International Journal of Computer Applications (0975 – 8887)
Volume 53– No.2, September 2012

End-to-End Performance of Multiple-Input-Multiple-Output Relay Transmission Link over Rayleigh Fading Channels

Kapil Gupta
Department of Electronics & Communication Engineering,
Faculty of Engineering and Technology, Mody Institute of Technology and Science (Deemed University)
Lakshmangarh, Dist. Sikar
Rajasthan (INDIA)

P.K.Ghosh
Department of Electronics & Communication Engineering,
Faculty of Engineering and Technology, Mody Institute of Technology and Science (Deemed University)
Lakshmangarh, Dist. Sikar
Rajasthan (INDIA)

ABSTRACT
In this work, we investigate the end-to-end performance of a multiple-input-multiple-output (MIMO) relay system over a flat Rayleigh fading channel to study the performance in terms of outage probability (OP) and the average bit error rate (ABER) of the digital receivers. New closed form expressions for the statistics of the received signal-to-noise ratio (SNR) for both amplify-and-forward and decode-and-forward systems are obtained from the novel approach of moment generating function (MGF). Comparisons between amplify-and-forward and decode-and-forward systems are also presented. The calculated results reveal that the performance of MIMO-antenna relay system improves significantly for both modes with the increase of number of input antennas (M) and output antennas(N). It is also observed that both the system performance improves significantly (roughly 3 dB) when the number of input antennas (M) is varied from $\mathrm{M} = \mathrm{N}$ to $\mathrm{N} + 1$ at both low and high SNR regimes.

Keywords
Diversity, Rayleigh fading, Moment Generating Function, Amplify-and-Forward, Decode-and-Forward.

1. INTRODUCTION
The coverage area of a transmitter can be extended by relaying. Cooperative diversity is a promising technique, due to its ability to combat the effect of deep fading in a wireless channel [1]. With the help of single or multiple relay links, it formed a virtual antenna array and achieves spatial diversity at the receiver. Cellular and ad-hoc wireless communication systems are some applications of cooperative diversity [2].

The end-to-end performance of a relayed transmission link has been studied analytically over Rayleigh fading channel with single reception and transmission antenna system at the relay [1]. The capacity increment of MIMO systems over the single antenna system attracted researchers to work on multiple antennas at the relay node [3]. However, deployment of multiple antennas at the mobile terminals often encounters various implementation problems because wireless terminals are expected to become smaller in dimension and lighter in weight in future. In contrast to mobile terminals, accommodating less number of antennas on infrastructure-based fixed relays is feasible [4], and the single antenna relay can be considered as a special case of this system. The end-to-end performance of such infrastructure-based relaying is studied for the decode-and-forward (DF) relaying scheme and subsequently examined the achievable cooperative diversity in Ref. [4]. Ref. [5] presented, the performance of selection combining (SC) based multi-antenna fixed relay for both amplify-and-forward (AF) and decode-and-forward relaying. All these papers mostly focus on multiple input antennas relay, to receive information from the source and single output antenna for transmitting information from the relay to the destination. Capacity bounds of a MIMO channel were explained in [3]. Comparison of different signaling and routing methods for MIMO relay network is shown in [6]. Outage probability is calculated in closed form for MIMO relay in [7] and approximate bit error rate (BER) with maximum ratio combining MRC/STBC is presented in [8], are some related works. However, in fixed relay systems, a more general form of relay structure such as MIMO antennas are achieving gain [6-8]. This paper addresses the end-to-end performance of such an infrastructure-based MIMO-antennas relay system over flat Rayleigh fading channel.

In this work, we consider a MIMO-relay transmission link where the relay is equipped with multiple diversity antennas (M) at its input and employs maximum ratio combining (MRC) technique for reception from the source. The onward transmission to the destination is done via multiple transmit antennas (N) using maximum ratio transmission (MRT). We assume that the complete channel state information (CSI) for both the source-relay link and the relay-destination link is known at the relay**.**

The organization of the paper is as follows. In section II, we derive a close form expression for the moment generating function (MGF) of the destination signal-to-noise ratio (SNR) for a two-hop MIMO-relay link in a flat Rayleigh fading environment. A general formula relating the MGF of a random variable to the MGF of its inverse is established to derive the MGF of the destination SNR. The infrastructure-based relaying system and channel models are introduced in section III. Here, the expressions of important statistics like the probability density function (pdf), the cumulative density function (cdf), and the output SNR moments are derived in closed forms. Section IV provides the expressions for various performance metrics of the system. In section V, we study the various performance metrics, namely, the outage probability (OP) and the average bit error rate (ABER) of the amplify-and-forward (A&F) relay systems and compare with the decode-and-forward (D&F) relaying systems. Finally, the concluding remarks are provided in section VI.

