A New Class sc*g-Set Weaker Form of Closed Sets in Topological Spaces

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ABSTRACT
In this paper, we have introduced a new classes of closed sets, as weaker forms of closed sets namely sc*g-closed sets and continuous functions in topological spaces.

Keywords
scg-closed, scg- continuous functions.

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1. INTRODUCTION

In 1970, Levine[1] first considered the concept of generalized closed(briefly,g-closed) sets were defined and investigated. Negaveni[10]investigated weakly generalized closed sets(sg-closed sets). Tong [8] and Hatir et al [4] introduced B-sets and t-sets and α*-sets respectively. t-sets and α*-sets are weak forms of open sets. In this paper, we have introduced a new class of sets called scg-closed, sc*g-closed,sc(s) g-closed Sets and study some of their properties.

2. PRELIMINARIES

Definition: 2.1 A subset of a topological space(X, τ) is called

(i) Generalized closed(g-closed)[1] if cl(A) $\subseteq$ U whenever A $\subseteq$ U, and U is open in X.

(ii)Semi-generalizedclosed(sg-closed)[9]if scl(A) $\subseteq$ U whenever A $\subseteq$ U, and U is semi open in X.

(iii) Generalized semi preclosed[12]if spcl(A) $\subseteq$ U whenever A $\subseteq$ U, and U is open in X.

(iv) Weakly generalized closed[10] if cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U, and U is open in X.

Definition: 2.2 For a subset A of X, the preclosure of A[11], denoted by pcl(A), is defined as the intersection of all preclosed sets containing A in X and the preinterior of A[11], denoted by pint(A), is defined as the union of the preopen sets contained in A in X. semi closure of A[16], denoted by scl(A), semi interior of A, denoted by sint(A) are defined similarly.

Result: 2.3 For a subset S of X

(a) pcl(A) = A U cl(int(A))[11]
(b) pint(A) = A $\cap$ int(cl(A))[11]
(c) scl(A) = A U int(cl(A))[17]
(d) sint(A) = A $\cap$ cl(int(A))[17]

Definition: 2.4 For a subset S of X is called

(a) a t-set in X[8] if int(A) = int(cl(A)),
(b) a B-set in X[8] if A = G $\cap$ F where G is open and F is a t-set in X,
(c) an α*-set in X[4] if int(A) = int(cl(int(A))),
(d) a C-set(Due to Sundaram)[7] if A= G $\cap$ F where G is g-open and F is a t-set in X,
(e) a C-set (Due to Hatir, Noiri and Yuksel)[4] if A = G $\cap$ F where G is open and F is an α*-set in X,
(f) a C*-set in X[5] if A= G $\cap$ F where G is g-open and F is a α*-set in X,

Definition: 2.5 A map f: X →Y is called

(a) semicontinuous[1] if f^{-1}(F) is semiclosed in X for each closed set F in Y,
(b) generalized continuous (g-continuous)[14] if f^{-1}(F) is g-closed in X for each closed set F in Y,
(c) α-generalized continuous (αg-continuous)[15] if f^{-1}(F) is αg-closed in X for each closed set F in Y,
(d) generalized semicontinuous (gs-continuous)[6] if f^{-1}(F) is gs-closed in X for each closed set F in Y,
(e) closed map if for each closed set F in X, f(F) is a closed set in Y.
(f) open map if for each open set F in X, f(F) is a open set in Y.

3. sc*g SETS IN TOPOLOGICAL SPACES

Definition: 3.1 A subset A of X is called

(i) a scg-closed set if scl(A) $\subseteq$ U whenever, A $\subseteq$ U and U is C-set in X.

Remark : 3.2

(i) The complement of scg-closed set is scg-open set in X.

Theorem : 3.3 Every closed set in X is sc*g-closed set in X but not conversely.

Proof :- Let A be a closed set in X. Let U be a sc*g-closed set such that A $\subseteq$ U. Since A is closed, cl(A) = A, cl(A) $\subseteq$ U. But scl(A) $\subseteq$ cl(A) $\subseteq$ U. Hence scl(A) $\subseteq$ U. Therefore A is sc*g-closed set in X.

The converse of the above theorem need not be true as seen from the following example.

Example : 3.4 Consider the topological space X = {a,b,c} with the topology τ = { φ ,X,{a, b}}.

