Digital Simulation of Limit Cycle in Second and Higher Order Analog Filter using MATLAB

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ABSTRACT
The non-linearities like saturation, dead zone and relay etc. of electronic components like op-amp leads to limit cycles in the electronic system. Limit cycles describe the oscillations of non-linear electronic systems. This paper examines the development of a graphical technique, leading to the use of computer graphics, for systematic analysis of limit cycles in Second and Higher Order Electronic Filter Systems. Its accuracy has been sustained by comparing with the results from digital simulation.

Keywords— Phasor Diagram, Limit Cycles, Electronic Filter System

1. INTRODUCTION
If the demand for a digital system is heavy, the most effective approach to design is through VLSI implementation. The VLSI implementation of filters can become cost effective if the number of distinct chips needed is kept low, since, then a high demand can be created for each chip. An electronic filter is a frequency selective electronic circuit that passes signals of specified band of frequencies and attenuates the signal of frequencies outside the band. Electronic filters are used in circuits, which require the separation of communication signals according to their frequencies. Electronic filters are widely used in, signal processing and in one form or another in almost all sophisticated electronic instruments. The VLSI implementation of electronic filters composed of passive components, active components, crystals, independent sources etc. The electronic filters can be classified into many configurations but mainly are of two types analog and digital. Electronic filters are classified according to their frequency domain behavior, which was specified in terms of their magnitude and phase response. The most commonly used classification for filters based on frequency domain is:

(i) Low Pass (ii) High Pass
(iii) Band Pass (iv) Band Reject

2. INTRODUCTION TO LIMIT CYCLE
The limit cycles describe the oscillations of nonlinear electronic system. The existence of a limit cycle corresponds to an oscillation of fixed amplitude and period. The limit cycle problem is an important issue that is needed to be solved when dealing with filters. The problem of limiting cycle results in marginal stability. The main reason to rely on the relatively completely stable dynamics is that, it often seems exceedingly difficult to control and practically exploit behaviours such as limit cycles. It is important to note that a limit cycle, in general is an undesirable characteristic of an electronic circuits. It may be tolerated only if its amplitude is within specified limits. Determination of existence of limit cycles is not an easy task as these may depend upon both type and amplitude of the input signal. Prediction of limit cycles is very important because limit cycles can occur in any kind of system. The constant oscillation associated with the limit cycles can cause deteriorating performance or even failure of the system hardware. A graphical technique, suitable for computer graphics, has been developed for prediction of a limit cycle. The method is derived from the basic concept of phasor relationships between the system variables.

3. MATHEMATICAL MODEL
The block diagram of a general 2 x 2, two dimensional system Patra K.C. (1993), Nikiforuk. P.N. (1968) consists of two electronic filter subsystems S_1 and S_2. Each subsystem contains a non-linearity like saturation, dead zone etc. and a stable or neutral part i.e. electronic filter like LPF, HPF etc. connected in series. This is a most general structure representation of two-dimensional system Patra. K.C. (1994). The complete two dimensional structure system is shown in Fig:1.

Where

C	: Amplitude of sub-system output
R	: Input Amplitude to the subsystem from the other
N(X) = Y/X	:Describing Function of the non-linear elements
X	: Amplitude of input to non-linear element
Y	: Amplitude of non-linear element output (Approximated by fundamental component)
θ_e	: Phase shift of the subsystem closed loop response
θ_l	: Phase shift of the subsystem closed loop response
ω	: Angular Frequency in rad/second

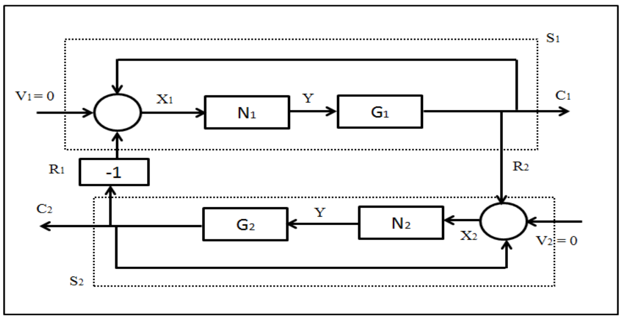

Fig: 1 General 2 x 2 Electronic Filter System

The phasor diagram for the general 2x2 Electronic Filter system in autonomous state is shown in Fig:4.2. For a fixed frequency 'ω', the angles θ_{L1} and θ_{L2} are fixed.