2. THE MGF OF INVERSE RANDOM VARIABLE

Let $\boldsymbol{z} = \frac{1}{X}$ be the inverse random variable of X, where X is a positive random variable with the pdf $\boldsymbol{p_X(x)}$. The MGF $M_Z(s)$ of Z can be written as

$$M_z(s) = E_z(\exp(-sz)) = \int_o^\infty e^{-sz}\, p_Z(z)dz$$
$$= \int_0^\infty e^{-S/x}\, p_X(x)dx = \int_0^\infty \sum_{k=0}^\infty \frac{\left(-\frac{s}{x}\right)^k}{\Gamma(k+1)} p_X(x)dx \qquad (1)$$

where $E_z(.)$ is the expectation value of $(.)$. By the change of order of integration and summation we may write,

$$M_z(s) = \sum_{k=0}^\infty \frac{(-s)^k}{\Gamma(k+1)} \int_0^\infty x^{-k}\, p_X(x)dx$$
$$= \sum_{k=0}^\infty \frac{(-s)^k}{\Gamma(k+1)} \mu_{-k} \qquad (2)$$

where the inverse-moment of X is defined as [9]

$$\mu_{-k} = \frac{1}{\Gamma(k)} \int_0^\infty s^{k-1}\, M_X(s)ds \qquad (3)$$

Thus,

$$M_z(s) = \sum_{k=0}^\infty \frac{(-1)^k s^k}{\Gamma(k)\,\Gamma(k+1)} \int_0^\infty s^{k-1} M_X\,(s)ds \qquad (4)$$

This novel equation relates the MGF of an inverse random variable to the MGF of the original random variable. It is easy to show that (4) indeed converges and in some special cases can be represented in closed form.

3. SYSTEM AND CHANNEL MODEL

3.1 Description of received signal over MIMO link

An infrastructure–based fixed wireless relaying system is shown in Fig.1 where source S is transmitting signal for destination D through the relay terminal R . The relay terminal R is equipped with M number of receiving antennas for reception of transmitted signal from source terminal S and N transmitting antenna to convey the signal to the destination terminal D after suitable processing (A&F or D&F) at the relay. Maximum ratio combining (MRC) is used for receiving the signal at relay and maximum ratio transmission (MRT) is used for conveying to the destination D . We also assume that full channel-state information (CSI) is available at the relay.

The combiner output, equals to the sum of the SNRs in the individual branches in MRC which is given by [10]

$$\gamma_{mrc} = \sum_{i=1}^M \gamma_i \qquad (5)$$

where the instantaneous signal-to-noise ratio (SNR) of the i-th branch is, $\gamma_i = \frac{E_s}{N_0}\left(\alpha_i^2\right)$, i = 1, 2,..,$M$ with α_i being the Rayleigh fading amplitude of the channel between source terminal and the relay antennas (i = 1, 2,…,M) , E_s is the energy of the transmitted signals and N_0 is the one-sided noise power spectral density per branch.

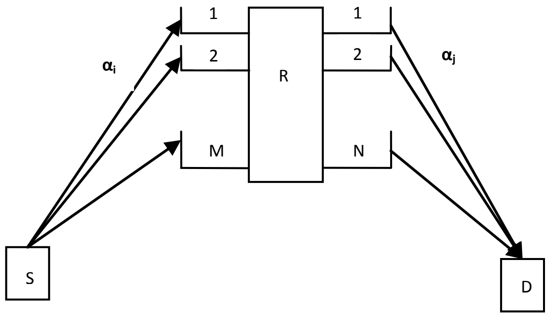

Fig1. Infrastructure–based fixed wireless relaying system

We assume that the channel is frequency non-selective slowly varying such that it is constant for the transmitted symbols interval. α_i is Rayleigh distributed, α_i^2 is the exponentially distributed random variables. We also assume that the input signal-to-noise ratio (SNR) is same for all diversity branches (i.e., $\overline{\gamma_1} = \overline{\gamma_2} = \cdots = \overline{\gamma_M} = \overline{\gamma_s}$), where $\overline{\gamma_i}$ is the average SNR of the i-th link. The output SNR $\gamma_{s,r}$,of the MRC that is the SNR between S and R, has the probability density function given by [10]

$$p_{\Gamma_{s,r}}(\gamma_{s,r}) = \frac{\gamma_{s,r}^{M-1}}{\overline{\gamma_s}^M \Gamma(M)} e^{\frac{-\gamma_{s,r}}{\overline{\gamma_s}}} \qquad (6)$$

where, Γ(.) is the Gamma function defined in [11].

