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A Modified Approach for Ranking Non-Normal p-norm Trapezoidal Fuzzy numbers

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ABSTRACT
Ranking fuzzy numbers is a prerequisite for the decision making problem. In order to rank fuzzy quantities many researchers proposed and analyzed different techniques on triangular and trapezoidal fuzzy numbers. However, no one can claim their method is a satisfactory one. In this paper a modified distance based approach called signed distance proposed by Yao and Wu [9] is discussed. This proposed approach is free from computational complexity in the process of decision making, optimization and forecasting problems. Some Numerical examples are used to illustrate the proposed approach.

Mathematics Subject Classification
90C08,90C90.

Keywords
Non-Normal p-norm trapezoidal fuzzy numbers – Ranking function – Signed distance.

1. INTRODUCTION
Since the pioneering work of Zadeh [1], fuzzy set theory has been applied to many areas, in particular to decision making problems. Ordering of $\geq$ and $\leq$ are possible in the real number system. Since fuzzy numbers are represented by different formats it is very difficult to say which one is larger and which one is smaller. An efficient approach for ordering the fuzzy numbers is by the use of ranking function from the set of fuzzy numbers F ($\mathbb{R}$) into $\mathbb{R}$. Ranking of fuzzy numbers is very important for decision makers to take up their decision in an optimum result. This is to be done based on ranking or comparison.

The method of ranking was first proposed by Jain [2]. Yager [3] proposed four indices which may be employed for the purpose of ordering fuzzy quantities [1]. In [4] an approach is presented for the ranking of fuzzy numbers. Cheng [6] presented a method for ranking fuzzy numbers by using distance method. Chu and Tsao [8] proposed a method for ranking fuzzy numbers with an area between the centroid point and original point. Chen and Tang [13] proposed the ranking formula for the non-normal p-norm trapezoidal fuzzy numbers based on integral value proposed by Liou and Wang [5]. Abbasbandy and Hajiari [11] introduced a new approach for ranking of trapezoidal fuzzy numbers based on the left and right spreads at some α-level of trapezoidal fuzzy number. Chen and Chen [12] presented a method for fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads.

Amit Kumar et.al [14] presented a method for ranking non-normal p-norm trapezoidal fuzzy numbers with different heights proposed by Chen and Tang [13]. Abbasbandy and Asady [10] proposed ranking of fuzzy numbers by signed distance and Yao and Wu [9] presented a method on ranking fuzzy numbers based on decomposition principle and signed distance. In this paper a modified approach is proposed for ranking non-normal p-norm trapezoidal fuzzy numbers by signed distance proposed by Yao and Wu [9].

2. PRELIMINARIES

2.1 Basic Definitions:
In this section some basic definitions are reviewed as follows:

Definition 2.1[4]: The characteristic function μ_A of a crisp set. A⊆X assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range. i.e. $\mu_{\tilde{A}}$: X → [0, 1]. The assigned value indicates the membership grade of the element in the set A. The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \epsilon X\}$ defined by $\mu_{\tilde{A}}(x)$ for each X is called a fuzzy set.

Definition 2.2[4]: A fuzzy number $\tilde{A}$ = (a, b, c, d) is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x\epsilon(-\infty, a] \cup [d, +\infty) \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \frac{x-d}{c-d}, & c \leq x \leq d \end{cases}$$

Definition 2.3: A fuzzy set $\tilde{A}$ defined on the Universal set of real numbers $\mathbb{R}$ is said to be generalized trapezoidal fuzzy number if its membership function has the following characteristics:

1. $\mu_{\tilde{A}}(x)$: $\mathbb{R}$ → [0, w] is continuous

2. $\mu_{\tilde{A}}(x)$ = 0 for all $x\epsilon(-\infty, a] \cup [d, +\infty)$

3. $\mu_{\tilde{A}}(x)$ is strictly increasing on [a, b] and strictly decreasing on [c, d]

4. $\mu_{\tilde{A}}(x)$ = w for all $x\epsilon[b, c]$ where $0 < w \leq 1$

Definition 2.4[12]: A generalized trapezoidal fuzzy number $\tilde{A}$ = (a, b, c, d, w) is said to be a non-normal trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x\epsilon(-\infty, a] \cup [d, +\infty) \\ w\left(\frac{x-a}{b-a}\right), & a \le x \le b \\ w\ , & b \le x \le c, 0 < w < 1 \\ w\left(\frac{x-d}{c-d}\right), & c \le x \le d \end{cases}$$

The generalized trapezoidal fuzzy number is said to be a normal trapezoidal fuzzy number if w = 1.