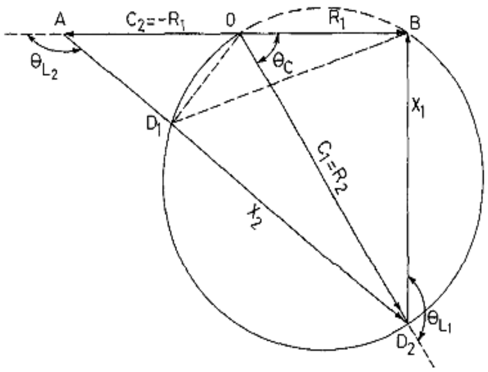

Fig: 2 Phasor Diagram of 2 x 2 Electronic Filter System

The sides of the triangle OBD_2 correspond to subsystem 1. For a given amplitude R_1, the sides OB subtend a constant angle $\theta' = 180^0 - \theta_{L1}$ constraining the points O, B and D_2 to lie in a circle as shown in Fig. 2. Thus, for a given R_1 and ω, the output C_1 will be given by the side OD_2 if the conditions mentioned below are satisfied simultaneously:

i) The Point D_2 must lie on the segments ODB of circle subtending an angle $180^0 - \theta_{L1.}$
ii) $OD_2 = Y_1G_1$, $BD_2 = X_1$, where the relation between X_1 and Y_1 is the Describing Function of non-linear element and G_1 is a known function of ω.

Similar arguments hold good for the triangle OAD_1 representing parameters of the subsystem2.

For the system limit cycling in the autonomous state, it can be expressed by inspecting of Fig: 4.1 that

$C_1 = R_2$ and $R_1 = - C_2$

Both the triangles are superimposed in a fashion shown in Fig: 2 to satisfy the condition stated above. Once triangle OD_2B and circle through points O, D and B are determined, then for a particular R_1 and ω, OA is drawn opposite to OB and AD_2 represent the phasor X_2 drawn at an angle θ_{L2} with phasor C_2. The intersection of this straight line AD_i (representing X_2) with the circle, as shown in Fig: 4.2, would represent possible self oscillation if the conditions mentioned below are satisfied:

i) The Point D_i (i = 1, 2 implying double interaction) lies on the intersection of segment OD_iB of a circle subtending an angle $(180^0 - \theta_{L1})$ on OB and a straight line ADi making an angle θ_{L2} with OA.
ii) $OD_i = C_1 = Y_1G_i$; i = 1,2 $BD_i = X_1$, where $Y_1 = N_1X_{1.}$
iii) $OA = C_2 = Y_2G_2$ and $AD_i = X_2$, where $Y_2 = N_2X_2$

4. NORMALIZED PHASOR DIAGRAM

For various possible values of R_1, it is necessary to construct separate phasor diagrams to check the above conditions for a fixed frequency. If all these quantities are normalized with respect to R1, a single phasor diagram (for a particular value of frequency) can be used for checking possibility of limit cycle. This diagram is termed as normalized phasor diagram and hence the name of the technique, is shown in Fig:3.

From the diagram we can drive:

$$OA = \frac{C_2}{R_1} = -1.0$$

$$OB = \frac{R_1}{R_1} = 1.0$$

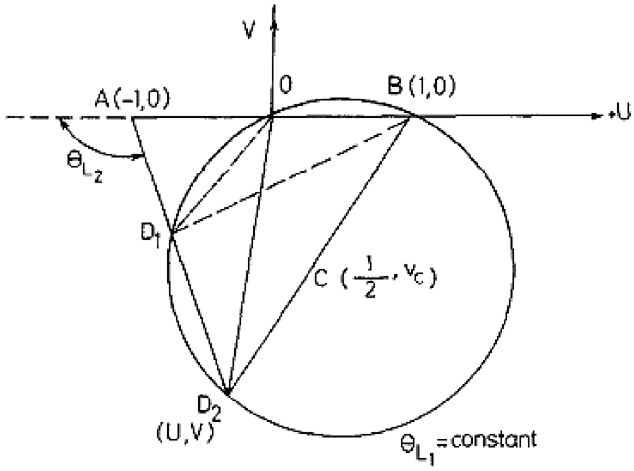

Fig: 3 Normalized Phasor Diagram for second order 2 x 2 Electronic Filter System

$$OD_i = \frac{C_1}{R_1} = \frac{C_1}{C_2}, \qquad i = 1 \; or \; 2$$

$$BD_i = \frac{X_1}{R_1}$$

$$AD_i = \frac{X_2}{R_1}$$

In Fig: 3, C is the center of circle OBD_i of radius r = OC. Selecting O as the origin in Fig: 4.3, the co-ordinates of the points D_i can be determined.