Assume that $\alpha_j, j = 1,2, \ldots\ldots, N,$ the fading amplitude of the channel between the j-th antenna at R and D is of Rayleigh type, and the signal-to-noise ratio is the same for all diversity branches. In the similar fashion, the pdf of $\gamma_{r,d}$, the signal-to-noise ratio between R and D can be written as (replacing M by N and $\gamma_{s,r}$ by $\gamma_{r,d}$ in the eq.(6))

$$p_{\Gamma_{r,d}}(\gamma_{r,d}) = \frac{\gamma_{r,d}^{N-1}}{\overline{\gamma_d}^N \Gamma(N)} e^{\frac{-\gamma_{r,d}}{\overline{\gamma_d}}} \qquad (7)$$

At the relay terminal before re-transmission by the choice of the appropriate gain, the overall SNR Γ_{eq} at the receiving terminal D can be very closely upper bounded as [1],

$$\Gamma_{eq} = \frac{\Gamma_{s,r}\,\Gamma_{r,d}}{\Gamma_{s,r} + \Gamma_{r,d}} \qquad (8)$$

where $\Gamma_{s,r}$ is the output signal-to-noise ratio of the MRC, and $\Gamma_{r,d}$ is the instantaneous SNR between R and D with MRT. Eq. (8) is preferred for performance analysis in dual-hop relayed transmission links due to its mathematical tractability [1].

3.2 Derivation of the MGF over MIMO link of received signal.

We define the random variable γ as

$$\Gamma_{eq} = \frac{1}{\frac{1}{\Gamma_{s,r}} + \frac{1}{\Gamma_{r,d}}} \qquad (9)$$

The pdf of $\Gamma_{s,r}$ and $\Gamma_{r,d}$ are given in equations (6) and (7), respectively. Taking $\Gamma_{s,r} = \frac{1}{X_1}$ and $\Gamma_{r,d} = \frac{1}{X_2}$ as two independent random variables, the moment MGF of $X = X_1 + X_2$ can be written as

$$M_X(s) = M_{X_1}(s) M_{X_2}(s) \tag{10}$$

Taking $\overline{\gamma_s} = \frac{1}{\beta_1}$ and $\overline{\gamma_d} = \frac{1}{\beta_2}$ we have

$$M_{X_1}(s) = \int_0^\infty e^{-sx_1} \frac{x_1^{-M-1} {\beta_1}^M}{\Gamma(M)} e^{-\frac{\beta_1}{x_1}} dx_1$$
$$= \frac{2(\sqrt{\beta_1 s})^M K_M(2\sqrt{s\beta_1})}{\Gamma(M)} \tag{11}$$

and

$$M_{X_2}(s) = \int_0^\infty e^{-sx_2} p_{X_2}(x_2) dx_2 = \frac{2(\sqrt{\beta_2 s})^N K_N(2\sqrt{s\beta_2})}{\Gamma(N)} \tag{12}$$

where K_V is the modified Bessel function of second kind of order V.
Therefore, equation (10) may be written as

$$M_X(s) = \frac{4(\sqrt{\beta_1 s})^M K_M(2\sqrt{s\beta_1})(\sqrt{s\beta_2})^N K_N(2\sqrt{s\beta_2})}{\Gamma(M)\Gamma(N)} \tag{13}$$

Now using [12, Eq. 1.11.51], we can write the MGF $M_{\Gamma_{eq}}(s)$ as

$$M_{\Gamma_{eq}}(s) = \sum_{k=0}^{\infty} \frac{(-s)^k}{\Gamma(k+1)} \left(\frac{1}{\Gamma(k)} \int_0^\infty s^{k-1} M_X(s) ds\right) \tag{14}$$

After algebraic manipulation, equation (13) leads to

$$M_{\Gamma_{eq}}(s) = \sum_{k=0}^{\infty} \frac{(-s)^k}{\Gamma(k+1)} \frac{{\beta_1}^{N-k} {\beta_2}^N \Gamma(k+M+N)\Gamma(k+M)\Gamma(k+N)}{\Gamma(M)\Gamma(N)\Gamma(2k+M+N)}$$
$${}_2F_1\left(k+M+N, k+M; 2k+M+N; 1-\frac{\beta_2^2}{\beta_1^2}\right) \tag{15}$$

Here, ${}_2F_1(.,.;.;.)$ is the Gauss' hypergeometric function [11]. Unfortunately Eq. (15) is in the form of infinite sum for general values of β_1 and β_2. However, for the special case when $\overline{\gamma_d} = \overline{\gamma_s} = \bar{\gamma}$ or equivalently, $\beta_1 = \beta_2 = \beta = \frac{1}{\bar{\gamma}}$, (the symmetric branch SNR), $M_{\Gamma_{eq}}(s)$ can be reduced to a very compact form as follows:

$$M_{\Gamma_{eq}}(s) = {}_3F_2\left(M, N, M+N, \tfrac{1}{2}(M+N+1), \tfrac{1}{2}(M+N); -\frac{s}{4\beta}\right) \tag{16}$$

where, ${}_3F_2(.,.,.;.;.)$ is the generalized hypergeometric function defined in [11].

