International Journal of Computer Applications (0975 – 8887)
Volume 52– No.3, August 2012

Total Domination Number and Chromatic Number of a Fuzzy Graph

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ABSTRACT

A subset S of V is called a domination set in G if every vertex in V-S is adjacent to at least one vertex in S. A dominating set is said to be Fuzzy Total Dominating set if every vertex in V is adjacent to at least one vertex in S. Minimum cardinality taken over all total dominating set is called as fuzzy total domination number and is denoted by γ_{ft} (G). The minimum number of colours required to colour all the vertices such that adjacent vertices do not receive the same colour is the chromatic number χ(G). For any graph G a complete sub graph of G is called a clique of G. In this paper we find an upper bound for the sum of the fuzzy total domination and chromatic number in fuzzy graphs and characterize the corresponding extremal fuzzy graphs.

General Terms
G(μ,σ) be simple undirected fuzzy graph

Keywords
Fuzzy Total Domination Number, Chromatic Number, Clique, Fuzzy Graphs

1. INTRODUCTION

The study of dominating sets in graphs was begun by Ore and Berge, the domination number, total domination number are introduced by Cockayne and Hedetniemi. A Mathematical framework to describe the phenomena of uncertainty in real life situation is first suggested by L.A.Zadeh in 1965.
Research on the theory of fuzzy sets has been witnessing an exponential growth; both within mathematics and in its applications. This ranges from traditional mathematical subjects like logic, topology, algebra, analysis etc. consequently fuzzy set theory has emerged as potential area of interdisciplinary research and fuzzy graph theory is of recent interest.
The fuzzy definition of fuzzy graphs was proposed by Kaufmann [4], from the fuzzy relations introduced by Zadeh [9]. Although Rosenfeld[5] introduced another elaborated definition, including fuzzy vertex and fuzzy edges. Several fuzzy analogs of graph theoretic concepts such as paths, cycles connectedness etc. The concept of domination in fuzzy graphs was investigated by A.Somasundram, S.Somasundram [6]. A. Somasundram presented the concepts of independent domination, total domination, connected domination and domination in cartesian product and composition of fuzzy graphs([7][8]).

Several authors have studied the problem of obtaining an upper bound for the sum of a domination parameter and a graph theoretic parameter and characterized the corresponding extremal graphs. In [10], Paulraj Joseph J and Arumugam S proved that γ+k≤p. In[9], Paulraj Joseph J and Arumugam S proved that γ_c (G)+ $\chi \leq$ p+1. They also characterized the class of graphs for which the upper bound is attained. They also proved similar results for γ and γ_t. In[14], Mahadevan G introduced the concept the complementary perfect domination number γ_{cp} and proved that γ_{cp}(G)+ $\chi \leq$ 2n-2, and characterized the corresponding external graphs. In[15], S.Vimala and J.S.Sathya proved that γ_t (G)+χ(G)=2n-5. They also characterised the class of graphs for which the upper bound is attained. In this paper we obtain sharp upper bound for the sum of the fuzzy total domination number and chromatic number and characterize the corresponding extremal fuzzy graphs.

2. PRELIMINARIES

If X is collection of objects denoted generically by x, then a Fuzzy set $\tilde{A}$ in X is a set of ordered pairs: $\widetilde{\mathrm{A}} = \{(\mathrm{x}, \mu_{\widetilde{\mathrm{A}}}(\mathrm{x}))/ \mathrm{x} \in \mathrm{X}\}$, $\mu_{\widetilde{\mathrm{A}}}(\mathrm{x})$ is called the membership function of x in $\widetilde{\mathrm{A}}$ that maps X to the membership space M (when M contains only the two points 0 and 1). Let E be the (crisp) set of nodes. A fuzzy graph is then defined by, $\widetilde{\mathrm{G}}(x_i, x_j) = \{(x_i, x_j), \mu_{\widetilde{\mathrm{G}}}(x_i, x_j)/(x_i, x_j) \in \mathrm{E} \times \mathrm{E}\}$. $\widetilde{\mathrm{H}}(x_i, x_j)$ is a Fuzzy Sub graph of $\widetilde{\mathrm{G}}(x_i, x_j)$ if $\mu_{\widetilde{\mathrm{H}}}(x_i, x_j) \leq \mu_{\widetilde{\mathrm{G}}}(x_i, x_j)\ \forall (x_i, x_j) \in \mathrm{E} \times \mathrm{E}$, $\widetilde{\mathrm{H}}(x_i, x_j)$ is a spanning fuzzy sub graph of $\widetilde{\mathrm{G}}(x_i, x_j)$ if the node set of $\widetilde{\mathrm{H}}(x_i, x_j)$ and $\widetilde{\mathrm{G}}(x_i, x_j)$ are equal, that is if they differ only in their arc weights.

