

Reliability and Cost-Benefit Analysis of a Single Unit System with Degradation and Inspection at Different Stages of Failure subject to Weather Conditions

M. S. Kadyan

Department of Statistics & O.R.,
Kurukshetra University, Kurukshetra (India)

Promila

BPS Mahila Poltechnic Institute, BPS Mahila
Vishwavidalaya, Khanpur Kalan (India)

ABSTRACT

The main object of this paper is to develop a reliability model of a single-unit system operating under two weather conditions-normal and abnormal. There is a single server who visits the system immediately whenever needed and plays the dual role of inspection and repair. The unit does not work as new after repair at complete failure and so called the degraded unit. The unit is inspected at its partial failure to know the possibility of on-line repair as well as at its complete and degraded failure stages to reveal the feasibility of repair. Repair and inspection activities are stopped in abnormal weather while system remains operative. The rate of change of weather conditions and failure rates of the units are exponentially distributed whereas the inspection time and repair time distributions are taken as general. Various expressions for reliability and cost-benefit measures are derived using regenerative point technique. The numerical results for a particular case are also obtained to depict the behavior of mean time to system failure (MTSF), availability and profit of the system graphically.

Keywords: Single Unit System, Degradation, Inspection, Weather conditions and Cost-Benefit Analysis.

2000 Mathematics Subject Classification: Primary 90 B25 and Secondary 60K10

1. INTRODUCTION

Recently, the reliability models of single-unit systems operating under different weather conditions have been proposed by the researchers including Nakagawa and Osaki [1], Chander and Bansal [3], Malik and Barak [5] and Renbin and Zaiming [8] considering the concepts of different failure modes, inspection, on-line repair, replacement of the components at certain levels of damages, immediate arrival of the server, random appearance and disappearance of the server from the system in normal mode. Most of these models have been analyzed in detail using regenerative point technique under the assumptions that

- (i) Operation of the system are not possible in abnormal weather
- (ii) Unit works as new after repair.
- (iii) Repair of the unit is always feasible.

But, in real life, these assumptions are not always true. It is observed that whenever operation of the system is stopped due to abnormal weather, the system may have increased down time and therefore suffers a loss. But this does not mean that this loss cannot be minimized, it can be done by operating the system under appropriate care of the server in abnormal weather.

The unit may have increased failure rate after repair if it is repaired by an ordinary server and thus called a degraded unit.

Also, sometimes repair of the degraded unit is not feasible due to its excessive use and increased cost of maintenance. In such cases, the failed degraded unit may be replaced by new unit in order to avoid the unnecessary expenses of repair and this can be revealed by inspection.

In view of the above facts, here a reliability model is developed for a single-unit system operating under two weather conditions – normal and abnormal. There is a single server who visits the system immediately whenever needed and plays the dual role of inspection and repair. The unit does not work as new after repair at complete failure and so called the degraded unit. The unit is inspected at its partial failure to know the possibility of on-line repair as well as at its complete and degraded failure stages to reveal the feasibility of repair. Repair and inspection activities are stopped in abnormal weather while system remains operative. The rate of change of weather conditions and failure rates of the unit are exponentially distributed whereas the inspection time and repair time distributions are taken as general. To make cost-benefit analysis, the expressions for reliability and economic measures are derived using regenerative point technique. The numerical results for a particular case are also obtained to depict the behavior of MTSF, availability and profit of the system model graphically.

2. Notations:

| | | |
|-----------------------------------|---|--|
| E | - | Set of regenerative states. |
| N_0/D_0 | - | New unit/Degraded unit is operative. |
| $P_{wi}/P_{ui}/P_{ur}/P_{urd}$ | - | New unit is partially failed and operative but waiting for inspection/under on-line inspection/under on-line repair/ under repair in down state. |
| P_{wr}/P_{wrd} | - | New unit is partially failed and operating but waiting for repair/waiting for repair in down state due to abnormal weather |
| $F_{ui}/F_{wi}/F_{ur}/F_{wr}$ | - | New Unit is completely failed and under inspection/waiting for inspection/under repair/waiting for repair. |
| $DF_{ui}/DF_{wi}/DF_{ur}/DF_{wr}$ | - | Degraded unit failed and under inspection /waiting for inspection/under repair/ waiting for repair. |
| $r_1/r_2/r_3$ | - | The constant failure rate of the normal unit/partially failed unit/degraded unit. |
| $g(t)/g_1(t)/g_2(t)$ | - | Repair rate of the normal unit after complete failure/partial failure/degraded unit |