3.3 Derivation of pdf, cdf and moments of received SNR

Using [14, Eq.(07.27.26.0004.01)], we express the MGF as given in (16) as

$$M_{\Gamma_{eq}}(s) = \frac{\Gamma\left(\frac{M+N+1}{2}\right)\Gamma\left(\frac{M+N}{2}\right)}{\Gamma(M)\Gamma(N)\Gamma(M+N)} G_{3,3}^{1,3}\left[\frac{s}{4\beta}\Big|_{0,1-\frac{1}{2}(M+N+1),1-\frac{1}{2}(M+N)}^{M,1-N,1-M-N}\right] \tag{17}$$

where $G[\cdot]$ is the Meijer-G function [11]. The inverse Laplace transform of $M_{\Gamma_{eq}}(s)$ leads to the expression for the pdf of Γ_{eq} as

$$P_{\Gamma_{eq}}(\gamma_{eq}) = \frac{\Gamma\left(\frac{M+N+1}{2}\right)\Gamma\left(\frac{M+N}{2}\right)}{\Gamma(M)\Gamma(N)\Gamma(M+N)} G_{2,3}^{3,0}\left[4\beta\,\gamma_{eq}\Big|_{0,1-\frac{1}{2}(M+N+1),1-\frac{1}{2}(M+N)}^{M,1-N,1-M-N}\right] \tag{18}$$

To find the cumulative distribution function (cdf) $F_{\Gamma_{eq}}(\gamma_{eq})$ of Γ_{eq}, we use the relation $F_{\Gamma_{eq}}(\gamma_{eq}) = \int_0^{\gamma_{eq}} p_\Gamma(\gamma) d\gamma$. The cdf is thus given by

$$F_{\Gamma_{eq}}(\gamma_{eq}) = \frac{\Gamma\left(\frac{M+N+1}{2}\right)\Gamma\left(\frac{M+N}{2}\right)}{\Gamma(M)\Gamma(N)\Gamma(M+N)} G_{3,4}^{3,1}\left[4\beta\gamma_{eq}\Big|_{M,N,M+N,0}^{1,\frac{1}{2}(M+N+1),\frac{1}{2}(M+N)}\right] \tag{19}$$

As a check, for $M = N$, Eq. (19) can then be written as

$$F_{\Gamma_{eq}}(\gamma_{eq}) = \frac{\Gamma\left(M+\frac{1}{2}\right)}{\Gamma(M)\Gamma(2M)} G_{3,4}^{3,1}\left[4\beta\gamma_{eq}\Big|_{M,M,2M,0}^{1,M+\frac{1}{2},M}\right] \tag{20}$$

Using functional relationship of the Meijer-G function [11], the above equation reduces to

$$F_{\Gamma_{eq}}(\gamma_{eq}) = \frac{\Gamma\left(M+\frac{1}{2}\right)}{\Gamma(M)\Gamma(2M)} 4\beta\gamma_{eq}\, G_{2,3}^{2,1}\left[4\beta\gamma_{eq}\Big|_{M-1,2M-1,-1}^{0,M-\frac{1}{2}}\right] \tag{21}$$

and with the aid of the property of gamma function, namely,

$$\sqrt{\pi}\Gamma(2z) = 2^{2z-1}\Gamma(z)\Gamma(z+\tfrac{1}{2}) \tag{22}$$

we have

$$F_{\Gamma_{eq}}(\gamma_{eq}) = \frac{\sqrt{\pi}}{2^{2z-3}\Gamma^2(M)} \beta\gamma_{eq}\, G_{2,3}^{2,1}\left(4\beta\gamma_{eq}\Big|_{M-1,2M-1,-1}^{0,M-1/2}\right) \tag{23}$$

which is the same form of Eq. (36) of [13].
The n-th moments of the received SNR at the destination (i.e., the n-th moment of Γ_{eq}) can be obtained as

$$E({\Gamma_{eq}}^n) = \frac{(M+N)_n (M)_n (N)_n}{\left(\frac{M+N+1}{2}\right)_n \left(\frac{M+N}{2}\right)_n} \times \frac{1}{(4\beta)^n} \tag{24}$$

where $(a)_n = a(a+1)\ldots\ldots(a+n-1), (a)_0 = 1,\ a \neq 0,$ is the Pochhammer symbol. For example, with n = 1, and $M = N$,

$$E(\Gamma_{eq}) = \frac{M^2}{\beta(2M+1)} = \frac{M^2}{(2M+1)} \overline{\gamma_{eq}}. \tag{25}$$