The set {a, b} is sc*g-closed sets but not a closed set in X.

Theorem : 3.5 The Union of two sc*g-closed sets is sc*g-closed in X.

Proof :- Let A and B be sc*g-closed sets in X. Let U be a sc*g-closed set in X such that AUB ⊆ U. Then A ⊆ U and B ⊆ U. Since A and B are sc*g-closed set, scl(A) ⊆ U and scl(B) ⊆ U. Hence scl(AUB) = scl(A) U scl(B) ⊆ U.

Therefore AUB is sc*g-closed in X.

Theorem : 3.6 Every g-closed set in X is sc*g-closed set in X but not conversely.

Proof :- Let A be a g-closed set in X. Let A ⊆ U, U is a sc*g-closed set, and U is a C* set,

U = P I Q where P is g-open, Q is α*-set. Hence U ⊆ P and U ⊆ Q, A ⊆ U ⊆ P, P is g-open. Now, cl(A) ⊆ P, scl(A) ⊆ cl(A) ⊆ U ⊆ P. Hence scl(A) ⊆ U. Therefore A is sc*g-closed set in X.

The converse of the above theorem need not be true as seen from the following example.

Example : 3.7 Consider the topological space X = {a,b,c} with the topology τ = { φ ,X,{a, b}}. The set {a, b} is sc*g-closed set but not a g-closed set in X.

Theorem : 3.8 Every gs-closed set in X is sc*g-closed set in X but not conversely.

Proof :- Let A be a gs-closed set in X. Let A ⊆ U, and U is a C* set, U =P I Q where P is g-open, Q is α*-set. Therefore U ⊆ P and U ⊆ Q, A ⊆ U ⊆ P,

P is g-open. Now, cl(A) ⊆ P, scl(A) ⊆ cl(A) ⊆ U ⊆ P. Hence scl(A) ⊆ U. Therefore A is sc*g-closed set in X.

The converse of the above theorem need not be true as seen from the following example.

Example : 3.9 Consider the topological space X = {a,b,c} with the topology τ = { φ ,X,{a, b}}. The set {a, b} is sc*g-closed set but not a gs-closed set in X.

Theorem : 3.10 Every αg-closed set in X is sc*g-closed set in X but not conversely.

Proof :- Let A be a αg-closed set in X. Let A ⊆ U, Since U is a C* set, U =P I Q where P is g-open, Q is α*-set. Therefore U ⊆ P and U ⊆ Q, A ⊆ U ⊆ P, P is g-open. Now, cl(A) ⊆ P, scl(A) ⊆ cl(A) ⊆ U ⊆ P. Hence scl(A) ⊆ U.

Therefore A is sc*g-closed set in X.

The converse of the above theorem need not be true as seen from the following example.

Example : 3.11 Consider the topological space X = {a,b,c} with the topology τ = { φ ,X,{a, b}}. The set {a, b} is sc*g-closed set but not a αg-closed set in X.

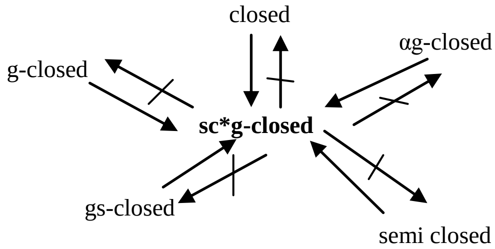
Theorem : 3.12 Every semi closed set in X is sc*g-closed set in X but not conversely.

Proof :- Let A be a semi closed set scl(A) ⊆ U, A ⊆ U, U is a C* set.Since cl(int(A)) ⊆ U. Therefore AU cl(int(A)) ⊆ U. Hence scl(A) ⊆ U. Therefore A is sc*g-closed set in X.

The converse of the above theorem need not be true as seen from the following example.

Example : 3.13 Consider the topological space X = {a,b,c} with the topology τ = { φ ,X,{a, b}}. The set {a, b} is sc*g-closed set but not semi closed set in X.

We illustrate the relations between various sets in the following diagram:



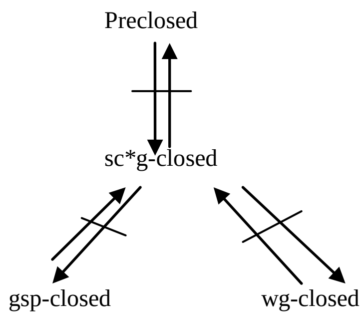
In the above diagram none of the implications can be reversed.

