

3-Stage Specially Structured Flow Shop Scheduling to Minimize the Rental Cost, Set Up Time Separated from Processing Time Including Transportation Time

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ABSTRACT

This article describes the development of a new heuristic algorithm which guarantees an optimal solution for specially structured flow shop problem with n-jobs,3- machines, to minimize the rental cost under specified rental policy in which set up times are separated from processes time, including transformation time. Further the processing times are not merely random but bear a well defined relationship to one another. Most of literature emphasized on minimization of idle time/ make span. But minimization of make span may not always lead to minimize rental cost of machines. Objective of this work is to minimize the rental cost of machines under a specified rental policy irrespective of make span.

Keywords: Specially structured flow shop scheduling. Rental policy, Processing time, Rental cost, Transportation time, Set up time, Utilization Time.

Mathematical subject classification: 90B30, 90B35.

1. INTRODUCTION

Scheduling is a decision making practice that is used on a regular basis in manu facturing and service industries. Its aim is to optimize one or more objectives with the allocation of resources to task over given time periods.

The time that a job spends on a machine include three phases viz setup, processing and removal. In the majority of investigation dedicated to production planning and scheduling, set up time considered to be negligible. But considering set up time separate from processing time have great impact on performance measure. As when there exists idle time on the second machine than the setup time for a job on a second machine can be performed prior to the completion time of this job on the first machine. Further the transportation times (loading time, moving time and unloading etc.) from one machine to another are also not negligible and therefore must be included in the job processing. However, in some application, transportation time have major impact on the performance measures considered for the scheduling problem so they need to considered separately. In a flow shop scheduling each job has the same routing throw machines and the sequence of operations is fixed. In a specially structured flow shop scheduling the data is not merely random but bears a well defined relation with one another. Gupta J.N.D [6] studied two stage specially structured flow shop scheduling problem. The basic study of flow shop scheduling was developed by Johnson [7]. Then work was developed by Singh and Deepak [15], Pandian & Rajendran [12], Yoshida

and Hitomi [17] etc. while considering various parameters. Maggu and Das [9] consider a two machine flow shop problem with transportation time of the jobs. Yoshida and Hitomi [17] studied the optimal two stage production scheduling with setup time separated from processing time. Gupta Deepak [5] et.al. Studied three stage specially structured flow shop scheduling problems to minimize the rental cost under a specified rental policy. Present Paper extends the study made by Gupta et al [5] by introducing the transportation time and setup time

In this paper we presents a specially structured flow shop scheduling model to minimize the utilization time of the machines and hence their rental cost under specified rental policy in which the setup times are separated from processing time, including transportation time. Most of the work emphasize on minimization of make span. Here we have discussed the algorithm which shows that minimization of make span does not always lead to minimize rental cost of machines.

2. PRACTICAL SITUATION

The majority of scheduling research assumes set up as negligible or part of processing time. While this assumption adversely affects solution quality for many applications which require explicit treatment of setup, includes work to prepare the machine for processing. This includes obtaining tools, positioning work-in-process material, return tooling, cleaning up, setting the required jigs and fixtures, adjusting tools and inspecting material and hence significant.

In our day to day working in factories and industrial production concern different jobs are processed on various machines. These jobs are required to process in machines A,B,C,---- in a specified order. When the machine on which jobs are to be processed are planted at different places the transportation time (which include loading time, moving time, and unloading time etc.) has a significant role in production concern. To established a new business or a manufacturing plant or a company one needs huge amount of money to purchase various machines, due to non liquidity of funds one cannot afford to buy all the expensive machinery prefer to take on rent. Renting is an affordable and quick solution for up gradation to new technology, saving working capital and best use of limited resources.

3. NOTATIONS

- S : Sequence of jobs 1, 2, 3, ..., n
 S_k : Sequence obtained by applying Johnson's procedure, $k = 1, 2, 3, \dots, r$.
 M_j : Machine j , $j = 1, 2, 3$.
 a_{ij} : Processing time of i^{th} job on machine M_j
 s_{ij} : Set up time of i^{th} job on machine M_j
 $t_{ij}(S_k)$: Completion time of i^{th} job on machine M_j
 $t_{i1 \rightarrow 2}$: Transportation time of i^{th} job on 1st machine to 2nd machine.
 $t_{i2 \rightarrow 3}$: Transportation time of i^{th} job from 2nd machine to 3rd machine
 $U_j(S_k)$: Utilization time for which machine M_j is required.
 $R(S_k)$: Total rental cost for the sequence S_k of all machine
 C_j : Rental cost per unit time of j^{th} machine.