- β/β_1 - Constant rate of change of weather form normal to abnormal/abnormal to normal
- $ah(t)/bh(t)$ - Rate of change of partially failed unit under inspection to see the feasibility under on-line repair / under repair in down state
- $p_1h_1(t)/q_1h_1(t)$ - Rate of change of degraded failed unit under inspection to see the feasibility of repair / replacement
- $p_2h_2(t)/q_2h_2(t)$ - Rate of change of complete failed unit under inspection to see the feasibility of repair / replacement
- $q_{ij}(t)/Q_{ij}(t)$ - Probability density function (pdf) and cumulative distribution function (cdf) of first passage time from regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in $(0, t]$
- m_{ij} - The unconditional mean time taken by the system to transit from any regenerative state S_i when it (time) is counted from epoch of entrance in to the state S_j . Mathematically, it can be written as $m_{ij} = \int_0^{\infty} t d[Q_{ij}(t)] = -q_{ij}'(0)$
- μ_i - Mean sojourn time in state S_i which is given by $\mu_i = E(T) = \int P(T_i > t) dt = \sum_j m_{ij}$, where T denotes the time to system failure.
- $M_i(t)$ - Probability that the system initially up in the regenerative state S_i is up at time t without passing through any other regenerative state
- $W_i(t)$ - Probability that the server is busy at an instant t , given that the system entered into the regenerative state S_i at $t = 0$
- ®/© - Symbol of Laplace Stieltjes Convolution/ Laplace convolution.
- **/* - Symbols for Laplace Stieltjes transform(LST)/ Laplace transform(LT)

The possible transition states along with transition rates for the model are shown in figure 1.

| | | | | | | | |
|----------|----------|-----------|----------|-----------|-----------|-----------|-----------|
| S_0 | S_1 | S_2 | S_3 | S_4 | S_5 | S_6 | S_7 |
| N_0 | P_{ui} | P_{urd} | P_{ur} | P_{wi} | P_{wrd} | P_{wr} | F_{ui} |
| S_8 | S_9 | S_{10} | S_{11} | S_{12} | S_{13} | S_{14} | S_{15} |
| F_{wi} | F_{ur} | F_{wr} | D_0 | DF_{ui} | DF_{ur} | DF_{wi} | DF_{wr} |

3. Transition probabilities and mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements

$$p_{ij} = Q_{ij}(\infty) = \int q_{ij}(t) dt \text{ as}$$

$$\begin{aligned}
 P_{01} = P_{52} = P_{87} = P_{10,9} = P_{11,12} = P_{14,12} = P_{15,13} = 1 \\
 p_{12} = bh^*(\beta + r_2), \quad p_{13} = ah^*(\beta + r_2) \\
 p_{14} = \frac{\beta[1 - h^*(\beta + r_2)]}{\beta + r_2}, \quad p_{17} = \frac{r_2[1 - h^*(\beta + r_2)]}{(\beta + r_2)}, \\
 p_{20} = g_1^*(\beta), \quad p_{25} = [1 - g_1^*(\beta)], \\
 p_{36} = \frac{\beta[1 - g_1^*(\beta + r_2)]}{(\beta + r_2)}, \quad p_{30} = g_1^*(\beta + r_2) \\
 p_{37} = \frac{r_2[1 - g_1^*(\beta + r_2)]}{(\beta + r_2)}, \quad p_{41} = \frac{\beta_1}{\beta_1 + r_2}, \\
 p_{48} = \frac{r_2}{\beta_1 + r_2}, \quad p_{63} = \frac{\beta_1}{\beta_1 + r_2}, \\
 p_{68} = \frac{r_2}{\beta_1 + r_2}, \quad p_{7,0} = q_2 h_2^*(\beta) \\
 p_{7,9} = p_2 h_2^*(\beta), \quad p_{78} = [1 - h_2^*(\beta)], \\
 p_{9,10} = [1 - g^*(\beta)], \quad p_{9,11} = g^*(\beta), \\
 p_{12,0} = q_1 h_1^*(\beta), \quad p_{12,13} = p_1 h_1^*(\beta) \\
 p_{12,14} = [1 - h_1^*(\beta)], \quad p_{13,11} = g_2^*(\beta), \\
 p_{13,15} = [1 - g_2^*(\beta)], \tag{1}
 \end{aligned}$$