Example : 3.15 Consider the topological space X = {a,b,c} with the topology τ = { φ ,X,{a, b}}. In this topology {a}, {b} are preclosed set but not sc*g-closed set and the set {a, b}is sc*g-closed set but not a preclosed set.

Example : 3.16 Consider the topological space X = {a,b,c} with the topology τ = { φ ,X,{a, b}}. In this topology {a}, {b} are gsp-closed set but not sc*g-closed set and the set {a, b}is sc*g-closed set but not a gsp-closed set.

Example : 3.17 Consider the topological space X = {a,b,c} with the topology τ = { φ ,X,{a, b}}. In this topology {a}, {b} are wg-closed set but not sc*g-closed set and the set {a, b}is sc*g-closed set but not a wg-closed set.

Remark : 3.14 From the above examples we obtain the following diagram with independent sets.



4. sc*g CONTINUOUS MAPPINGS IN TOPOLOGICAL SPACES

In this section we introduce αcg continuous mappings.

Definition : 4.1 A map f : X →Y from a topological space X in to a topological space Y is called sc*g-continuous if the inverse image of every closed set in Y is sc*g-closed in X.

Theorem : 4.2 If a map f : X →Y is continuous, then it is sc*g-continuous but not conversely.

Proof: Let f : X →Y be a continuous map. Let F be any closed set in Y. Then the inverse image $f^{-1}(F)$ is closed in Y. Since every closed set is sc*g-closed set, $f^{-1}(F)$ is sc*g-closed set in X. Therefore, f is sc*g-continuous.

The converse of the above theorem need not be true as seen from the following example.

Example : 4.3 Let X = Y = {a,b,c} , τ = { φ ,X,{a, b}}, σ = { φ ,Y,{a}}. Let f : (X, τ) →(Y, σ) be the identity map. Then f is sc*g-continuous but not continuous. For, the closed set {b, c} in Y , but f^{-1}({b, c}) = {b, c} is not closed in X. Therefore, f is not continuous.

Theorem : 4.4 If a map f : X →Y is g-continuous, then it is sc*g-continuous but not conversely.

Proof: Let f : X →Y be a g-continuous map. Let F be any g-closed set in Y, then the inverse image $f^{-1}(F)$ is g-closed in Y. Since every g-closed set is sc*g-closed set, $f^{-1}(F)$ is sc*g-closed set in X. Therefore, f is sc*g-continuous.

The converse of the above theorem need not be true as seen from the following example.

Example : 4.5 Let X = Y = {a,b,c} , τ = { φ ,X,{a, b}}, σ = { φ ,Y,{c}}. Let f : (X, τ) →(Y, σ) be the identity map. Then f is sc*g-continuous but not g-continuous. For, the closed set {a, b} in Y, but f^{-1}({a, b}) = {a, b} is not g-closed in X. Therefore, f is not g-continuous.

Theorem : 4.6 If a map f : X →Y is αg-continuous, then it is sc*g-continuous but not conversely.

Proof: Let f : X →Y be a αg-continuous map. Let F be any αg-closed set in Y, then the inverse image $f^{-1}(F)$ is αg-closed in Y. Since every αg-closed set is s*cg-closed set, $f^{-1}(F)$ is sc*g-closed set. Therefore, f is sc*g-continuous.

The converse of the above theorem need not be true as seen from the following example.

Example : 4.7 Let X = Y = {a,b,c} , τ = { φ ,X,{a, b}}, σ = { φ ,Y,{c}}. Let f : (X, τ) →(Y, σ) be the identity map. Then f is sc*g-continuous but not αg-continuous. For, the closed set {a, b} in Y, but f^{-1}({a, b}) = {a, b} is not αg-closed in X. Therefore, f is not αg-continuous.

Theorem : 4.8 If a map f : X →Y is semi continuous, then it is sc*g-continuous but not conversely.

Proof: Let f : X →Y be a semi continuous map. Let F be any semi closed set in Y, then the inverse image $f^{-1}(F)$ is semi closed in Y. Since every semi closed set is s*cg-closed set, $f^{-1}(F)$ is sc*g-closed set. Therefore, f is sc*g-continuous.

