag{14}$$

6. PREFERENCE BOUNDS ON THE DECISION VARIABLES

Since the objectives of level DMs are conflicting, cooperation between the level DMs is necessary in order to reach compromise optimal solution. Each DM tries to reach maximum profit with the consideration of benefit of other. Cooperation between the DMs is reflected by the relaxations provided by the level DMs on both decision variables.

Let $(x_{1j}^* - r_{1j}^-)$ and $(x_{1j}^* + r_{1j}^+)$ ($j = 1, 2, \ldots, n_1$) be the lower and upper bounds of decision variable x_{1j} ($j = 1, 2, \ldots, n_1$) provided by the ULDM. $\overline{X}_1^* = (x_{11}^*, x_{12}^*, ..., x_{1n_1}^*, x_{1n_1+1}^*, ..., x_{1n}^*)$ is the individual best solution of the non-linear membership function $\mu_1(\overline{X})$ of ULDM when calculated in isolation subject to the equivalent deterministic system constraints.

Similarly, $(x_{2j}^* - r_{2j}^-)$ and $(x_{2j}^* + r_{2j}^+)$ ($j = 1, 2, \ldots, n_2$) be the lower and upper bounds of decision variables x_{2j} ($j = 1, 2, \ldots, n_2$) provided by the LLDM. $\overline{X}_2^* = (x_{21}^*, x_{22}^*, \ldots, x_{2n_2}^*, x_{2n_2+1}^*, \ldots, x_{2n}^*)$ is the individual best solution of the non-linear membership function $\mu_2(\overline{X})$ of LLDM when calculated in isolation subject to the equivalent deterministic system constraints. Therefore, preference bounds on the decision variable can presented as follows:

$$(x_{1j}^* - r_{1j}^-) \leq x_{1j} \leq (x_{1j}^* + r_{1j}^+) \; (j = 1, 2, \ldots, n_1) \tag{15}$$

$$(x_{2j}^* - r_{2j}^-) \leq x_{2j} \leq (x_{2j}^* + r_{2j}^+) \; (j = 1, 2, \ldots, n_2) \tag{16}$$

Here, r_{1j}^-, r_{1j}^+ ($j = 1, 2, \ldots, n_1$) and r_{2j}^-, r_{2j}^+ ($j = 1, 2, \ldots, n_2$) are all of non-negative values and these are not necessarily same.

7. FORMULATION OF FGP MODEL OF CCLPLFBLPP

The CCLPLFBLPP reduces to the following problem:

Max $\mu_1^*(\overline{X})$, Max $\mu_2^*(\overline{X})$

subject to

$(x^*_{1j} - r^-_{1j}) \le x_{1j} \le (x^*_{1j} + r^+_{1j})$, (j = 1, 2, …, n_1)

$(x^*_{2j} - r^-_{2j}) \le x_{2j} \le (x^*_{2j} + r^+_{2j})$, (j = 1, 2, …,$n_2$)

$\overline{X} \in X$ (17)

According to Pramanik and Dey, it can be written [23] as:

$\mu^*_i + d^-_i = 1$, i = 1, 2. (18)

d^-_1 and d^-_2 are the negative deviational variables. Now, two FGP models are formulated as follows:

Model-I

min λ (19)

subject to

$\mu^*_1(\overline{X}) + d^-_1 = 1,$

$\mu^*_2(\overline{X}) + d^-_2 = 1,$

$\lambda \ge d^-_1,$

$\lambda \ge d^-_2,$

$0 \le d^-_1 \le 1,$

$0 \le d^-_2 \le 1,$

$(x^*_{1j} - r^-_{1j}) \le x_{1j} \le (x^*_{1j} + r^+_{1j})$, (j = 1, 2, …, n_1)

$(x^*_{2j} - r^-_{2j}) \le x_{2j} \le (x^*_{2j} + r^+_{2j})$, (j = 1, 2, …,$n_2$)

$\overline{X} \in X$

Model –II (20)

Min $\xi = \sum_{i=1}^{2} d^-_i$

subject to

$\mu^*_1(\overline{X}) + d^-_1 = 1,$

$\mu^*_2(\overline{X}) + d^-_2 = 1,$

$0 \le d^-_1 \le 1,$

$0 \le d^-_2 \le 1,$

$(x^*_{1j} - r^-_{1j}) \le x_{1j} \le (x^*_{1j} + r^+_{1j})$, j = 1, 2, …, n_1

$(x^*_{2j} - r^-_{2j}) \le x_{2j} \le (x^*_{2j} + r^+_{2j})$, j = 1, 2, …,$n_2$

$\overline{X} \in X$

8. USE OF DISTANCE FUNCTION TO DETERMINE COMPROMISE SOLUTION

For multi objective programming, the objectives are incommensurable and conflicting in nature. The aim of decision makers is to find out the compromise solution which is as near as possible to the ideal solution points in the decision making context. Here, we use the Euclidean distance function [24] of the type

$$D_2 = \left[\sum_{i=1}^{2}(1-\mu^*_i)^2\right]^{1/2} \quad (21)$$

The solution with the minimum distance is considered as the best compromise optimal solution.