4. PERFORMANCE OF MIMO-ANTENNA RELAY

4.1 Outage Probability (OP)

The outage probability of an *amplify-and-forward* relaying system is defined as the probability that the instantaneous SNR γ falls bellow some prescribed threshold (γ_{th}). Mathematically, it is given by

$$P_{op} = \Pr[0 \leq \gamma \leq \gamma_{th}] = F_{\Gamma}(\gamma_{th}) \qquad (26)$$

Using (19) and (26), the outage probability for *amplify-and -forward* systems can be shown to be given by

$$P_{op} = \frac{\Gamma\left(\frac{M+N+1}{2}\right)\Gamma\left(\frac{M+N}{2}\right)}{\Gamma(M)\Gamma(N)\Gamma(M+N)} G_{3,4}^{3,1}\left(4\beta\gamma_{th}\Big|_{M,N,M+N,0}^{1,\frac{1}{2}(M+N+1),\frac{1}{2}(M+N)}\right) \qquad (27)$$

For *decode-and-forward* relaying the outage occurs when both $S \rightarrow R$ and $R \rightarrow D$ links are in outage and can be shown to be given by [1]

$$P_{op} = 1 - \int_{\gamma_{th}}^{\infty} p_{\Gamma_{s,r}}(\gamma)d\gamma \int_{\gamma_{th}}^{\infty} p_{\Gamma_{r,d}}(\gamma)d\gamma$$
$$= 1 - \left(\frac{\Gamma(M,\beta\gamma_{th})}{\Gamma(M)}\right)\left(\frac{\Gamma(N,\beta\gamma_{th})}{\Gamma(N)}\right) \qquad (28)$$

where $\Gamma(.,.)$ is the incomplete gamma function[11].
It may be noted that the lower bound of the outage probability for the fixed relay MIMO-antenna system can be achieved for fixed M by letting $N \rightarrow \infty$. If N goes to infinity, the SNR of the $R \rightarrow D$ link can be much larger compared to that of the $S \rightarrow R$ link and the overall received SNR is dictated only by the $S \rightarrow R$ link. The lower bound of outage probability for both *amplify-and-forward* relaying and *decode-and-forward relaying* systems can be written as

$$P_{op,lb} = 1 - \left(\frac{\Gamma(M,\beta\gamma_{th})}{\Gamma(M)}\right) \qquad (29)$$

4.2 Average Bit Error Rate (ABER)

The average bit error rate (ABER) or average symbol error rate (ASER) of various digital modulation schemes over the MIMO relayed link can be derived by adopting the MGF-based approach as discussed in [15]. For example, let us consider the ABER of binary differential phase-shift keying (DPSK). For *amplify-and-forward* relaying, the ABER $P_b(E)$ is given by

$$P_b(E) = \frac{1}{2} M_{\Gamma_{eq}}(E) \qquad (30)$$

For d*ecode-and-forward relaying* the ABER can be shown to be given by [1]

$$P_b(E) = P_b(E_1) + P_b(E_2) - 2P_b(E_1)P_b(E_2) \qquad (31)$$

where $P_b(E_1)$ is the ABER for $S \rightarrow R$ link and is given by

$$P_b(E_1) = \int_0^{\infty} \frac{1}{2} e^{\gamma_{s,r}} p_{\Gamma_{s,r}}(\gamma_{s,r}) d\gamma_{s,r} = \frac{1}{2}\left(\frac{\beta_1}{1+\beta_1}\right)^M \qquad (32)$$

The ABER for R→D link is given as

$$P_b(E_2) = \int_0^{\infty} \frac{1}{2} e^{\gamma_{r,d}} p_{\Gamma_{r,d}}(\gamma_{r,d}) d\gamma_{r,d} = \frac{1}{2}\left(\frac{\beta_2}{1+\beta_2}\right)^N \qquad (33)$$

Using (32) and (33), the ABER for *decode-and-forward relaying* can be rewritten as

$$P_b(E) = \frac{1}{2}\left(\frac{\beta_1}{1+\beta_1}\right)^M + \frac{1}{2}\left(\frac{\beta_2}{1+\beta_2}\right)^N - 2 \times \frac{1}{2}\left(\frac{\beta_1}{1+\beta_1}\right)^M \times \frac{1}{2}\left(\frac{\beta_2}{1+\beta_2}\right)^N \qquad (34)$$