The converse of the above theorem need not be true as seen from the following example.

Example : 4.9 Let X = Y = {a,b,c} , τ = { φ ,X,{a}}, σ = { φ ,Y,{a, b}}. Let f : (X, τ) →(Y, σ) be the identity map. Then f is sc*g-continuous but not semi continuous. For, the closed set {b, c} in Y but f^{-1}({b, c}) = {b, c} is not semi closed in X. Therefore, f is not semi continuous.

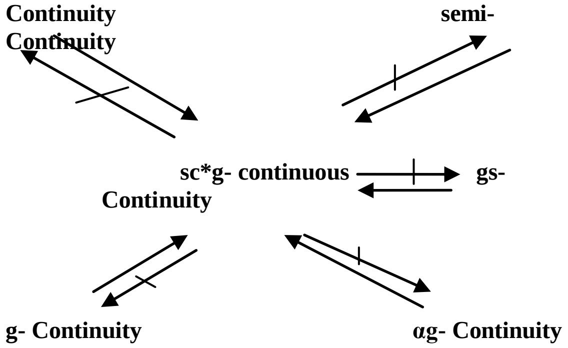
Theorem : 4.10 If a map f : X →Y is gs-continuous, then it is sc*g-continuous but not conversely.

Proof: Let f : X →Y be a gs-continuous map. Let F be any gs-closed set in Y, then the inverse image $f^{-1}(F)$ is gs-closed in Y. Since every gs-closed set is sc*g-closed set, $f^{-1}(F)$ is sc*g-closed set. Therefore, f is sc*g-continuous.

The converse of the above theorem need not be true as seen from the following example.

Example : 4.11 Let X = Y = {a,b,c} , τ = { φ ,X,{a, b}}, σ = { φ ,Y,{c}}. Let f : (X, τ) →(Y, σ) be the identity map. Then f is sc*g-continuous but not gs-continuous. For, the closed set {a, b} in Y, but f^{-1}({a, b}) = {a, b} is not g-closed in X. Therefore, f is not gs-continuous.

We illustrate the relations between various generalizations of continuous functions in the following diagram:



In the above diagram none of the implications can be reversed.

5 . sc*g-closed maps and sc*g-open maps in Topological spaces

Malghan [17] introduced and investigated some properties of generalized closed maps in topological spaces. The concept of generalized open map was introduced by Sundaram [6]. Biswas [18]defined semi open mappings as a generalization of open mappings and studied several of its properties.

In this section, we introduced the concepts of sc*g-closed maps and sc*g-open maps in topological spaces.

Definition :5.1 A map f : X →Y is called sc*g-closed map if for each closed set F in X, f(F) is a sc*g-closed set in Y.

Theorem : 5.2 If f : X →Y is a closed map, then it is sc*g-closed but not conversely.

Proof : Since every closed set is sc*g-closed the result follows.

The converse of the above theorem need not be true as seen from the following example.

Example : 5.3 Consider the topological spaces X=Y={a, b, c} with topologies τ ={ φ ,X,{a}} and σ = { φ ,Y,{a, b}}. Here C(X, τ) = { φ ,X,{b, c}}, C(Y, σ) = { φ ,Y,{c}}and sc*g-(Y, σ) = { φ ,Y,{c}, {a, b},{b, c},{a, c}}. Let f be the identity map from X onto Y. Then f is sc*g-closed but not a closed map, since for the closed set {b, c} in (X, τ), f({b, c}) = {b, c} is not a closed set in Y.

Definition : 5.4 A map f : X →Y is called a sc*g-open map if f(U) is sc*g-open in Y for every open set U in X.

Theorem : 5.5 If f : X →Y is an open map, then it is sc*g-open but not conversely.
Proof : Let f : X →Y be an open map. Let U be any open set in X. Then f(U) is an open set in Y. Then f(U) is sc*g-open, since every open set is sc*g-open. Therefore f is sc*g-open.

The converse of the above theorem need not be true as seen from the following example.

Example : 5.6 Consider the topological spaces X=Y={a, b, c} with topologies τ ={ φ ,X,{a}} and σ = { φ ,Y,{a, b}}. Here sc*g-(Y, σ) = { φ ,Y,{a}, {b},{c},{a, b}}. Then the identity function f: X →Y is sc*g-open but not open, since for the open set {a} in (X, τ), f({a}) = {a} is sc*g-open but not open in (Y, σ). Therefore f is not an open map.