Let G(μ,σ) be simple undirected fuzzy graph. The degree of any vertex u in G is the number of edges incident with u and is denoted by d(u). The minimum and maximum degree of a vertex is denoted by δ(G) and Δ(G) respectively, P_n denotes the path on n vertices. The vertex connectivity κ(G) of a graph G is the minimum number of vertices whose removal results in a disconnected graph. The chromatic number χ is defined to be the minimum number of colours required to colour all the vertices such that adjacent vertices do not receive the same colour. For any graph G a complete sub graph of G is called a clique of G. The number of vertices in a largest clique of G is called the clique number of G.

A subset S of V is called a dominating set in G, if every vertex in V-S is adjacent to at least one vertex is S. The minimum cardinality taken over all minimal dominating sets in G is called the domination number of G and is denoted by γ. A dominating set S is said to be fuzzy total dominating set if every vertex in V is adjacent to at least one vertex in S. Minimum cardinality taken over all total dominating set is called as fuzzy total domination number and is denoted by γ_{ft} (G). We use the following previous results.

2.1 Theorem: [1]: For any connected graph G, γ_t (G) $\leq n$

2.2 Theorem: [2]: For any connected graph G, χ(G)≤ Δ(G)+1.

3. MAIN RESULTS

3.1 Theorem: For any connected fuzzy graph G, γ_t (G)+ χ(G)≤2n and the equality holds if and only if G≅K_1

Proof: γ_t (G)+χ(G) ≤ n+Δ+1=n+(n-1)+1≤2n. If γ_t (G)+χ(G)=2n the only possible case is γ_t (G)=n and χ(G)=n, Since χ(G)=n, G=K_n , But for K_n, γ_t (G)=1, so that G≅K_1. Converse is obvious.

3.2 Theorem: For any connected fuzzy graph G, γ_t (G)+χ(G)=2n-1 and the equality holds if and only if G≅K_2

Proof: Assume that γ_t (G)+χ(G)=2n-1. This is possible only if γ_t (G)=n and χ(G)=n-1 (or) γ_t (G)=n-1 and χ(G)=n.

Case (i) Let γ_t (G)=n and χ(G)=n-1.

Since χ(G)= n-1, G contains a clique K on n-1 vertices. Let x be a vertex of G-K_{n-1}. Since G is connected the vertex x is adjacent to one vertex u_i for some i in K_{n-1} {u_i} is γ_t – set, so that γ_t (G)=1, we have n=1. Then $\chi = 0$, which is a contradiction. Hence no fuzzy graph exists.

Case (ii) Let γ_t (G)=n-1 and χ(G)=n

Since χ(G)=n, G=K_n , But for K_n , γ_t (G)=1, so that n=2, χ =2 Hence G≅K_2. Converse is obvious.

3.3 Theorem: For any connected fuzzy graph G, γ_t (G)+χ(G)=2n-2 and the equality holds if and only if G≅K_3

Proof: Assume that γ_t (G)+χ(G)=2n-2. This is possible only if γ_t (G)=n and χ(G)=n-2 (or) γ_t (G)=n-1 and χ(G)=n-1 (or) γ_t (G)=n-2 and χ(G)=n.

Case (i) Let γ_t (G)=n and χ(G)=n-2.

Since χ (G)= n-2, G contains a clique K on n-2 vertices. Let S={x,y}∈V-S. Then $< S >= K_2$ or $\overline{K_2}$

Subcase (a) Let $< S >= K_2$ Since G is connected, x is adjacent to some u_i of K_{n-2}. Then {x,u_i} is γ_t - set, so that γ_t (G)=2 and hence n=2. But χ(G)=n-2=0. Which is a contradiction. Hence no fuzzy graph exists.