Definition 2.5[13]: A non-normal fuzzy number $\tilde{A}$ = (a, b, c, d, w)$_p$ is said to be a non-normal p-norm trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x\epsilon(-\infty, a] \cup [d, +\infty) \\ w\left[1 - \left(\frac{x-b}{a-b}\right)^p\right]^{\frac{1}{p}} & a \le x \le b \\ w\ , & b \le x \le c, \\ w\left[1 - \left(\frac{x-c}{d-c}\right)^p\right]^{\frac{1}{p}} & c \le x \le d \end{cases}$$

where p is a positive integer.

The left and right inverse functions of $\mu_{\tilde{A}}(x)$ are,

$$\left. \begin{aligned} L_{\tilde{A}}^{-1}(y) &= b + (a-b)\left[1 - \left(\frac{y}{w}\right)^p\right]^{\frac{1}{p}}, 0 \le y \le w \\ R_{\tilde{A}}^{-1}(y) &= c + (d-c)\left[1 - \left(\frac{y}{w}\right)^p\right]^{\frac{1}{p}}, 0 \le y \le w \end{aligned} \right\} \quad \ldots\ldots (1)$$

2.2 Arithmetic Operations:

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1 : w_1)_p$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2 : w_2)_p$ be two non-normal p-norm trapezoidal fuzzy numbers defined on the universal set of real numbers ℝ. Then the arithmetic operations between $\tilde{A}_1$ and $\tilde{A}_2$ are

i. $\tilde{A}_1 \oplus \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2 : w)_p$

ii. $\tilde{A}_1 \ominus \tilde{A}_2 = (a_1 - a_2, b_1 - b_2, c_1 - c_2, d_1 - d_2 : w)_p$

iii. $\tilde{A}_1 \otimes \tilde{A}_2 = (a_1 \times a_2, b_1 \times b_2, c_1 \times c_2, d_1 \times d_2 : w)_p$

iv. $\tilde{A}_1 \oslash \tilde{A}_2 = (a_1 / d_2, b_1 / c_2, c_1 / b_2, d_1 / a_2 : w)_p$ where w = min(w$_1$, w$_2$)

v. $\lambda\tilde{A}_1 = \begin{cases} (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1 : w_1)_p, \lambda \ge 0 \\ (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1 : w_1)_p, \lambda < 0 \end{cases}$

While considering more than two non-normal p-norm trapezoidal fuzzy numbers let w = min (w$_1$, w$_2$, w$_3$).

2.3 Ranking function:

As fuzzy numbers are represented by possibility distributions, they may overlap with each other and hence it is not possible to order them. It is true that fuzzy numbers are frequently partial order and cannot be compared like real numbers. An efficient approach for comparing fuzzy numbers is by the use of a ranking function. A ranking function is a map from the set of fuzzy numbers F(ℝ) into a real line.

Definition 2.6[9]: For $\tilde{A}$, $\tilde{B}$ ∈ F(ℝ) with α-cut ($0 \le \alpha \le 1$) define the signed distance of $\tilde{A}$, $\tilde{B}$ as follows:
d ($\tilde{A}$, $\tilde{B}$) = $\frac{1}{2}\int_0^1 [\tilde{A}_L(\alpha) + \tilde{A}_R(\alpha) - \tilde{B}_L(\alpha) - \tilde{B}_R(\alpha)]d\alpha$
where d ($\tilde{A}$, $\tilde{B}$) means the distance of $\tilde{B}$ to $\tilde{A}$,
A(α) = $[A_L(\alpha), A_R(\alpha)]$ and B(α) = $[B_L(\alpha), B_R(\alpha)]$.