It can be easily verified that

$$\begin{aligned}
 P_{01} = P_{12} + P_{13} + P_{14} + P_{17} = P_{20} + P_{25} = P_{30} + P_{36} + P_{37} = P_{41} + P_{48} \\
 = P_{52} = P_{63} + P_{68} = P_{70} + P_{78} + P_{79} = P_{87} = P_{9,10} + P_{9,11} = P_{10,9} \\
 = P_{11,12} = P_{12,0} + P_{12,13} + P_{12,14} = P_{13,11} + P_{13,15} = P_{14,12} = P_{15,13} = 1 \tag{2}
 \end{aligned}$$

The mean sojourn times μ_i in the state S_i is given by

$$\begin{aligned}
 \mu_0 = \int_0^{\infty} P(T > t) dt = \frac{1}{r_1} \\
 \mu_1 = \left[\frac{1 - h^*(\beta + r_2)}{(\beta + r_2)} \right], \quad \mu_2 = \left[\frac{1 - g_1^*(\beta)}{\beta} \right] \\
 \mu_3 = \left[\frac{1 - g_1^*(\beta + r_2)}{(\beta + r_2)} \right], \quad \mu_4 = \frac{1}{\beta_1 + r_2}, \\
 \mu_5 = \frac{1}{\beta_1} = \mu_8 = \mu_{10} = \mu_{14} = \mu_{15}, \quad \mu_6 = \frac{1}{\beta_1 + r_2}, \\
 \mu_7 = \left[\frac{1 - h_2^*(\beta)}{\beta} \right], \quad \mu_9 = \frac{1 - g^*(\beta)}{\beta}, \\
 \mu_{11} = \frac{1}{r_3}, \quad \mu_{12} = \frac{1 - h_1^*(\beta)}{\beta},
 \end{aligned}$$

$$\mu_{13} = \left[\frac{1 - h_2^*(\beta)}{\beta} \right]. \quad (3)$$

The unconditional mean time taken by the system to transit from any regenerative. State S_i when time is counted from epoch of entrance into state S_j is given by

$$m_{ij} = \int t dQ_{ij}(t) = - \left[\frac{d}{ds} (Q_{ij}^{**}(s)) \right]_{s=0}$$

We have

$$\begin{aligned} \mu_0 &= m_{01}, & \mu_1 &= m_{12} + m_{13} + m_{14} + m_{17}, \\ \mu_2 &= m_{20} + m_{25}, & \mu_3 &= m_{30} + m_{36} + m_{37}, \\ \mu_4 &= m_{41} + m_{48}, & \mu_5 &= m_{52}, \\ \mu_6 &= m_{63} + m_{68}, & \mu_7 &= m_{70} + m_{78} + m_{79}, \\ \mu_8 &= m_{87}, & \mu_9 &= m_{9,10} + m_{9,11}, \\ \mu_{10} &= m_{10,9} & \mu_{12} &= m_{12,0} + m_{12,13} + m_{12,14}, \\ \mu_{11} &= m_{11,12} & \mu_{13} &= m_{13,11} + m_{13,15} \\ \mu_{14} &= m_{14,12} & \mu_{15} &= m_{15,13} \end{aligned} \quad (4)$$

4. Reliability and Mean Time to System Failure (MTSF)

Let $\phi_i(t)$ be the cdf of first passage time from the regenerative state i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relation for $\phi_i(t)$:

$$\phi_i(t) = \sum_j Q_{i,j}(t) \otimes \phi_j(t) + \sum_k Q_{i,k}(t) \quad (5)$$

Where j is an un-failed regenerative state to which the given regenerative state i can transit and k is a failed state to which the state i can transit directly.