Subcase (b) Let $< S >= \overline{K_2}$ Since G is connected, x is adjacent to some u_i of K_{n-2}. Then y is adjacent to the same u_i of K_{n-2}. Then {u_i} γ_t - set, so that γ_t (G)=1 and hence n=1. But χ(G)=n-2=negative value. Which is a contradiction. Hence no fuzzy graph exists, or y is adjacent to u_j of K_{n-2} for i≠j. In this case {u_i,u_j} γ_t - set, so that γ_t (G)=2 and hence n=2. But χ(G)=0. Which is a contradiction. Hence no fuzzy graph exists.

Case (ii) Let γ_t (G)=n-1 and χ(G)=n-1.

Since χ (G)= n-1, G contains a clique K on n-1 vertices. Let x be a vertex of G-K_{n-1}. Since G is connected, x is adjacent to one vertex u_i for some i in K_{n-1}, so that γ_t (G) = 1, we have n=2. Then $\chi = 1$, which is a contradiction. Hence no fuzzy graph exists.

Case (iii) Let γ_t (G)=n-2 and χ(G)=n

Since χ(G)=n, G=K_n , But for K_n , γ_t (G) = 1,so that n=3, χ = 3 Hence G≅K_3. Converse is obvious.

3.4 Theorem: For any connected fuzzy graph G, γ_t (G)+ χ(G)=2n-3 and the equality holds if and only if G≅P_3,K_4

Proof: Assume that γ_t (G)+ χ(G)=2n-3. This is possible only if γ_t (G)=n and χ(G)=n-3 (or) γ_t (G)=n-1 and χ(G)=n-2 (or) γ_t (G)=n-2 and χ(G)=n-1(or) γ_t (G)=n-3 and χ(G)=n.

Case (i) Let γ_t (G)=n and χ(G)=n-3.

Since χ (G)= n-3, G contains a clique K on n-3 vertices. Let S={x,y,z}∈V-S. Then $< S >= K_3$, $\overline{K_3}$, K_2∪K_1,P_3

Subcase (i) Let $< S >= K_3$. Since G is connected, x is adjacent to some u_i of K_{n-3}. Then {x,u_i} is γ_t - set, so that γ_t (G)=2 and hence n=2. But χ(G)=n-3=negative value. Which is a contradiction. Hence no fuzzy graph exists.

Subcase (ii) Let $< S >= \overline{K_3}$ Since G is connected, one of the vertices of K_{n-3} say u_i is adjacent to all the vertices of S or two vertices of S or one vertex of S. If u_i for some i is adjacent to all the vertices of S, then {u_i} in K_{n-3} is a γ_t -set of G, so that γ_t (G)=1 and hence n=1. But χ(G)=n-3=negative value. Which is a contradiction. Hence no fuzzy graph exists. Since G is connected u_i for some i is adjacent to two vertices of S say x and y and z is adjacent to u_j for i≠j in K_{n-3}, then {u_i,u_j} in K_{n-3} is γ_t -set of G, so that γ_t (G)=2 and hence n=2. But χ(G)=n-3=negative value. Which is a contradiction. Hence no fuzzy graph exists. If u_i for some i is adjacent to x and u_j is adjacent to y and u_k is adjacent to z, then {u_i,u_j,u_k} for i≠j≠k in K_{n-3} is a γ_t -set of G. so that γ_t (G)=3 and hence n=3. But χ(G)=n-3=0. Which is a contradiction. Hence no fuzzy graph exists.

Subcase (iii) Let $< S >= P_3 = \{x, y, z\}$. Since G is connected, x(or equivalently z) is adjacent to u_i for some i in K_{n-3}. Then {x,y,u_i} is a γ_t -set of G. so that γ_t (G)=3 and hence n=3. But χ(G)=n-3=0. Which is a contradiction. Hence no fuzzy graph exists. If u_i is adjacent to y then {u_i,y} is a γ_t -set of G. so that γ_t (G)=2 and hence n=2. But χ(G)=n-3=negative value. Which is a contradiction. Hence no fuzzy graph exists.