For symmetric case with $\beta_1 = \beta_2 = \beta = \frac{1}{\bar{\gamma}}$, Eq. (34) reduces to

$$P_b(E) = \frac{1}{2}\left[\left(\frac{1}{(1+\bar{\gamma})}\right)^M + \left(\frac{1}{(1+\bar{\gamma})}\right)^N - \left(\frac{1}{(1+\bar{\gamma})}\right)^{M+N}\right] \qquad (35)$$

Following the same reasoning as in case of outage probability, the lower bound of ABER of DPSK for both *amplify-and-forward* relaying and *decode-and-forward relaying* systems can be obtained for fixed M by letting N→∞ and is given by

$$P_{b,lb}(E) = \frac{1}{2}\left(\frac{\beta}{1+\beta}\right)^M \qquad (36)$$

It may be noted that the above derivation for ABERs of DPSK for both *amplify-and-forward* relaying and *decode-and-forward relaying* are general and can easily be extended to other kind of digital modulation schemes.

5. NUMERICAL RESULTS

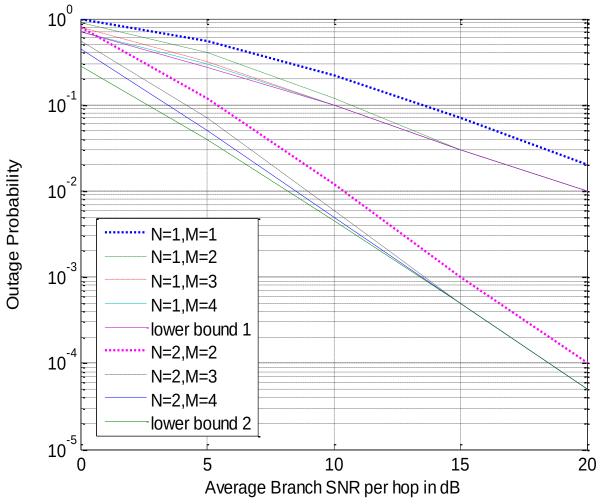

Fig 2. Outage Probability versus Average Branch SNR for amplify-and–forward relaying mode.

Fig. 2 shows the outage probability of the MIMO-antenna relay in symmetrical networks for different number of antennas (M, N) at the relay using amplify-and-forward (AF) protocol. It is observed that the MIMO-relay systems yield significant gains over the referenced single antenna system$(\mathrm{M} = 1, \mathrm{N} = 1)$). For a comparison, at an outage probability of 10^{-1}, the SNR requirements over the conventional single antenna relaying are obtained as 2.8 dB $(\mathrm{M} = 2, \mathrm{N} = 1)$, and 10.5 dB $(\mathrm{M} = 2, \mathrm{N} = 2)$, approximately. Thus the gain improvement is significant in MIMO-antenna relay. It is also observed that for a fixed outage probability (for fixed N, say $(\mathrm{N} = 2)$ the additional gains for $\mathrm{M} =$ 3 and 4 are approximately 2 dB and 1dB, respectively relative to $\mathrm{M} = 2$. Thus the gain of the system increases only marginally for values of $(\mathrm{M} > 3, \mathrm{N} = 2)$ over$(\mathrm{M} = 3, \mathrm{N} = 2)$. If M is increased still further, the gain values attain saturation, the lower bound of performance value, and no further gain will be possible with $N = 2$. Keeping M fixed and varying N yields similar curves as interchanging M and N does not change eq. (19).

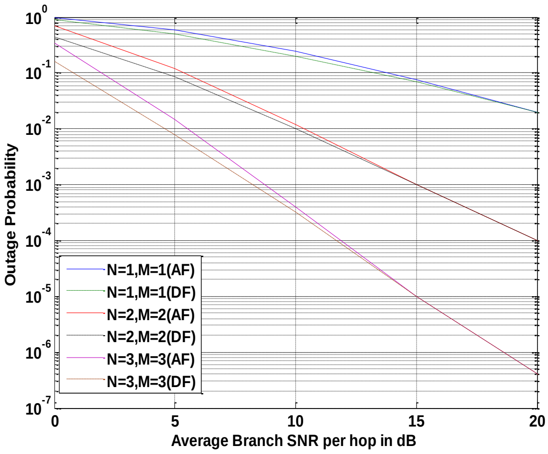

Fig. 3. Comparision of outage probabilities of amplify-and –forward relaying and of decode-and-forward relaying system.