Taking LST of above relation (5) and solving for $\phi_0^*(s)$,

we have

$$R^*(s) = \frac{1 - \phi_0^*(s)}{s} \quad (6)$$

The reliability of the system model can be obtained by taking inverse LT of (6)

The mean time to system failure is given by

$$MTSF = \lim_{s \rightarrow 0} R^*(s) = \frac{N_{11}}{D_{11}} \quad (7)$$

where

$$\begin{aligned} N_{11} &= [Z_{22} + p_{12}(1 - p_{36}p_{63})(\mu_2 + p_{25}\mu_5)], \\ D_{11} &= [(1 - p_{36}p_{63})(1 - p_{14}p_{41}) - p_{12}(1 - p_{36}p_{63}) - p_{13}p_{30}] \\ Z_{22} &= [p_{20}(1 - p_{36}p_{63})(1 - p_{14}p_{41})\mu_0 + p_{20}(1 - p_{36}p_{63})(\mu_1 + p_{14}\mu_4) \\ &\quad + p_{13}p_{20}(\mu_3 + p_{36}\mu_6)] \end{aligned}$$

5. Steady state Availability

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state i at $t=0$. The recursive relations for $A_i(t)$ are given as

$$A_i(t) = M_i(t) + \sum_j q_{i,j}(t) \otimes A_j(t) \quad (8)$$

Where j is any successive regenerative state to which the regenerative state i can transit. We have

$$\begin{aligned} M_0 &= e^{-\eta t}, & M_1 &= e^{-(\beta+r_2)t} \bar{H}(t), \\ M_3 &= e^{-(\beta+r_2)t} \bar{G}(t), & M_4 &= e^{-(\beta_1+r_2)t}, \\ M_6 &= e^{-(\beta_1+r_2)t}, & M_{11} &= e^{-r_3 t}. \end{aligned} \quad (9)$$

Taking LT of relation (8) and solving for $A_0^*(s)$. The steady state availability can be determined as

$$A_0(\infty) = \lim_{s \rightarrow 0} s A_0^*(s) = \frac{N_{12}}{D_{12}} \quad (10)$$

where

$$\begin{aligned} N_{12} &= p_{20}p_{9,11}p_{12,0}p_{13,11}(1 - p_{78})[(1 - p_{36}p_{63})(\mu_0(1 - p_{14}p_{41}) + \mu_1 + p_{14}\mu_4) \\ &\quad + p_{13}(\mu_3 + p_{36}\mu_6)] + \mu_{11}p_{79}p_{9,11}p_{20}(1 - p_{12,14})p_{13,11} \\ &\quad [p_{13}(p_{36}p_{68} + p_{37}) + (1 - p_{36}p_{63})(p_{17} + p_{14}p_{48})] / \\ &\quad (1 - p_{36}p_{63})\{\mu_0(1 - p_{14}p_{41}) + \mu_1 + p_{14}\mu_4\} + p_{13}(\mu_3 + p_{36}\mu_6) \end{aligned}$$

and

$$\begin{aligned} D_{12} &= p_{9,11}p_{12,0}p_{13,11}(1 - p_{78})[(1 - p_{36}p_{63})(p_{20}((1 - p_{14}p_{41})\mu_0 + \mu_1 + p_{14}\mu_4) \\ &\quad + p_{12}(\mu_2 + p_{25}\mu_5)) + p_{13}p_{20}(\mu_3 + p_{36}\mu_6)] \\ &\quad + p_{20}[(1 - p_{36}p_{63})(1 - p_{12} - p_{14}p_{41}) - p_{13}p_{30}] \\ &\quad [p_{12,0}p_{13,11}(\mu_7p_{9,11} + p_{79}(\mu_9 + p_{9,10}\mu_{10} + p_{9,11}\mu_{11})) \\ &\quad + p_{9,11}(p_{12,13}(p_{13,11}\mu_{11} + \mu_{13} + p_{13,15}\mu_{15}) + p_{13,11}(\mu_{12} + p_{12,14}\mu_{14}))] \\ &\quad + \mu_8p_{9,11}p_{12,0}p_{13,11}p_{20}[p_{78}(p_{17}(1 - p_{36}p_{63}) + p_{13}p_{37}) \\ &\quad + (1 - p_{36}p_{63})p_{14}p_{48} + p_{13}p_{36}p_{68}] \end{aligned}$$