Subcase (iv) Let $< S >= K_2 \cup K_1$ Let xy be the edge and z be the isolated vertex of $K_2 \cup K_1$ Since G is connected, there exists a u_i in K_{n-3} is adjacent to x and z. Then {u_i} is γ_t -set of G, so that γ_t (G)=1 and hence n=1. But χ(G)=n-3=negative value. Which is a contradiction. Hence no fuzzy graph exists. If z is adjacent to u_j for some i≠j then {u_i,u_j} for i≠j is γ_t -set of G, so that γ_t (G)=2 and hence n=2. But χ(G)=n-3=negative value. Which is a contradiction. Hence no fuzzy graph exists.

Case (ii) Let γ_t (G)=n-1 and χ(G)=n-2.

Since χ(G)=n-2, G contains a clique K on n-2 vertices. Let S={x,y}∈V-S. Then $< S >= K_2$ or $\overline{K_2}$

Subcase (a) Let $< S >= K_2$ Since G is connected, x(or equivalently y) is adjacent to some u_i of K_{n-2}. Then {x,u_i} is γ_t - set, so that γ_t (G)=2 and hence n=3. But χ(G)=n-2=1 for which G is totally disconnected, which is a contradiction. Hence no fuzzy graph exists.

Subcase (b) Let $< S >= \overline{K_2}$ Since G is connected, x is adjacent to some u_i of K_{n-2}. Then y is adjacent to the same u_i of K_{n-2}. Then {u_i} is γ_t - set, so that γ_t (G)=1 and hence n=2. But χ(G)=n-2=0. Which is a contradiction. Hence no fuzzy graph exists. Otherwise x is adjacent to u_i of K_{n-2} for some i and y is adjacent to u_j of K_{n-2} for i≠j. In this $\{u_i,u_j\}$ γ_t - set, so that γ_t(G)=2 and hence n=3. But χ(G)=1 for which G is totally disconnected. Which is a contradiction. In this case also no fuzzy graph exists.

Case (iii) Let γ_t (G)=n-2 and χ(G)=n-1.

Since χ(G)=n-1, G contains a clique K on n-1 vertices. Let x be a vertex of K_{n-1}. Since G is connected the vertex x is adjacent to one vertex u_i for some i in K_{n-1} so that γ_t (G)=1, we have n=3 and $\chi = 2$ .Then K=K_2= uv. If x is adjacent to u_i, then G ≅ P_3.

Case (iv) Let γ_t (G)=n-3 and χ(G)=n

Since χ(G)=n, G=K_n, But for K_n, γ_t (G)=1, so that n=4, $\chi = 4$ Hence G≅K_4. Converse is obvious.

3.5 Theorem: For any connected fuzzy graph G, γ_t(G)+χ(G)=2n-4 and the equality holds if and only if G≅P_4,K_5 or the graph in figure 3.1

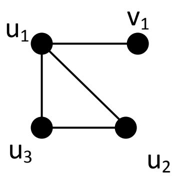

Figure 3.1

Proof: Assume that γ_t(G)+χ(G)=2n-4. This is possible only if γ_t(G)=n and χ(G)=n-4 (or) γ_t(G)=n-1 and χ(G)=n-3 (or) γ_t(G)=n-2 and χ(G)=n-2 (or) γ_t(G)=n-3 and χ(G)=n-1 (or) γ_t(G)=n-4 and χ(G)=n.

Case (i) Let γ_t (G)=n and χ(G)=n-4.

Since χ(G)=n-4, G contains a clique K on n-4 vertices. Let S = $\{v_1, v_2, v_3, v_4\}$. Then the induced subgraph <s> has the following possible cases K_4,$\overline{K}_4$,P_4,P_3UK_1,K_2UK_2,K_3UK_1,$K_{1,3}$

In all the above cases, it can be verified that no new fuzzy graphs exists.

Case(ii) Let γ_t (G)=n-1 and χ(G)=n-3.