For $\tilde{A}$, $\tilde{B}$ ∈ F(ℝ), d ($\tilde{A}$, $\tilde{B}$) = d ($\tilde{A}$, 0_1) - d ($\tilde{B}$, 0_1) where d($\tilde{A}$, 0_1) and d ($\tilde{B}$, 0_1) are the signed distances of $\tilde{A}$ and $\tilde{B}$ from 0_1 (y-axis) respectively.

Definition 2.7[9]: The ranking in F(ℝ) is defined as follows:

i. d ($\tilde{A}$, $\tilde{B}$) > 0 iff d($\tilde{A}$, 0_1) > d ($\tilde{B}$, 0_1) iff $\tilde{B} \prec \tilde{A}$

ii. d ($\tilde{A}$, $\tilde{B}$) < 0 iff d($\tilde{A}$, 0_1) < d ($\tilde{B}$, 0_1) iff $\tilde{A} \prec \tilde{B}$

iii d ($\tilde{A}$, $\tilde{B}$) = 0 iff d($\tilde{A}$, 0_1) = d ($\tilde{B}$, 0_1) iff $\tilde{B} \approx \tilde{A}$

where $\tilde{A}$ and $\tilde{B}$ are in F(ℝ) and d ($\tilde{A}$, $\tilde{B}$) is the difference of the two signed distances of fuzzy sets $\tilde{A}$ and $\tilde{B}$ from 0_1 (y-axis) .

Definition 2.8: Let $\tilde{A} = (a_1, b_1, c_1, d_1 : w_1)_p$ and $\tilde{B} = (a_2, b_2, c_2, d_2 : w_2)_p$ be two non-normal p-norm trapezoidal fuzzy numbers with different heights. We define the signed distance of $\tilde{A}$ and $\tilde{B}$ as follows:

$$d(\tilde{A}, \tilde{B}) = \frac{1}{2}\int_0^w \{[L_{\tilde{A}}^{-1}(y) + R_{\tilde{A}}^{-1}(y)] - [L_{\tilde{B}}^{-1}(y) + R_{\tilde{B}}^{-1}(y)]\}dy$$

where the inverse functions are defined as in (1) and d ($\tilde{A}$, $\tilde{B}$) means the distance of $\tilde{B}$ to $\tilde{A}$.

Preposition: Let $\tilde{A} = (a_1, b_1, c_1, d_1 : w_1)_p$ and $\tilde{B} = (a_2, b_2, c_2, d_2 : w_2)_p$ be two non-normal p-norm trapezoidal fuzzy numbers with different heights. Then we have

$$d(\tilde{A}, 0_1) = \frac{w}{2}[(a_1 - b_1 - c_1 + d_1) \frac{\Gamma\left(\frac{1}{p}+1\right)\Gamma\left(\frac{1}{p}\right)}{p\Gamma\left(\frac{2}{p}+1\right)} + (b_1 + c_1)]$$

$$d(\tilde{B}, 0_1) = \frac{w}{2}[(a_2 - b_2 - c_2 + d_2) \frac{\Gamma\left(\frac{1}{p}+1\right)\Gamma\left(\frac{1}{p}\right)}{p\Gamma\left(\frac{2}{p}+1\right)} + (b_2 + c_2)]$$

and $d(\tilde{A}, \tilde{B}) = \frac{w}{2}[(a_1 - b_1 - c_1 + d_1 - a_2 + b_2 - d_2 + c_2) \frac{\Gamma\left(\frac{1}{p}+1\right)\Gamma\left(\frac{1}{p}\right)}{p\Gamma\left(\frac{2}{p}+1\right)} + (b_1 + c_1 - b_2 - c_2)]$.