6. Busy Period Analysis

Let $B_i(t)$ be the probability that the server is busy at an instant 't' given that the system entered regenerative state i at $t=0$. The recursive relations for $B_i(t)$ are given as

$$B_i(t) = W_i(t) + \sum_j q_{i,j}(t) \otimes B_j(t) \quad (11)$$

where j is any successive regenerative state to which the regenerative state i can transit. We have

$$\begin{aligned} W_1(t) &= e^{-(\beta+r_2)t} \bar{H}(t), & W_2(t) &= e^{-\beta t} \bar{G}_1(t), \\ W_3(t) &= e^{-(\beta+r_2)t} \bar{G}_1(t), & W_7(t) &= e^{-(\beta)t} \bar{H}_2(t) \\ W_9(t) &= e^{-\beta t} \bar{G}(t), & W_{12}(t) &= e^{-\beta t} \bar{H}_1(t), \\ W_{13}(t) &= e^{-\beta t} \bar{G}_2(t). \end{aligned} \quad (12)$$

Taking LT of relation (11) and solving for $B_0^*(s)$. The busy period of the server can be obtained as.

$$B_0 = \lim_{s \rightarrow 0} s B_0^*(s) = \frac{N_{13}}{D_{12}} \quad (13)$$

Where

$$N_{13} = P_{9,1}P_{13,1}P_{12,0}(1-p_{78})[(1-p_{36}p_{63})(p_{20}W_1 + p_{12}W_2) + p_{20}p_{13}W_3] + p_{20}[p_{13}(p_{36}p_{68} + p_{37})(p_{17} + p_{14}p_{48})] \\ + [p_{13,1}p_{12,0}(p_{9,1}W_7 + p_{79}W_9) + p_{79}p_{9,1}(p_{13,1}W_{12} + p_{12,13}W_{13})]$$

and D_{12} is already specified.

7. Expected number of visits by the server

Let $N_i(t)$ be the expected number of visits by the server in $(0, t]$ given that the system entered the regenerative state i at $t=0$, we have the following recurrence relations for $N_i(t)$:

$$N_i(t) = \sum_j Q_{i,j}(t) \otimes [\delta_j + N_j(t)] \quad (14)$$

Where j is any regenerative state to which the given regenerative state i transits and $\delta_j=1$, if j is the regenerative state where the server does job afresh, otherwise $\delta_j=0$.

Taking LT of the relation (14) and solving for $N_0^{**}(s)$. The expected number of visits per unit time is given by

$$N_0 = \lim_{s \rightarrow 0} s N_0^{**}(s) = \frac{N_{14}}{D_{12}}, \quad (15)$$

Where

$$N_{14} = P_{20}P_{9,1}P_{12,0}P_{13,1}(1-p_{78})(1-p_{36}p_{63})(1-p_{14}p_{41}) + p_{79}p_{9,1}P_{20}P_{13,1}(1-p_{12,14})[p_{13}(p_{36}p_{68} + p_{37}) + (1-p_{36}p_{63})(p_{17} + p_{14}p_{48})]$$

and D_{12} is already specified.

8. Cost-Benefit Analysis

Profit incurred to the system model is given by

$$P = K_0A_0 - K_1B_0 - K_2N_2$$

where K_0 = fixed revenue per unit up time of the system.

K_1 = fixed cost per unit up time for which server is busy.

K_2 = fixed cost per visit by the server.

Particular Case

To show the importance of results and characterize the behavior of MTSF, availability and profit of the system, here we assume that repair times of the units and inspection times are exponential distributed. Probability density function of exponential distribution is given by

$$f(t) = \lambda e^{-\lambda t}$$

$$\text{Suppose } \begin{aligned} g(t) &= \theta e^{-\theta t} & g_1(t) &= \theta_1 e^{-\theta_1 t} \\ g_2(t) &= \theta_2 e^{-\theta_2 t} & h(t) &= \alpha e^{-\alpha t} \\ h_1(t) &= \alpha_1 e^{-\alpha_1 t} & h_2(t) &= \alpha_2 e^{-\alpha_2 t} \end{aligned}$$