4. DEFINITION

Completion time of i^{th} job on machine M_j is denoted by t_{ij} and is defined as:

$$t_{ij} = \max((t_{i-1,j} + s_{i,j}) \quad t_{i,j-1} + T_{i,j-1 \rightarrow j}) + a_{ij}; \quad j \geq 2, \quad j=2, 3$$

$$t_{i1} = \max((t_{i-1,2} + s_{i,2}) \quad t_{i,1} + T_{i,1 \rightarrow 2}) + a_{i2}$$

$$t_{i1} = \max((t_{i-1,3} + s_{i,3}, \quad t_{i2} + T_{i,2 \rightarrow 3}) + a_{i3}$$

5. RENTAL POLICY (P)

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required.

6. PROBLEM FORMULATION

Let some job i ($i = 1, 2, \dots, n$) are to be processed on three machines M_j ($j = 1, 2, 3$) under the specified rental policy P. Let a_{ij} be the processing time of i^{th} job on j^{th} machine s_{ij} be the set time of its job on j th machine and t_i and g_i be transportation time of i th job from machine M_1 to M_2 and from machine M_2 to M_3 respectively such that either $\min(a_{i1} + t_{i1 \rightarrow 2} - s_{i2}) \geq \max(a_{i1} + t_{i1 \rightarrow 2} - s_{i1})$

Or $\min(a_{i3} + g_{i2 \rightarrow 3} - s_{i2}) \geq \max(a_{i2} + t_{i2 \rightarrow 3} - s_{i3})$ for all $i, j, \quad i \neq j$

Our aim is to find the sequence $\{S_k\}$ of jobs which minimize the rental cost of the machines while minimizing the utilization time of machines.

The mathematical model of the problem in matrix form can be stated as:

Jobs	Machine M_1		$t_{i1 \rightarrow 2}$ t_i	Machine M_2		$t_{i2 \rightarrow 3}$ g_i	Machine M_3	
i	a_{i1}	s_{i1}	t_1	a_{i2}	s_{i2}	g_1	a_{i3}	s_{i3}
1	a_{11}	s_{11}	t_2	a_{12}	s_{12}	g_2	A_{13}	s_{13}
2	a_{21}	s_{21}	t_3	a_{22}	s_{22}	g_3	A_{23}	s_{23}
3	a_{31}	s_{31}	t_4	a_{32}	s_{32}	g_4	A_{33}	s_{33}
-	-	-		-	-		-	-
N	a_{n1}	s_{n1}	t_n	a_{n2}	s_{n2}	g_n	a_{n3}	s_{n3}

Table -1

Mathematically, the problem is stated as:

Minimize :

$$R(S_k) = \sum_{i=1}^n A_{i1} \times c_1 + u_2(S_k) \times c_2 + u_3(S_k) \times c_3$$

Subject to constraint: Rental Policy (P) i.e. our objective is to minimize utilization time of machine and hence rental cost of machines.

7. ALGORITHM

Step 1: Check the following structural relationship.

either $a_{i1} + t_i - s_{i2} \geq a_{i2} + t_i - s_{i2}$ for all $i, j \quad i \neq j$

or $a_{i2} + g_i - s_{i2} \geq a_{i2} + g_i - s_{i3}$ or both

i.e either $(a_{i1} + t_i - s_{i2}) \geq \max(a_{i2} + t_i - s_{i2})$

or $\min(a_{i2} + g_i - s_{i2}) \geq \max(a_{i2} + g_i - s_{i3})$ or both for all $i, j, \quad i \neq j$

If the conditions are satisfied then go to step 2 else modify the data.

Step 2: convert the problem into two machine problem. Let G and H be two fictitious machines having G_i and H_i as their processing times as:

$$G_i = a_{i1} + t_i + a_{i2} + g_i$$

$$H_i = t_i + a_{i1} + g_i + a_{i3}$$

Step 3: obtain new reduced problem with processing time G_i & H_i as follow: Then $G'_i = G_i + \max(s_{i1}, s_{i2})$

$$H'_i = H_i - s_{i3}$$

Step 4: Obtain the optimal sequence S_i (Say) to minimize the make span by applying Johnson's [1954] algorithm on machine G and H with processing time G'_i and H'_i respectively

Step 5: Obtain other feasible sequences by putting 2nd, 3rd, ..., nth jobs of sequence S_1 in first position respectively and all other jobs of S_1 in same order.