From triangle D_iCO,

The co-ordinates of the point C are $(0.5, -0.5/\tan\theta_{L1})$.

The radius of the circle $OC = 0.5/\sin\theta_{L1}$

The equation of the straight line AD_i is obtained as:

$$U = v \cot\theta_{L2} - 1 \qquad (1)$$

The co-ordinates of the intersection points, D_i of the circle and straight line AD_i are obtained as:

$$v_i = \frac{3\,cot\theta_{L2} + \cot\theta_{L1} + \sqrt{(3\cot\theta_{L2} + \cot\theta_{L1})^2 - 8cosec^2\theta_{L1}}}{2cosec^2\theta_{L2}} \qquad (2)$$

and

$$u_i = v_i / \tan\theta_{L2} - 1 \qquad (3)$$

It may be noted that co-ordinates of the point D_i are the functions of angle θ_{L1}, θ_{L2} and later are function of frequency. Hence, for specific frequency, if the co-ordinates are known of D_i, the other variables (X_1, X_2 or C_1, C_2) can be determined as:

$$OD_i = (u_i^2 + v_i^2)^2$$

$$OA = -1.0$$

$$BD_i = [(1-u_i)^2 + v_i^2]^{0.5}$$

$$AD_i = [(1+u_i)^2 + v_i^2]^{0.5} \qquad (4)$$

$$N_1(X_1) = Y_1/X_1 = C_1/(G_1X_1) = OD_i/(G_1BD_i)$$

(5)

$$N_2(X_2) = Y_2/X_2 = C_2 / (G_2X_2) = OA / (G_2AD_i) \quad (6)$$

$$X_1 / X_2 = BD_i / AD_i \quad (7)$$

Corresponding to the value of N_1 and N_2 determined from equation (4.5) and (4.6), the value of X_1 and X_2 can be found out from the describing function expressions of both non-linearities. The ratio of X_1 / X_2 is again obtained from the equation (4.7). This process is repeated for different values of 'ω'. The value of X_1 / X_2 calculated from various 'ω' are tabulated and a graph is plotted between these two parameters. The ratio X_1 / X_2 calculated from Equations (4.5 and 4.6) and (4.7) intersect at a point, this intersection point gave the frequency of the limit cycle. Once this limit cycle frequency is determined, the amplitude of other variables of interest can be determined.

5. IMPLEMENTATION

For the system shown in Fig: 1, the circuit element is LPF whose transfer function, $G_1(s) = 1.586/ (s^2 + 1.414s + 1)$ and high Pass Filter with transfer function, $G_2(s) = 1.586\, s^2 / (s^2 + 1.414s + 1)$. The non-linearity N1 and N2 both are saturation. The frequency 'ω' has been varied from 0.4 to 0.7 and the ratio X_1 / X_2 are computed from equation (5 & 6) and equation (7). The results of computation are tabulated in Table 1.

Table: 1 Results of Computation

Frequency 'ω'	θ_{L1}	θ_{L2}	X_1 / X_2 from eq. (5 & 6)	X_1 / X_2 from eq. (7)
0.4	- 33.96	- 33.96	1.0754	1.256
0.45	- 38.45	- 38.45	1.0596	1.157
0.5	- 43.32	- 43.32	1.0353	1.098
0.55	- 47.67	- 47.67	1.0223	1.046
0.6	- 52.98	- 52.98	0.9985	1.020
0.65	- 57.97	- 57.97	0.9852	0.9675
0.7	- 62.74	- 62.74	0.9723	0.9223

The calculated values of ratio X_1 / X_2 from equation (5 & 6) and equation (7) tabulated in Table 1 are plotted with respect to 'ω'. From the graph the intersection point of both the lines is determined, which gives the frequency of limit cycle. The plot is as shown in Fig: 4.

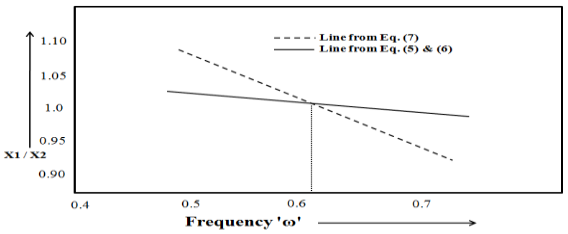

Fig: 4 Plot of X_1 / X_2 with respect to 'ω'

From the graph, the frequency of limit cycle is found to be 0.6183 rad/sec. The amplitude of limit cycle is calculated as $C_1 = 0.5835$, $C_2 = 0.3185$ from Equation (5 & 6).