9. SUMMARIZATION OF THE PROCESS FOR SOLVING CHANCE CONSTRAINED LPLFBLPP

To solve CCLPLFBLPP we use the following steps.

Step-1. Transform the chance constraints into equivalent deterministic constraints.

Step-2. Calculate individual best solution for each linear plus linear fractional objective function of the level DM subject to the equivalent deterministic constraints.

Step-3. Lower and upper tolerance limits are determined for each linear plus linear fractional objective function as stated in Section 4.

Step-4. Non linear membership functions are formulated by using individual best solutions subject to the equivalent deterministic system constraints.

Step-5. Find out the individual best solution for each of the non-linear membership functions subject to the equivalent deterministic constraints.

Step-6. Using first order Taylor's series, the non-linear membership functions are approximated into linear functions at the individual best solution point.

Step-7. Both level DMs express their choices for the upper and lower preference bounds on the decision variables controlled by them.

Step-8. Two FGP models are formulated and solved.

Step-9. Determine the Euclidean distance for two optimal compromise solutions obtained from two FGP Models.

Step-10. Select the solution with the minimum Euclidean distance as the best compromise optimal solution.

10. ILLUSTRATIVE EXAMPLES OF CCLPLFBLPP

To illustrate the proposed FGP approach, the following CCLPLFBLPP with maximization type objective function at each level is considered.

$$[\text{ULDM}] \max_{x_1} Z_1(\overline{X}) = 8 + x_2 + \frac{2x_1 - 3x_2}{x_1 + x_2} \quad (22)$$

$$[\text{LLDM}] \max_{x_2} Z_2(\overline{X}) = x_1 + 7 - \frac{x_1 - 2x_2}{3x_1 + x_2} \quad (23)$$

subject to

Pr $(4x_1 + 3x_2 \le d_1) \ge 1 - m_1$

(24)

Pr $(5x_1 + 2x_2 \ge d_2) \ge 1 - m_2$ (25)

$x_1 \ge 0,\ x_2 \ge 0,$ (26)

The mean, variance and the confidence levels are prescribed as follows:

E (d_1) = 2, var (d_1) = 1, $m_1 = 0.02$ (27)

E (d_2) = 4, var (d_2) = 2, $m_2 = 0.04$

(28)

Using (5) and (7), the chance constraints defined in (24) and (25) can be converted into equivalent deterministic constraints as:

$4x_1 + 3x_2 \leq 4.055$ (29)

$5x_1 + 2x_2 \geq 1.518055$ (30)

The individual best solution for each objective function of the level DM subject to the equivalent deterministic constraints is obtained as $Z_1^B = 10$, at $\overline{X}_1^B = (0.303611, 0)$, and $Z_2^B = 9$, at $\overline{X}_2^B = (0, 0.7590275)$.

The fuzzy goals appear as:

$$Z_1(\overline{X}) \underset{\sim}{\geq} 10,\ Z_2(\overline{X}) \underset{\sim}{\geq} 9 \qquad (31)$$

The lower tolerance limits are obtained as $Z_1^W = 5.759028$ and $Z_2^W = 6.970278$

Now, the non-linear membership function for ULDM and LLDM are constructed as follows:

$$\mu_1(\overline{X}) = \left\langle \begin{array}{ll} 1, & \text{if } Z_1(\overline{X}) \geq 10, \\ \dfrac{Z_1(\overline{X}) - 5.759028}{10 - 5.759028}, & \text{if } 5.759028 \leq Z_1(\overline{X}) \leq 10, \\ 0 & \text{if } Z_1(\overline{X}) \leq 5.759028 \end{array} \right\rangle \qquad (32)$$

$$\mu_2(\overline{X}) = \left\langle \begin{array}{ll} 1, & \text{if } Z_2(\overline{X}) \geq 9, \\ \dfrac{Z_2(\overline{X}) - 6.970278}{9 - 6.970278}, & \text{if } 6.970278 \leq Z_2(\overline{X}) \leq 9, \\ 0 & \text{if } Z_2(\overline{X}) \leq 6.970278 \end{array} \right\rangle \qquad (33)$$