Proof : By definition,

$$d(\tilde{A}, 0_1) = \frac{1}{2}\int_0^w \{[L_{\tilde{A}}^{-1}(y) + R_{\tilde{A}}^{-1}(y)]dy$$

$$= \frac{1}{2}\int_0^w \{[a_1 - b_1 - c_1 + d_1][\left[1 - \left(\frac{y}{w}\right)^p\right]^{\frac{1}{p}}] + (b_1 + c_1)\}\, dy$$

Setting $z = \left[1 - \left(\frac{y}{w}\right)^p\right]$, we have $dy = \frac{-w}{p}(1 - z)^{\frac{1}{p}-1} dz$ where $1 \leq z \leq 0$. Then $d(\tilde{A}, 0_1) = \frac{1}{2}\int_0^1 \{[a_1 - b_1 - c_1 + d_1] z^{\frac{1}{p}} [[1 - z]^{\frac{1}{p}-1} [\frac{w}{p}]\} dz + \frac{w}{2}[b_1 + c_1]$

$$= \frac{w}{2}[(a_1 - b_1 - c_1 + d_1) \square\square \frac{\Gamma\left(\frac{1}{p}+1\right)\Gamma\left(\frac{1}{p}\right)}{p\Gamma\left(\frac{2}{p}+1\right)} + (b_1 + c_1)]$$

Similarly, $d(\tilde{B}, 0_1)$

$$= \frac{w}{2}[(a_2 - b_2 - c_2 + d_2) \square\square \frac{\Gamma\left(\frac{1}{p}+1\right)\Gamma\left(\frac{1}{p}\right)}{p\Gamma\left(\frac{2}{p}+1\right)} + (b_2 + c_2)]$$

But $d(\tilde{A}, \tilde{B}) = d(\tilde{A}, 0_1) - d(\tilde{B}, 0_1)$. Hence the preposition.

Remark 1[7]: For all fuzzy numbers $\tilde{A}, \tilde{B}, \tilde{C}$ *and* $\tilde{D}$ we have the reasonable properties for the ordering of fuzzy quantities

i. $\tilde{A} \succ \tilde{B} \Rightarrow \tilde{A} \oplus \tilde{B} \succ \tilde{B} \oplus \tilde{C}$

ii. $\tilde{A} \succ \tilde{B} \Rightarrow \tilde{A} \ominus \tilde{B} \succ \tilde{B} \ominus \tilde{C}$

iii. $\tilde{A} \sim \tilde{B} \Rightarrow \tilde{A} \oplus \tilde{B} \sim \tilde{B} \oplus \tilde{C}$

iv. $\tilde{A} \succ \tilde{B}, \tilde{C} \succ \tilde{D} \Rightarrow \tilde{A} \oplus \tilde{C} \succ \tilde{B} \oplus \tilde{D}$

3. PROPOSED APPROACH:

For $\tilde{A} = (a_1, b_1, c_1, d_1 : w_1)_p$ and $\tilde{B} = (a_2, b_2, c_2, d_2 : w_2)_p$ two non-normal p-norm trapezoidal fuzzy numbers with different heights.

i. $d(\tilde{A}, \tilde{B}) > 0$ iff $d(\tilde{A}, 0_1) > d(\tilde{B}, 0_1)$ iff $\tilde{B} \prec \tilde{A}$

ii. $d(\tilde{A}, \tilde{B}) < 0$ iff $d(\tilde{A}, 0_1) < d(\tilde{B}, 0_1)$ iff $\tilde{A} \prec \tilde{B}$

iii $d(\tilde{A}, \tilde{B}) = 0$ iff $d(\tilde{A}, 0_1) = d(\tilde{B}, 0_1)$ iff $\tilde{B} \approx \tilde{A}$

4. NUMERICAL EXAMPLES:

Example 4.1: Let $\tilde{A}$ = (5, 7, 8, 9: 0.4) and $\tilde{B}$ = (5, 7, 8, 9: 0.6) be two non-normal p-norm trapezoidal fuzzy numbers w = min (0.4, 0.6) = 0.4 and p = 1

$d(\tilde{A}, 0_1) = 2.9$ and $d(\tilde{B}, 0_1) = 2.9$ (i.e.) $d(\tilde{A}, 0_1) = d(\tilde{B}, 0_1)$
$d(\tilde{A}, \tilde{B}) = d(\tilde{A}, 0_1) - d(\tilde{B}, 0_1) = 0$. Hence $\tilde{A} \approx \tilde{B}$.