A comparison of amplify-and-forward (AF) and decode-and-forward relaying systems for outage probability on various values of M and N is done in fig 3. It is found that the performance of decode-and-forward relaying is better than the amplify-and-forward relaying for the same values of M and N for low SNR values. However, the difference in error performance is not noticeable at high SNR values. The gap in error performance between AF and DF relaying in the low SNR regimes increase more rapidly for large values of M and N relative to low values of M and N.

Fig. 4 shows ABER plots of binary DPSK for amplify-and-forward MIMO-antenna relaying for various values of M and$N$. The graph shows clearly the improvement achieved at the destination employing MIMO-antenna relay in both the relaying schemes. Note that in case of amplify-and-forward system the improvement is significantly high for higher M and N as expected owing to diversity.

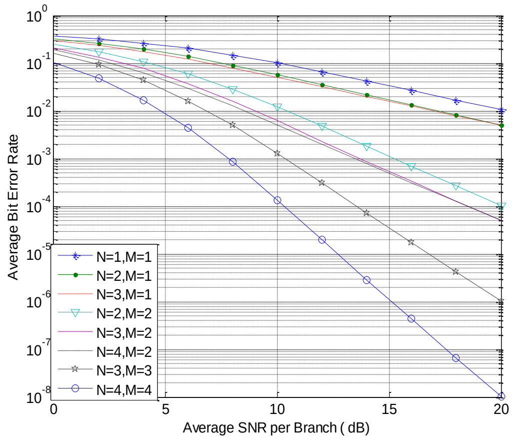

Fig 4. ABER for amplify-and–forward relaying mode

However, it is observed that keeping M fixed the additional improvement achieved for $N = M + 1$ compared to $N = M$, is insignificant for low SNR regimes and roughly 3 dB improvements in the high SNR regimes. Interestingly, increasing N even further $(N > M + 1)$ the additional improvement is insignificant at both low and high SNR regimes. For $N > M + 1$, the plots quickly align with the curves for $N = M + 1$ as the SNR increase. This behaviour of outage performance was also reported in [18] for the special case of $N = 1$.

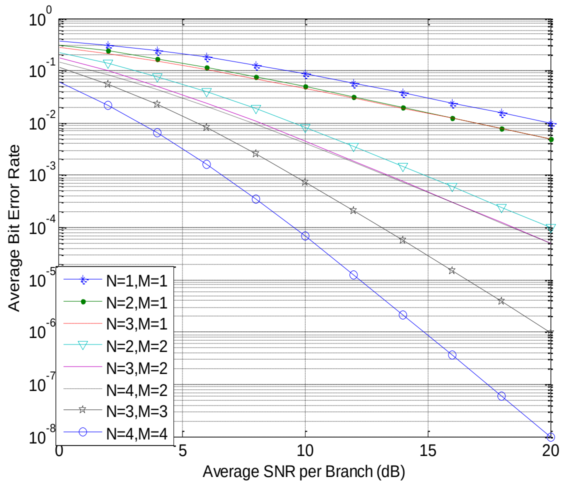

Fig 5. ABER for decode-and–forward relaying mode

Similar observations are also obtained for the decode-and-forward relaying systems as depicted in Fig. 5. This nature is highly expected as the SNR of $S \rightarrow R$ link is improved whereas overall performance is determined by both $S \rightarrow R$ and $R \rightarrow D$ links. As was for the case of outage probability, similar trend in ABER performance is expected for the case when N is fixed and M is varied.

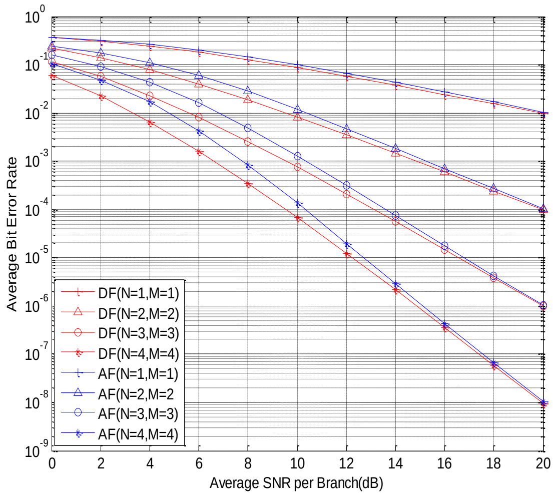

Fig 6. Comparison of ABER of amplify-and –forward relaying and of decode-and-forward relaying system.

Fig.6 serves to compare the variations of the ABERs for binary DPSK with the average SNR per branch for amplify-and-forward with that of decode-and-forward relaying over the flat Rayleigh fading channel. It is noted that the performance of a decode-and-forward relaying is better than the performance of the corresponding (i.e. of the same values of M and N) amplify-and-forward relaying only for the low SNR values, and the error performances are identical at high SNR values. The gaps in error performance between AF and DF relaying in the low SNR regimes also increase for large values of M and N.