Since χ(G)=n-3, G contains a clique K on n-3 vertices. Let S={x,y,z}∈V-S. Then <S>=K_3 ,$\overline{K_3}$, K_2∪K_1,P_3

Subcase (i) Let $< S >= K_3$. Since G is connected, x is adjacent to some u_i of K_{n-3}. Then $\{x,u_i\}$ is γ_t - set, so that γ_t(G)=2 and hence n=3. But χ(G)=n-3=0. Which is a contradiction. Hence no fuzzy graph exists.

Subcase (ii) Let $< S >= \overline{K_3}$ Since G is connected, one of the vertices of K_{n-3} say u_i is adjacent to all the vertices of S or two vertices of S or one vertex of S. If u_i for some i is adjacent to all the vertices of S, then $\{u_i\}$ in K_{n-3} is γ_t-set of G. so that γ_t(G)=1 and hence n=2. But χ(G)=n-3=negative value. Which is a contradiction. Hence no fuzzy graph exists. If u_i for some i is adjacent to two vertices of S say x and y then G is connected, z is adjacent to u_j for i≠j in K_{n-3}, then $\{u_i,u_j\}$ in K_{n-3} is γ_t-set of G, so that γ_t(G)=2 and hence n=3. But χ(G)=n-3=0. Which is a contradiction. Hence no fuzzy graph exists. If u_i for some i is adjacent to x and u_j is adjacent to y and u_k is adjacent to z, then $\{u_i,u_j,u_k\}$ for i≠j≠k in K_{n-3} is γ_t -set of G. so that γ_t(G)=3 and hence n=4. But χ(G)=1 for which G is totally disconnected. Which is a contradiction. Hence no fuzzy graph exists.

Subcase (iii) Let $< S >= P_3 = \{x, y, z\}$. Since G is connected, x(or equivalently z) is adjacent to u_i for some i in K_{n-3}. Then $\{x,y,u_i\}$ is γ_t-set of G. so that γ_t(G)=3 and hence n=4. But χ(G)=n-3=1. Which is a contradiction. Hence no fuzzy graph exists. If u_i is adjacent to y then $\{u_i,y\}$ is γ_t-set of G. so that γ_t(G)=2 and hence n=3. But χ(G)=n-3=0. Which is a contradiction. Hence no fuzzy graph exists.

Subcase (iv) Let $< S >= K_2 \cup K_1$ Let xy be the edge and z be a isolated vertex of $K_2 \cup K_1$ Since G is connected, there exists a u_i in K_{n-3} is adjacent to x and z also adjacent to same u_i Then $\{u_i\}$ is a γ_t-set of G. So that γ_t(G)=1 and hence n=2. But χ(G)=n-3=negative value. Which is a contradiction. Hence no fuzzy graph exists. If z is adjacent to u_j for some i≠j then $\{u_i,u_j\}$ for i≠j is a is a γ_t-set of G. so that γ_t(G)=2 and hence n=3. But χ(G)=n-3=0.Which is a contradiction. Hence no fuzzy graph exists.

Case (iii) Let γ_t (G)=n-2 and χ(G)=n-2.

Since χ (G)= n-2, G contains a clique K on n-2 vertices. Let S={x,y}∈V-S. Then $< S >= K_2$ or $\overline{K_2}$

Subcase (a) Let $< S >= K_2$. Since G is connected, x(or equivalently y) is adjacent to some u_i of K_{n-2}. Then $\{x,u_i\}$ is γ_t- set, so that γ_t(G)=2 and hence n=4. But χ(G)=n-2=2. Then G≅P_4.

Subcase (b) Let $< S >= \overline{K_2}$, since G is connected, x is adjacent to some u_i of K_{n-2}. Then y is adjacent to the same u_i of K_{n-2}. Then $\{u_i\}$ is γ_t- set, so that γ_t(G)=1 and hence n=3. But χ(G)=n-2=1.Which is a contradiction. Hence no fuzzy graph exists, or y is adjacent to u_j of K_{n-2} for i≠j. In this $\{u_i,u_j\}$ is γ_t- set, so that γ_t(G)=2 and hence n=4. But χ(G)=2.Then G≅P_4.

Case (iv) Let γ_t (G)=n-3 and χ(G)=n-1.