Let $\tilde{C}$ = (2, 3, 5, 7: 0.8), w = min (0.6, 0.4, 0.8) = 0.4
$\tilde{A} \oplus \tilde{C}$ = (7, 10, 13, 16: 0.4) and $\tilde{B} \oplus \tilde{C}$ = (7, 10, 13, 16:0.4)
We have $\tilde{A} \oplus \tilde{C} \sim \tilde{B} \oplus \tilde{C}$.

Example 4.2: Let $\tilde{A}$ = (6, 7, 9, 10: 0.6) and $\tilde{B}$ = (7, 8, 9, 10: 0.4) be two non-normal p-norm trapezoidal fuzzy numbers w = min (0.6, 0.4) = 0.4 and p = 1

$d(\tilde{A}, 0_1) = 3.2$ and $d(\tilde{B}, 0_1) = 3.4$ (i.e.) $d(\tilde{A}, 0_1) < d(\tilde{B}, 0_1)$
Now $d(\tilde{A}, \tilde{B}) = d(\tilde{A}, 0_1) - d(\tilde{B}, 0_1) = -0.2 < 0$. Hence $\tilde{A} \prec \tilde{B}$

Let $\tilde{C}$ = (2, 3, 5, 7: 0.8), w = min (0.6, 0.4, 0.8) = 0.4
$\tilde{A} \oplus \tilde{C}$ = (8, 10, 14, 17: 0.4) and $\tilde{B} \oplus \tilde{C}$ = (9, 11, 14, 17:0.4)
Then $d(\tilde{A} \oplus \tilde{C}, 0_1) = 4.95$ and $d(\tilde{B} \oplus \tilde{C}, 0_1) = 5.10$.
Therefore $\tilde{A} \oplus \tilde{C} \sim \tilde{B} \oplus \tilde{C}$.
We also have $\tilde{A} \prec \tilde{B} \Rightarrow \tilde{A} \ominus \tilde{C} \prec \tilde{B} \ominus \tilde{C}$.
Let $\tilde{D}$ = (10, 12, 15, 18: 0.5).
Here $\tilde{C} \succ \tilde{D} \Rightarrow \tilde{A} \oplus \tilde{C} \succ \tilde{B} \oplus \tilde{D}$.

Example 4.3: Let $\tilde{A} = (5, 7, 8, 11: 0.4)_2$ and $\tilde{B} = (3, 4, 6, 8: 0.6)_1$ be non-normal 2-norm trapezoidal fuzzy number and non-normal trapezoidal fuzzy number respectively. w = min (0.4, 0.6) = 0.4 and $p_1 > p_2$

$d(\tilde{A}, 0_1) = 3.16$ and $d(\tilde{B}, 0_1) = 2.2$ (i.e.) $d(\tilde{A}, 0_1) > d(\tilde{B}, 0_1)$
Now $d(\tilde{A}, \tilde{B}) = d(\tilde{A}, 0_1) - d(\tilde{B}, 0_1) = 0.96 > 0$. Hence $\tilde{A} \succ \tilde{B}$

Remark 2: $\tilde{A} = (a, b, c, d: w_1)_{p_1}$ and $\tilde{B} = (a, b, c, d: w_2)_{p_2}$ be non-normal p_1-norm and non-norm trapezoidal fuzzy numbers respectively. For w = min (w_1 , w_2)

i. $d(\tilde{A}, \tilde{B}) > 0$ iff $d(\tilde{A}, 0_1) > d(\tilde{B}, 0_1)$ iff $\tilde{A} \succ \tilde{B}$ provided a – b - c + d > 0 and $p_1 < p_2$

ii. $d(\tilde{A}, \tilde{B}) < 0$ iff $d(\tilde{A}, 0_1) < d(\tilde{B}, 0_1)$ iff $\tilde{A} \prec \tilde{B}$ provided a – b - c + d < 0 and $p_1 > p_2$ or a – b - c + d > 0 and $p_1 < p_2$

iii $d(\tilde{A}, \tilde{B}) = 0$ iff $d(\tilde{A}, 0_1) = d(\tilde{B}, 0_1)$ iff $\tilde{B} \approx \tilde{A}$ provided a – b - c + d = 0 whatever may be p_1 and p_2