6. CONCLUSION

We derived new closed form expressions of the MGF of the received signal at the destination for a symmetric dual-hop fixed amplify-and-forward MIMO-relay link for general values of number of receiving and transmitting antennas (M, N)at the relay. The statistics of the received SNR at the destination node such as the pdf, cdf and the moments are derived in closed forms and applied to study the outage probability and error performance. Finally, comparisons are made between the AF and DF relaying systems. The lower bounds of performance are also studied. The analysis reveals that relative to a single antenna relay, the MIMO-relay achieves very high performance gain in exchange of complexities and costs for more number of antenna deployment. The system performance for both the relaying types improves significantly (roughly 3 dB) when the number of input antennas (M) is varied from $M = N$ to $N + 1$ at both low and high SNR regimes. However, increasing $M > N + 1$, additional gain achieved is insignificant at low SNR regime and no gain improvement at high SNR regime. It is also shown that the best or optimum performance can be achieved when one additional antenna is installed at the output/input for a given number of input/output antenna(s) at the relay.

An important conclusions can be drawn here that for all practical purposes with MIMO-antenna relays the best error performance can be achieved when the relay has one additional transmitting antenna at the output than the number of receiving antennas at the input (i.e., $N = M + 1$) or the number of receiving antennas at the input is one more than the number of transmitting antennas at the output of the relay (i.e., $M = N + 1$).

7. ACKNOWLEDGEMENT

The authors acknowledge the facilities offered by the Department of ECE, Faculty of Engineering and Technology of Mody Institute of Technology and Science, Lakshmangarh, Dist. Sikar, Rajasthan (INDIA). One of the authors (PKG) wishes to thank Mr. Anup Dey of Kalyani Government Engineering College, Kolkata for his valuable suggestions.

8. REFERENCES

[1] Hasna, M.O., & Alouini, M.-S. 2003. End-to-end performance of transmission systems with relays over Rayleigh fading channels. IEEE Transaction on Wireless Communications, 2(6), 1126-1131.

[2] Sendonaris, A., Erkip, E., & Aazhang, B.2003. User cooperation diversity-part I. IEEE Transaction on Communication, 51(11)1939-1948.

[3] Wang, B., Zhang, J., & Host-Madsen, A. 2005. On the capacity of MIMO relay channels. IEEE Transaction on Information Theory, 51(1), 29–43.

[4] Adinoyi, A., & Yanikomeroglu,H. 2007. Cooperative Relaying in Multi-Antenna Fixed Relay Networks. IEEE Transaction on Wireless Communication, 6(2), 533-544.

[5] Dey, A., Ghosh, P. K., & Gupta, K. 2010. Performance of Multiple Selection Diversity Antennas at the Relay Input for Relaying System. In Proceedings of International Conference on Recent Trends in Information, Telecommunication and Computing (ITC), pp. 90-94, India.

[6] Fan, Y., & Thompson, J. 2007. MIMO configurations for relay channels: Theory and Practice. IEEE Transaction on Wireless Communication, 6(5),1774-1786.

[7] Yang, L., & Zhang, Q.T. 2009. Outage performance of MIMO relay channels with maximal ratio transmission. IEEE Electronics Letters, 45 (5).

[8] Ganwani, V., Dey, B. K., Sharma, G. V. V., Merchant. S. N. & Desai, U. B. 2009. Performance Analysis of Amplify and Forward Based Cooperative Diversity in MIMO Relay Channels. In Proceedings of IEEE 69th Vehicular Technology Conference (VTC), Spring (pp.1-5).

[9] Cresie, N., Davis, A.S., Folks, J.L. & Policello II, G.E.1981. The Moment-Generating Function and Negative Integer Moments. JSTOR, The American Statistician, 35(3), 148-150.

[10] Haykin, S., & Moher, M. 2005. Modern Wireless Communications. Pearson Education.

[11] Gradshteyn,I.S.,& Rzyhik,I.M. 1994. Table of Integrals, Series and Products (5th ed.). Academic Press.

[12] Oberhettinger, F. 1974. Tables of Mellin transforms, New York: Springer-Verlag.

[13] Hasna, M.O., & Alouini, M.-S. 2004. Harmonic Mean and End-to-End Performance of Transmission Systems With Relays", IEEE Transaction on Communication, 52(1), 1451–1458.

[14] The Wolfram function site: [online]. Available: http://Functions.wolfram.com.

[15] Simon, M. K., & Alouini, M.-S. 2000. Digital Communication over Fading Channels: A Unified Approach to Performance Analysis. New York: John Wiley & Sons.