Since χ (G)= n-1, G contains a clique K on n-1 vertices. Let x be a vertex of G-K_{n-1}. Since G is connected the vertex x is adjacent to one vertex u_i for some i in K_{n-1}, so that γ_t(G) = 1, we have n=4 and $\chi = 3$. Then K=K_3 Let u_1,u_2,u_3 be the vertices of K_3. Then x must be adjacent to only one vertex of G-K_3. Without loss of generality let x be adjacent to u_1.If d(x)=1, then G ≅ G_1.(in Fig 2.1)

Case (v) Let γ_t(G)=n-4 and χ(G)=n

Since χ(G)=n, G=K_n, But for K_n, γ_t (G)=1, so that n=5, $\chi = 5$. Hence G≅K_5. Converse is obvious.

Theorem 2.6 For any connected fuzzy graph G, γ_t(G)+χ(G)=2n-5 for any n>4, if and only if G is isomorphic to K_6,K_3(P_3),K_3(1,1,0),P_5,K_4(1,0,0,0),$K_{1,3}$ (or) any one of the following fuzzy graphs in the figure 3.2.

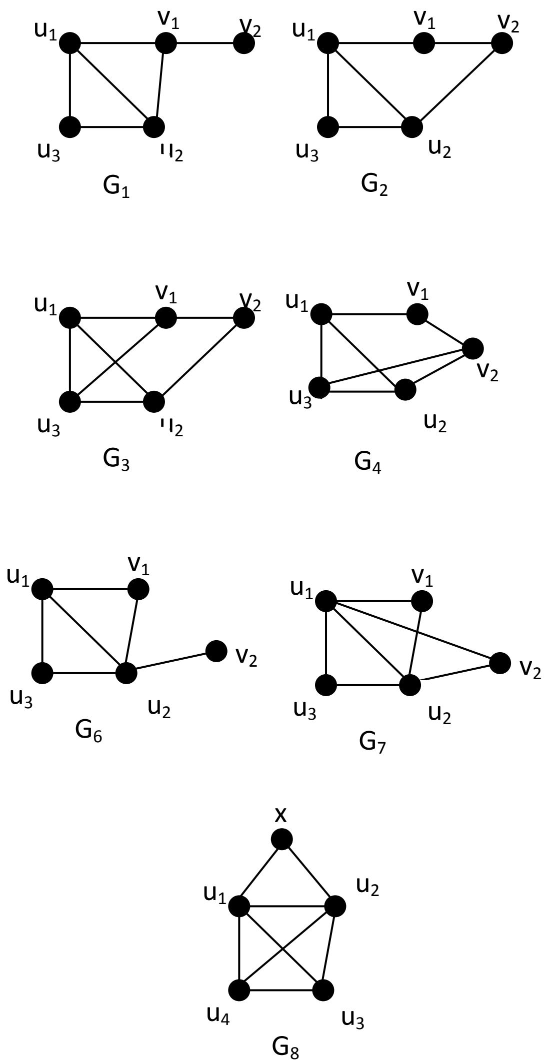

Figure 3.2

Proof: If G is any one of the graphs in the theorem, then it can be verified that γ_t (G)+ χ(G)=2n-5. Conversely set γ_t (G)+χ(G)=2n-5 then γ_t (G)=n and χ(G)=n-5 (or) γ_t (G)=n-1 and χ(G)=n-4 (or) γ_t (G)=n-2 and χ(G)=n-3 (or) γ_t (G)=n-3 and χ(G)=n-2 (or) γ_t (G)=n-4 and χ(G)=n-1 (or) γ_t (G)=n-5 and χ(G)=n

Case (i): Let γ_t (G)=n and χ(G)=n-5

Since χ(G)=n-5, G contains a clique K in n-5 vertices (or) does not contain a clique K on n-5 vertices. Let S={v_1,v_2,v_3,v_4,v_5}. Then the induced subgraph <S> has the following possible cases. K_5,$\overline{K}_5$,P_5,P_3∪P_2,P_3∪ $\overline{K}_2$,K_4∪K_1,P_4∪K_1,K_3∪K_2,K_3∪ $\overline{K}_2$.

In all the above cases, it can be verified that no new fuzzy graph exists.