Example 4.4: Let $\tilde{A} = (6, 7, 8, 10: 0.4)_2$ and $\tilde{B} = (6, 7, 8, 10: 0.6)_1$ be non-normal 2-norm trapezoidal fuzzy number and non-normal trapezoidal fuzzy number respectively.
w = min (0.4, 0.6) = 0.4 and $p_1 > p_2$; a – b – c + d = 1>0

$d(\tilde{A}, 0_1) = 3.16$ and $d(\tilde{B}, 0_1) = 3.1$ (i.e.) $d(\tilde{A}, 0_1) > d(\tilde{B}, 0_1)$
Now $d(\tilde{A}, \tilde{B}) = d(\tilde{A}, 0_1) - d(\tilde{B}, 0_1) > 0$. Hence $\tilde{A} \succ \tilde{B}$.

Example 4.5: Let $\tilde{A} = (5, 7, 8, 9: 0.4)_1$ and $\tilde{B} = (5, 7, 8, 9: 0.6)_2$ be non-normal trapezoidal fuzzy numbers and non-normal 2-norm trapezoidal fuzzy numbers respectively.
w = min (0.4, 0.6) = 0.4 and $p_1 < p_2$; a – b – c + d = - 1< 0

$d(\tilde{A}, 0_1) = 2.9$ and $d(\tilde{B}, 0_1) = 2.843$

(i.e.) $d(\tilde{A}, 0_1) > d(\tilde{B}, 0_1)$

Table 1.Comparison of the Ranking results for different methods

Methods	Set 1		Set 2		Set 3		Set 4		
	$\tilde{A}$	$\tilde{B}$	$\tilde{A}$	$\tilde{B}$	$\tilde{A}$	$\tilde{B}$	$\tilde{A}$	$\tilde{B}$	$\tilde{C}$
Cheng's method (1998)	0.5831	0.7071	0.461	0.5831	0.5831	0.5831	0.68	0.7257	0.7462
Chu's method (2002)	0.15	0.25	0.12	0.15	-0.15	0.15	0.2281	0.2624	0.2784
Murakami et al.'s Method (1983)	0.3	0.5	0.3	0.3	-0.3	0.3	0.44	0.5333	0.525
Yager's method (1978)	0.3	0.5	0.3	0.3	-0.3	0.3	0.44	0.5333	0.525
Chen-and –Chen's Method (2007)	0.4456	0.4884	0.3565	0.4456	0.4456	0.7473	0.3719	0.4155	0.3979
S.M.Chen.,J.H.Chen Method (2009)	0.2579	0.4298	0.2063	0.2579	-0.257	0.2579	0.3354	0.4079	0.4196
Proposed Method	0.3	0.5	0.24	0.24	-0.3	0.3	0.45	0.525	0.55

Now d ($\tilde{A}$, $\tilde{B}$) = d ($\tilde{A}$,0_1) - d($\tilde{B}$, 0_1) > 0. Hence $\tilde{A} \succ \tilde{B}$.

Example 4.6: Let $\tilde{A}$ = $(5, 7, 8, 9: 0.4)_2$ and $\tilde{B}$ = $(5, 7, 8, 9: 0.6)_1$ be non-normal 2-norm trapezoidal fuzzy numbers and non-normal trapezoidal fuzzy numbers respectively.
w = min (0.4, 0.6) = 0.4 and $p_1 > p_2$; a – b – c + d = - 1< 0

d ($\tilde{A}$,0_1) = 2.843 and d($\tilde{B}$, 0_1) = 2.9

(i.e.) d($\tilde{A}$, 0_1) < d($\tilde{B}$, 0_1)

Now d ($\tilde{A}$, $\tilde{B}$) = d ($\tilde{A}$,0_1) - d($\tilde{B}$, 0_1) < 0. Hence $\tilde{A} \prec \tilde{B}$.