9. CONCLUSION

The behaviour of mean time to system failure (MTSF) and availability of the system is shown in fig. 2 and fig. 3. These figures reveal that MTSF and availability decrease with the increase of abnormal weather rate (β) while their values increase by increasing the normal weather rate (β_1) and repair rate (θ_1) of partially failed unit. It may be noted that their values decrease by increasing the failure rates r_1 and r_2 . Fig. 4 indicates that profit of the system goes on decreasing with the increase of abnormal weather rate (β) for fixed values of other parameters. There is an increase in the value of profit of the system in case normal weather rate (β_1) and repair rate (θ_1) increase. Profit of the system decreases with the increase of failure rates r_1 and r_2 . On the basis of the results obtained for a particular case, it is concluded that the system model can be made more reliable and profitable to use by increasing the repair rate of the unit at its partial failure and replacement of the degraded unit at its failure by the new unit.

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State-Transition Diagram

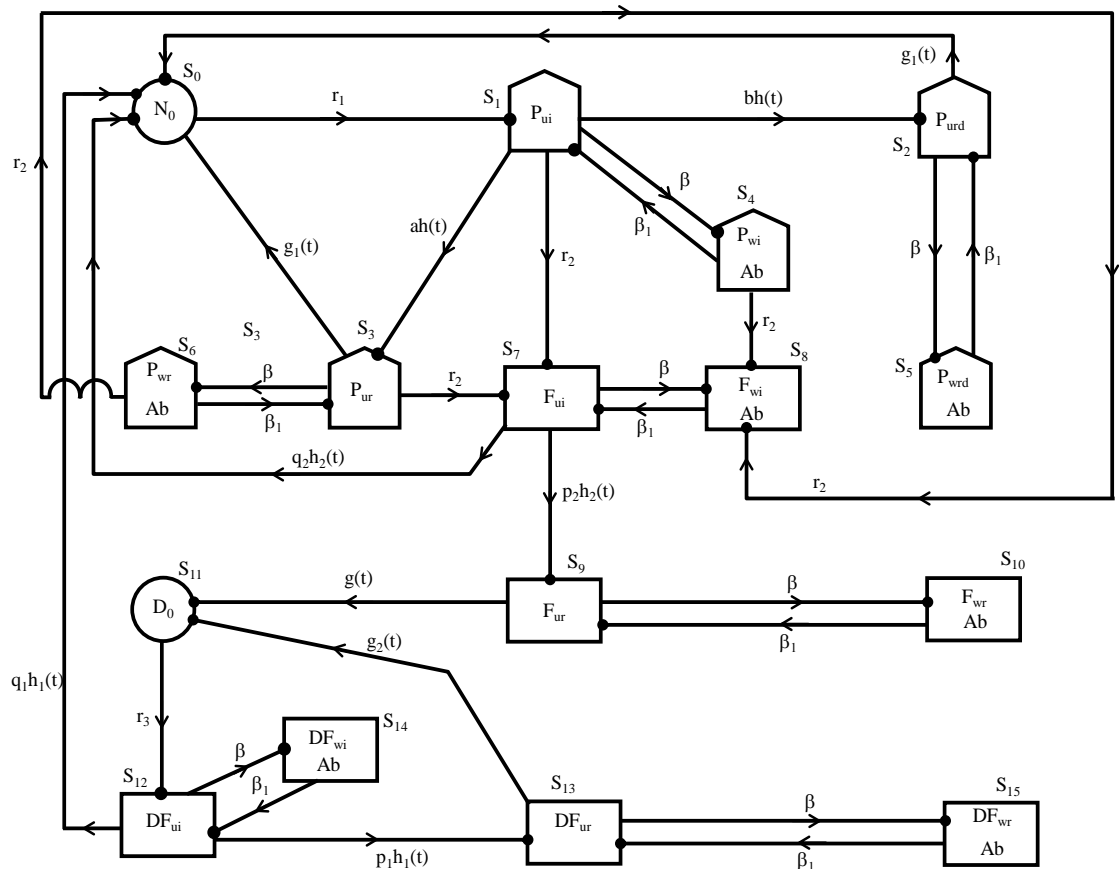


Fig.-1