Case (ii): Let γ_t(G)=n-1 and χ(G)=n-4.

Since χ(G)=n-4 G contains a clique K on n-4 vertices. Let S={v_1,v_2,v_3,v_4}. Then the induced subgraph <s> has the following possible cases K_4,$\overline{K}_4$,P_4,P_3UK_1, K_2UK_2,K_3UK_1

In all the above cases, it can be verified that no new fuzzy graph exists.

Case (iii): Let γ_t(G) = n-2 and χ(G)=n-3.

Since χ (G) = n-3, G contains a clique with n-3 vertices. Let S ={v_1, v_2, v_3} Then the induced subgraph <S> has the following possible cases K_3,$\overline{K}_3$,K_2∪K_1,P_3.

Subcase (i): Let <S>=K_3.Since G is connected there exists a vertex u_i in K_{n-3} which is adjacent to any one of {v_1,v_2,v_3} without loss of generality let u_i be adjacent to v_1, then {v_1, u_j} is γ_t - set of G, so that n=4. But χ(G)=1 which is a contradiction. Hence no graph exists.

Subcase (ii): Let <S>=$\overline{K}_3$. Let {v_1,v_2,v_3} be the vertices of $\overline{K}_3$.Since G is connected all the vertices of $\overline{K}_3$ are adjacent to one vertex say u_i, in K_{n-3} (or) 2 vertices of $\overline{K}_3$ are adjacent to the vertex u_i and remaining one vertex of $\overline{K}_3$ is adjacent to u_j for i≠ j inK_{n-3} (or) all the vertices of $\overline{K}_3$ are adjacent to the distinct vertices of K_{n-3}. If all the vertices of $\overline{K}_3$ are adjacent to one vertex say u_i in K_{n-3}, the {v_1,u_i} is γ_t - set of G, so that n = 4, χ(G)=1 which is a contradiction. Hence no fuzzy graph exists. If two vertices of $\overline{K}_3$ are adjacent to the vertex u_i and the remaining one is adjacent to u_j i≠j in K_{n-3} then {u_i,u_j,v_3} is γ_t -set of G. So that n=4 which is a contradiction. Hence no graphs exists. If all the vertices of $\overline{K}_3$ are adjacent to the vertices u_i, u_j, u_k i≠j≠k respectively.Then {u_i,u_j,v_3} is γ_t -set of G. Hence n=5,χ(G)=2 which is a contradiction. Hence no graph exists.

Subcase (iii): Let <S> =P_3. Since G is connected there exists a vertex u_i is adjacent to any one of {v_1,v_3} or v_2. If u_i is adjacent to any of the {v_1,v_3}, then {v_1,v_2,u_i} is γ_t -set of G. Hence n=5 so that K=K_2. Let u_1, u_2 are the vertices of K_2. Let v_1 be adjacent to u_i and if d(v_1)=d(v_2)=2, d(v_3)=1 then G≅ P_5. If v_2 is adjacent to u_i then{u_i,v_2} is γ_t - set of G. Hence n=4 χ(G) = 1 which is a contradiction. Hence no graph exists. If the degree of the vertices is increased then no new fuzzy graphs exists.

Subcase (iv): Let <S> = K_2∪K_1. Let {v_1,v_2} be the vertices of K_2 and v_3 be the isolated vertex. Since G is connected, there exists a vertex u_i is adjacent to any one of {v_1,v_2} and v_3 (or) u_i is adjacent to any one of {v_1,v_2} and u_j for i≠j is adjacent to v_3. If u_i is adjacent to any one of {v_1,v_2} and v_3,{u_i,v_1} is γ_t -set of G. Hence n=4 and χ(G)=1 which is a contradiction. Hence no graph exists.

If u_i is adjacent to any one of {v_1,v_2} and u_j j≠i is adjacent to v_3. In this case {v_1,u_i,u_j} is γ_t -set of G. Hence n=5,χ(G)=2. Then G≅ P_5.If the degree of the vertices is increased then no new fuzzy graphs exists.

Case (iv): Let γ_t (G)=n-3, χ(G)=n-2.