The detailed results using the above Graphical Method are tabulated in Table 2.

System (Filter Type)		Name of Non-Linearity		Graphical Method Results		
G_1	G_2	N_1	N_2	ω	C_1	C_2
LPF	HPF	Saturation	Saturation	0.6183	0.5835	0.3185
LPF (III order)	HPF (III order)	Saturation	Saturation	0.5135	0.6229	0.2578

Table: 2 Results of Graphical Method

6. IMPLEMENTATION USING MATLAB

The MATLAB 7.0 version is used as the simulation software for simulating the system. The SIMULINK block diagram consists of some major components which are as follows:

The major components of the SIMULINK model are:

(i) Sub-system1 and sub-system2 containing the linear time invariant continuous part and non-linear time invariant parts.

(ii) A signal generator for giving initial excitation signal

(iii) Scope to view the wave form.

The SIMULINK model is simulated to obtain the time history plots shown in Fig:5 to Fig:8. From the time history plot frequency and amplitude of the limit cycles for a particular Electronic system is computed.

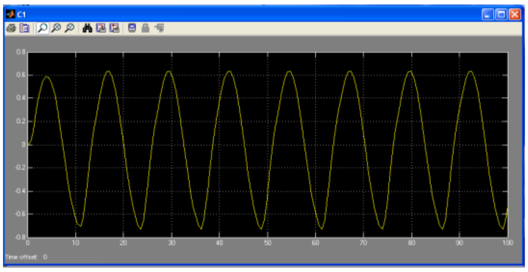

Fig: 5 Output C1 of LPF with Saturation Non-Linearity

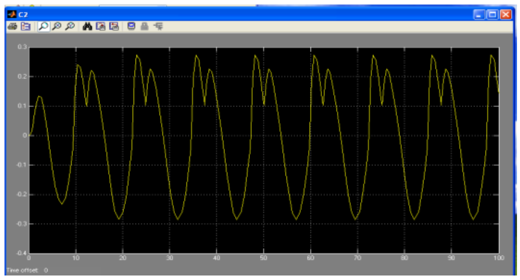

Fig: 6 Output C2 of HPF with Saturation Non-Linearity

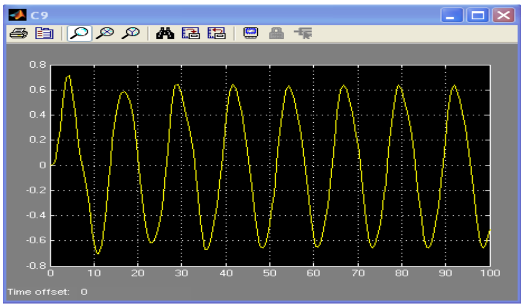

Fig: 7 Output C9 of III order LPF with Saturation Non-Linearity

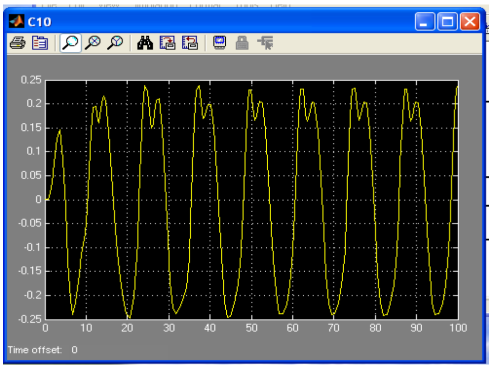

Fig: 8 Output C10 of III order HPF with Saturation Non-Linearity

From the time history plots shown in Fig: 5 – 8 frequency and amplitude of the Limit cycles for different electronic filter system are compared which is as shown in Table-3.

Table: 3 MATLAB / SIMULINK Results

System (Filter Type)		Name of Non-Linearity		MATLAB / SIMULINK Results		
G_1	**G_2**	**N_1**	**N_2**	**ω**	**C_1**	**C_2**
LPF	HPF	Saturation	Saturation	0.5969	0.5778	0.3125
LPF (III order)	HPF (III order)	Saturation	Saturation	0.5235	0.6429	0.2378

7. CONCLUSION

A graphical method leading to the use of computer graphics has been developed. It provides a useful insight into the nature of a limit cycle in such a system. The method is particularly elegant for systems incorporating relays with ideal two state characteristics. The technique can also be extended for the analysis of complex oscillations during the process of signal stabilization.

The use of SIMULINK for oscillation prediction provides an ideal tool for comparing the accuracy of DF solutions for more complicated system.

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