Let the sequence be:

S_2, S_3, \dots, S_n .

Step 6: Compute CT (S_k); $k= 1,2,\dots,r$ by making in – out table for sequences S_k ($k= 1,2,\dots,r$).

Step 7: Calculate $\sum A_{i1}$, $U_2(S_k)$ & $U_3(S_k)$ of 1st, 2nd and 3rd machines respectively.

Step 8: Calculate

$$R(S_k) = \sum_{i=1}^n A_{i1} \times c_1 + u_2(S_k) \times c_2 + u_3(S_k) \times c_3$$

Where C_1 , C_2 and C_3 are the rental cost per unit time of machines M_1 , M_2 and M_3 respectively.

8. NUMERICAL ILLUSTRATION

Consider 5 jobs, 3 machines flow shop problem in which processing times, set up times with transportation times are given in the table. The rental cost per unit time for machines M_1 , M_2 and M_3 are 4 units, 5 units and 2 units respectively under the rental policy P.

Jobs	Machine M_1		$t_{i1 \rightarrow 2}$	Machine M_2		$t_{i2 \rightarrow 3}$	Machine M_3	
	a_{i1}	s_{i1}	t_i	a_{i2}	s_{i2}	g_i	a_{i3}	s_{i3}
i	a_{i1}	s_{i1}	t_i	a_{i2}	s_{i2}	g_i	a_{i3}	s_{i3}
1	15	6	4	18	2	1	50	3
2	18	3	2	13	1	2	43	5
3	30	2	1	20	4	5	60	1
4	11	5	3	15	4	3	35	2
5	9	1	5	25	2	1	65	4

Table :2

Solution: As per step 1: The condition

$$\max (a_{i2} + g_i - s_{i3}) \leq \min (a_{i2} + g_i - s_{i2}) \text{ for all } i, j \text{ } i \neq j$$

Satisfies

Thus as per step 2: Convert the problem in two machine problem G and H with G_i & H_i as the processing time as defined in step 2.

Table : 3

i	1	2	3	4	5
G'_i	44	38	60	37	42
H'_i	70	55	85	54	52

As per step 3: New reduced problem is :

Table : 4

Jobs	Machine M_1	Machine M_2
i	G_i	H_i
1	38	73
2	35	60
3	56	86
4	32	56
5	40	96

As per step 4: Optimal sequence S_1 by Johnson method is

$$S_1 : 4 - 2 - 5 - 1 - 3$$

As per step 5: Other feasible sequence are

$$S_2 : 2 - 4 - 5 - 1 - 3$$

$$S_3 : 5 - 4 - 2 - 1 - 3$$

$$S_4 : 1 - 4 - 2 - 5 - 3$$

$$S_5 : 3 - 4 - 2 - 5 - 1$$

From in – out tables for these sequences we have:

$$\text{For } S_1: CT(S_1) = 299; \quad \sum_{i=1}^n A_{i1} = 98; U_2(S_1) = 105;$$

$$U_3(S_1) = 267 \quad R(S_1) = 1451.$$

$$\text{For } S_2: CT(S_1) = 302; \quad \sum_{i=1}^n A_{i1} = 98; U_2(S_2) = 101;$$

$$U_3(S_2) = 267 \quad R(S_2) = 1431.$$

$$\text{For } S_3: CT(S_1) = 307; \quad \sum_{i=1}^n A_{i1} = 98; U_2(S_3) = 105;$$

$$U_3(S_3) = 267 \quad R(S_3) = 1451.$$

$$\text{For } S_4: CT(S_1) = 305; \quad \sum_{i=1}^n A_{i1} = 98; U_2(S_4) = 100;$$

$$U_3(S_4) = 267 \quad R(S_4) = 1426.$$

$$\text{For } S_5: CT(S_1) = 321; \quad \sum_{i=1}^n A_{i1} = 94; U_2(S_5) = 101;$$

$$U_3(S_5) = 267 \quad R(S_5) = 1411.$$

Therefore $\min \{R(S_k)\} = R(S_5) = 1411$ units and is for sequences S_5 . Hence the sequences $S_5 : 3 - 4 - 2 - 5 - 1$ is optimal sequences with min rental cost 1411 units although the total elapsed time is not min.

9. CONCLUSION

The algorithm proposed here for specially structured three stage flow shop scheduling problem is more efficient as compared to the algorithm proposed by Johnson(1954) to find an optimal sequence to minimize the utilization time of the machines and hence their rental cost. The study may further be extended by considering various parameters like breakdown effect, job block criteria etc.

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