Example 4.7: Let $\tilde{A}$ = $(6, 7, 8, 10: 0.4)_1$ and $\tilde{B}$ = $(6, 7, 8, 10: 0.6)_2$ be non-normal trapezoidal fuzzy numbers and non-normal 2-norm trapezoidal fuzzy numbers respectively.
w = min (0.4, 0.6) = 0.4 and $p_1 < p_2$; a – b – c + d > 0

d ($\tilde{A}$,0_1) = 3.1 and d($\tilde{B}$, 0_1) = 3.16 (i.e.) d($\tilde{A}$, 0_1) < d($\tilde{B}$, 0_1)

Now d ($\tilde{A}$, $\tilde{B}$) = d ($\tilde{A}$,0_1) - d($\tilde{B}$, 0_1) < 0. Hence $\tilde{A} \prec \tilde{B}$.

Example 4.8: Let $\tilde{A}$ = $(5, 7, 8, 10: 0.4)_1$ and $\tilde{B}$ = $(5, 7, 8, 10: 0.6)_2$ be non-normal trapezoidal fuzzy numbers and non-normal 2-norm trapezoidal fuzzy numbers respectively.
w = min (0.4, 0.6) = 0.4 and $p_1 > p_2$; a – b – c + d = 0

d ($\tilde{A}$,0_1) = 3 and d($\tilde{B}$, 0_1) = 3 (i.e.) d($\tilde{A}$, 0_1) = d($\tilde{B}$, 0_1)

Now d ($\tilde{A}$, $\tilde{B}$) = d ($\tilde{A}$,0_1) - d($\tilde{B}$, 0_1) = 0. Hence $\tilde{A} \approx \tilde{B}$.

5. COMPARISON OF THE RANKING RESULTS FOR DIFFERENT METHODS[12]:

Set 1: $\tilde{A}$ =(0.1,0.3,0.3,0.5: 1) $\tilde{B}$ = (0.3,0.5,0.5,0.7:1);

Set 2: $\tilde{A}$ =(0.1,0.3,0.3,0.5: 0.8) $\tilde{B}$ = (0.1,0.3,0.3,0.5:1);

Set 3: $\tilde{A}$ = (-0.5,-0.3,-0.3,-0.1: 1) $\tilde{B}$ = (0.1,0.3,0.3,0.5:1);

Set 4: $\tilde{A}$ =(0,0.4,0.6,0.8: 1) $\tilde{B}$ = (0.2,0.5,0.5,0.9:1)

and $\tilde{C}$ = (0.1,0.6,0.7,0.8: 1);

From the Table 1, we can see that:

1. In Set 1, for fuzzy trapezoidal numbers all the previous methods and our Proposed method gets the same Ranking order:$\tilde{A} \prec \tilde{B}$.

2. In Set 2, for the Fuzzy trapezoidal numbers Cheng's method (1998), Chu's method (2002), Murakami et al.'s method (1983), Chen – and –Chen's method (2007) and S.M.Chen, J.H.Method (2009) get an incorrect ranking order. Yager's method (1978) and the proposed method get the same ranking order : $\tilde{A} \approx \tilde{B}$. Since their methods are true only for equal heights. But our proposed method gives the correct ranking order for different heights.

3. In Set 3, for the Fuzzy trapezoidal numbers Cheng's method(1998) get incorrect ranking order. Chu's method (2002), Murakami et al.'s method (1983), Chen – and –Chen's method (2007) and S.M.Chen, J.H.Method (2009), Yager's method (1978) and The Proposed method get the same ranking order: $\tilde{A} \prec \tilde{B}$.

4. In Set 4, for the Fuzzy trapezoidal numbers Cheng's method (1998), Chu's method (2002), S.M.Chen, J.H.Method (2009) and The Proposed method get the same ranking order : $\tilde{C} \succ \tilde{B} \succ \tilde{A}$. But Murakamiet al.'s method(1983), Chen – and –Chen's method (2007) and Yager's method (1978) get the same ranking order: $\tilde{B} \succ \tilde{C} \succ \tilde{A}$. Our proposed method considers the fact that it is suitable for fuzzy number with different heights and also spreads of the numbers.

6. CONCLUSION:

In this paper a modified approach is proposed for the ranking of non-normal p-norm trapezoidal fuzzy numbers proposed by Yao and Wu [9]. For the validation of the results of the proposed approach some numerical examples are illustrated. Our modified approach satisfies the reasonable properties for the ordering of fuzzy quantities given by Wang and Kerre[7].

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