Since χ(G)=n-2, G contains a clique K on n-2 vertices. If G contains a clique K on n-2 vertices. Let S={v_1,v_2}∈V(G)–V(K). Then the induced sub graph <S> has the following possible cases.

Subcase (i): Let <S> = K_2. Since G is connected, there exists a vertex u_i in K_{n-2} is adjacent to any one of {v_1,v_2}. Then {u_i,v_1} is a γ_t -set of G. Hence γ_t (G)=2,so that n=5,χ(G)=3. Hence K≅K_3. Let u_1,u_2,u_3 be the vertices of K_3.Let u_1 be adjacent to v_i.

If $d(v_1)=2$ $d(v_2)=1$ then $G \cong K_3(P_3)$ If $d(v_1)=3$ $d(v_2)=1$ $G \cong G_1$ Let u_1 be adjacent to v_1 and u_2 be adjacent to v_2 . If $d(v_1)=d(v_2)=2$ the n $G \cong G_2$ If $d(v_1)=3$ $d(v_2)=2$ then $G \cong G_3$ If $d(v_1)=2$, $d(v_2)=3$ then $G \cong G_4$. If the degree of the vertices is increased then no new fuzzy graphs exists.

Subcase (ii): Let<S> = $\overline{K}_2$. Since G is connected all the vertices of $\overline{K}_2$ are adjacent to one vertex say u_i in K_{n-2} (or) distinct vertices in K_{n-2}. If all the vertices of $\overline{K}_2$ are adjacent to one vertex say u_i in K_{n-2}. In this case $\{u_i\}$ is γ_t -set of G. Since γ_t (G)=1 so that n=4 χ(G)=2. Hence $K \cong K_2$. Let u_1, u_2 be the vertices of K_2. Let u_1 be adjacent to both v_1 and v_2 then $G \cong K_{1,3}$. If the two vertices of $\overline{K}_2$ are adjacent to the distinct of K_{n-2}. In this case $\{u_i,u_j\}$ for $i \neq j$ forms γ_t - set of G. Hence n=5,χ(G)=3. So that $K \cong K_3$. Let $\{u_1, u_2, u_3\}$ be the vertices of K_3. Let u_1 be adjacent to v_1 and u_2 be adjacent to v_2 then $G \cong K_3(1,1,0)$. If $d(v_1)=1$ and $d(v_2)=2$ than $G \cong G_5$. If $d(v_1)=2$ $d(v_2)=1$ then $G \cong G_6$ If $d(v_1)=2$ $d(v_2)=2$ then $G \cong G_7$. If the degree of the vertices is increased then no new fuzzy graphs exists.

Case (v): Let γ_t (G)=n-4 χ(G)=n-1

Since χ(G)=n-1, G contains a clique K on n-1 vertices. Let x be a vertex of G-K_{n-1}. Since G is connected the vertex x is adjacent to one vertex u_i of K_{n-1} so that γ_t (G)=1.Hence n=5 χ(G)=4, so that $K \cong K_4$.Let $\{u_1, u_2\ u_3, u_4\}$be the vertices of K_4.Without loss of generality let x be adjacent to u_1, of K_4, then $G \cong K_4(P_2)$. If $d(v_1)=2$ then $G \cong G_8$. If $d(v_1)=3$ then $G \cong G_9$

Case (vi): Let γ_t (G)=n-5 χ(G)=n

Since χ(G)=n,G≅Kn. But for K_n γ_t (G)=1, so that n=6. Hence $G \cong K_6$.

4. CONCLUSION

In this paper, upper bound of the sum of total domination and chromatic number is proved. In future this result can be extended to various domination parameters. The structure of the graphs had been given in this paper can be used in models and networks. The authors have obtained similar results with large cases of graphs for which $\boldsymbol{\gamma_t}$ (G)+$\boldsymbol{\chi}$(G)=2n-6, $\boldsymbol{\gamma_t}$ (G)+$\boldsymbol{\chi}$(G)=2n-7,$\boldsymbol{\gamma_t}$ (G)+$\boldsymbol{\chi}$(G)=2n-8

5. ACKNOWLEDGMENTS

We extend our thanks the Lord Almighty to give the knowledge to do this work.

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