The non-linear membership functions $\mu_1(\overline{X})$ and $\mu_2(\overline{X})$ are linearized at their individual best solution point at $\overline{X}_1^B$ = (0.303611, 0), $\overline{X}_2^B$ = (0, 0.7590275) and we obtain equivalent linear membership functions as follows:

$\mu_1^*(\overline{X}) = 1 + (x_1 - 0.303611) * 0 + (x_2 - 0) * (0.303611-5) / (4.240972*0.33611),$ (34)

$\mu_2^*(\overline{X}) = 1 + (x_1-0) * (0.7590275-7) / (2.029722*0.7590275) + (x_2 - 0.7590275) \times 0$ (35)

Let $0 \leq x_1 \leq 0.5$ and $0 \leq x_2 \leq 1$ be the preference bounds provided by the level DMs.

By using two FGP models (19) and (20), the optimal compromise solutions (See Figure –1, Fifure-2 and Figure-3) are presented in the Table1.

Table 1. The optimal solutions obtained from two FGP models of the problem

Model No.	μ_1, μ_2	x_1, x_2	Z_1, Z_2	D_2
FGP-I	0.435 0.265	0.2102191 0.2334797	7.6024 7.5073	0.928
FGP-II	0.603 0.157	0.2468549 0.1418902	8.3169 7.3289	0.932

Comparing Euclidean distance D_2 (see Table 1), we conclude that model I offers better optimal solution than Model II for this problem.

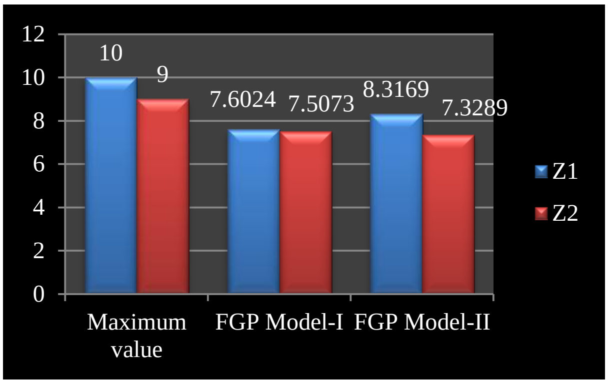

Figure 1. Comparison of optimal solutions

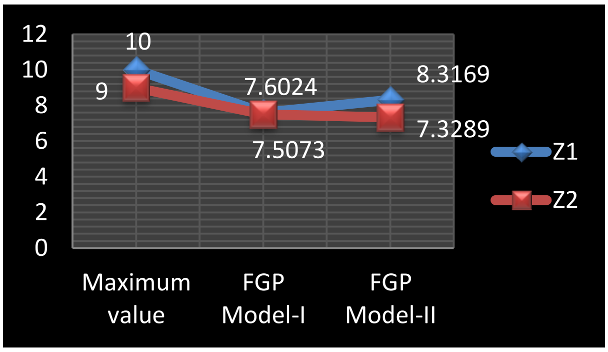

Figure 2. Comparison of optimal solutions

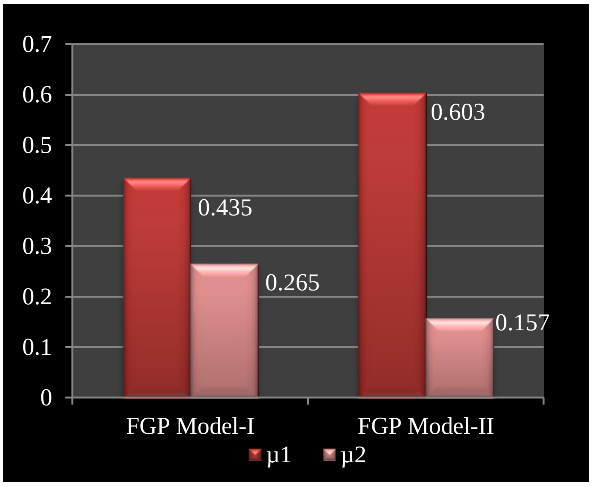

Figure 3. Comparison of obtained resulting membership values

11. CONCLUSION

In this paper, we present CCLPLFBLPP in simple way. The proposed approach is very easy to understand. In the proposed approach, chance constraints are transformed into equivalent deterministic constraints and linear plus linear fractional bi-level programming problem is converted into linear bi-level programming problem by using the first order Taylor's series approximation.

For the further study, chance constrained multi-level linear plus linear fractional programming problem can be solved by extending the proposed approach. In the hierarchical decision making context, the proposed approach can be also applied for solving chance constrained linear plus linear fractional decentralized multi-level multi-objective programming problems.

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