Theorem : 5.7 A map f : X →Y is sc*g-closed iff for each subset S of Y and for each open set U containing $f^{-1}(S)$ there is a sc*g-open set V of Y such that S ⊆ V and $f^{-1}(V)$ ⊆ U.
Proof : Suppose f is sc*g-closed. Let S be a subset of Y and U is an open set of X such that
$f^{-1}(S)$ ⊆ U. Then V = Y – f(X–U) is a sc*g-open set containing S such that $f^{-1}(V)$ ⊆ U. Conversely, suppose that F is a closed set in X. Then f^{-1}(Y- f(F)) = X – F and X – F is open. By hypothesis, there is a sc*g-open set V of Y such that Y- f(F) ⊆ V and $f^{-1}(V)$ ⊆ X – F. Therefore F ⊆ X - $f^{-1}(V)$. Hence
Y – V ⊆ f(F) ⊆ f(X - $f^{-1}(V)$) ⊆ Y – V, which implies
f(F) = Y – V. Since Y – V is sc*g-closed and thus f is sc*g-closed map.

Theorem : 5.8 If f : X →Y is g-continuous and sc*g-closed and A is a sc*g-closed set of X, then f(A) is sc*g-closed in Y.
Proof : Let f(A) ⊆ O where O is an g-open set of Y. Since f is g-continuous, $f^{-1}(O)$ is an g-open set containing A. Hence cl(A) ⊆ $f^{-1}(O)$ as A is a sc*g-closed set. Since f is sc*g-closed f(cl(A)) is a sc*g-closed set contained in the g-open set O, which implies that cl(f(cl(A)) ⊆ O and hence cl(f(A)) ⊆ O. So f(A) is a sc*g-closed set in Y.

Corollary : 5.9 If f : X →Y is continuous and closed and A is a sc*g-closed set of X, then f(A) is sc*g-closed in Y.
Proof : Since every continuous map is g-continuous and every closed map is sc*g-closed, by above theorem the result follows.

Theorem : 5.10 If f : X →Y is a closed map and h : Y →Z is a sc*g-closed then h : Y →Z is sc*g-closed then h O f : X →Z is a sc*g-closed.
Proof : If f : X →Y is a closed map and h : Y →Z is a sc*g-closed map. Let V be any closed set in X. Since f : X →Y is closed, f(V) is closed in Y and since h : Y →Z is sc*g-closed, h(f(V)) is a sc*g-closed set in Z. Therefore h O f : X →Z is a sc*g-closed map.

Theorem : 5.11 If f : X →Y is a sc*g-closed and A is closed set in X. Then f_A : A→Y is sc*g-closed.

Proof : Let V be closed set in A. Then V is closed in X. Therefore it is sc*g-closed in X. By Theorem 5.8, f(V) is sc*g-closed. That is $f_A(V)$ = f(V) is sc*g-closed in . Therefore f_A : A→Y is sc*g-closed.

The next theorem shows that normality is preserved under continuous and sc*g-closed maps.

Theorem : 5.12 If f : X →Y is a continuous, sc*g-closed map from a normal space X onto a space Y, then Y is normal.
Proof : Let A, B be disjoint closed set Y. Then $f^{-1}(A)$, $f^{-1}(B)$ are disjoint closed sets of X. Since X is normal, there are disjoint open sets U, V in X such that $f^{-1}(A)$ ⊆ U and $f^{-1}(B)$ ⊆ V. By Theorem 5.7, and since f is sc*g-closed, there are sc*g-closed sets G, H in Y such that A ⊆ G,
B ⊆ H and $f^{-1}(G)$ ⊆ U and $f^{-1}(H)$ ⊆ V. Since U, V are disjoint, int(G) and int(H) are disjoint open sets. Since G is sc*g-open, A is closed and A ⊆ G ⟹ A ⊆ int(G). Similarly B ⊆ int(H). Hence Y is normal.

6. CONCLUSION

In this paper we have introduced sc*(s)g-closed sets and continuous functions in topological spaces and studied some of its basic properties. Also we have studied the relationship between sc*g-closed sets and continuous functions with some of the sets already exist. in topological spaces.

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