Operations on Intuitionistic Fuzzy Hypergraphs

R.Parvathi
Department of Mathematics
Vellalar College for Women
Erode, Tamilnadu, India.

S.Thilagavathi
Department of Mathematics
Vellalar College for Women
Erode, Tamilnadu, India.

M.G.Karunambigai
Department of Mathematics
Sri Vasavi College, Erode
Tamilnadu, India.

ABSTRACT
Hypergraph is a graph in which an edge can connect more than two vertices. Hypergraphs can be applied to analyze architecture structures and to represent system partitions. The concept of hypergraphs was extended to fuzzy hypergraph. In this paper, we extend the concepts of fuzzy hypergraphs into that of intuitionistic fuzzy hypergraphs. Based on the definition of intuitionistic fuzzy graph, operations like complement, join, union, intersection, ringsum, cartesian product, composition are defined for intuitionistic fuzzy graphs. The authors further proposed to apply these operations in clustering techniques.

Keywords
Intuitionistic fuzzy hypergraph (IFHG), Complement, Union, Join, Intersection, Ringsum, Cartesian product, Composition.

1. INTRODUCTION
Hypergraph theory, originally developed by C.Berge in 1960, is a generalization of graph theory. The concept of hypergraphs can model more general types of relations than binary relations. The notion of hypergraphs has been extended in fuzzy theory and the concept of fuzzy hypergraphs was proposed by Lee-Kwang and S.M.Chen. The concept of an intuitionistic fuzzy graph (IFG) was introduced by Atanassov [1,2,3,4]. The authors have already introduced the concept of intuitionistic fuzzy hypergraph [7]. Operations on IFGs have also been analyzed by the authors [5,6]. Akram [8] applied the concepts in [7] to a real-life problem with a numerical example.

2. PRELIMINARIES
Definition 2.1
An intuitionistic fuzzy graph (IFG) is of the $G = \langle V, E\rangle$ where (i) $V = \{v_1, v_2, ..., v_n\}$ such that $\mu_1 : V \to [0, 1]$ and $\gamma_1 : V \to [0, 1]$ denote the degree of membership and non-membership of the element $v_i \in V$ respectively and

$$0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1 \quad (1)$$

for every $v_i \in V$, (i = 1, 2, …, n), (ii) $E \subseteq V \times V$ where $\mu_2 : V\times V \to [0,1]$ and $\gamma_2 : V \times V \to [0,1]$ are such that

$$\mu_2(v_i, v_j) \le \mu_1(v_i).\mu_1(v_j) \quad (2)$$

$$\gamma_2(v_i, v_j) \le \gamma_1(v_i).\gamma_1(v_j) \quad (3)$$

and $0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1 \quad (4)$

for every $(v_i, v_j) \in E$.

Definition 2.2
An IFHG H is an ordered pair $H = \langle V, E\rangle$ where (i) $V = \{v_1, v_2, ..., v_n\}$, a finite set of vertices (ii) $E = \{E_1, E_2, ..., E_n\}$, a family of intuitionistic fuzzy subsets of *V*. . (iii) $E_j = \{(v_i, \mu_j(v_i), \gamma_j(v_i)) : \mu_j(v_i), \gamma_j(v_i) \ge 0$ and $0 \le \mu_j(v_i) + \gamma_j(v_i) \le 1$, j = 1, 2 ,…, m (iv) $E_j \ne \varphi$ j = 1,2,…,m (v) $\cup_j \text{supp}(E_j) = V$, j=1, 2 ,…, m

Here the edges E_j are IFSs. $\mu_j(x_i)$ and $\gamma_j(x_i)$ denote the degree of membership and non-membership of the vertex v_i to edge E_j. Thus, the elements of the incidence matrix of IFHG are of the form $(a_{ij}, \mu_j(x_i), \gamma_j(x_i))$. The sets *V* and *E* are crisp sets.

Notations
The triple $\langle v_i, \mu_{1i}, \gamma_{1i}\rangle$ denotes the degree of membership and non-membership of the vertex v_i. The triple $\langle e_{ij}, \mu_{2ij}, \gamma_{2ij}\rangle$ denotes the degree of membership and non-membership of the edge $e_{ij} = (v_i, v_j)$ on V. That is,

$$\mu_{1i} = \mu_1(v_i), \gamma_{1i} = \gamma_1(v_i) \text{ and } \mu_{2ij} = \mu_2(v_i, v_j), \gamma_{2ij} = \gamma_2(v_i, v_j).$$

Note 2.3
(i) When $\mu_{2ij} = \gamma_{2ij} = 0$, for some *i* and *j*, then there is no edge between v_i and v_j. Otherwise there exists edge between v_i and v_j. (ii) If one of the inequalities (1) or (2) or (3) or (4) is not satisfied, then *G* is not an IFHG.

Definition 2.4

An IFHG , $G = \langle V, E \rangle$ is said to be a semi - μ strong IFHG if $\mu_{2ij} = \mu_{1i}.\mu_{1j}$ for every *i* and *j*.

Definition 2.5

An IFHG , $G = \langle V, E \rangle$ is said to be a semi - γ strong IFHG if $\gamma_{2ij} = \gamma_{1i}.\gamma_{1j}$ for every *i* and *j*.

Definition 2.6

An IFHG $G = \langle V, E \rangle$ is said to be a strong IFHG if $\mu_{2ij} = \mu_{1i}.\mu_{1j}$ and $\gamma_{2ij} = \gamma_{1i}.\gamma_{1j}$ for all $(v_i, v_j) \in E$.

Definition 2.7

An IFHG , $G = \langle V, E \rangle$ is said to be a complete - μ strong IFHG if $\mu_{2ij} = \mu_{1i}.\mu_{1j}$ and $\gamma_{2ij} < \gamma_{1i}.\gamma_{1j}$

Definition 2.8

An IFHG , $G = \langle V, E \rangle$ is said to be a complete - γ strong IFHG if $\mu_{2ij} < \mu_{1i}.\mu_{1j}$ and $\gamma_{2ij} = \gamma_{1i}.\gamma_{1j}$ for all *i* and *j*.

Definition 2.9

An IFHG , $G = \langle V, E \rangle$ is said to be *a complete* IFHG if $\mu_{2ij} = \mu_{1i}.\mu_{1j}$ and $\gamma_{2ij} = \gamma_{1i}.\gamma_{1j}$, for every $(v_i, v_j) \in V$.

3. COMPLEMENT OF AN IFHG

Definition 3.1

The complement of an IFHG $G = \langle V, E \rangle$ is an IFHG, $\overline{G} = \langle \overline{V}, \overline{E} \rangle$, where

- $\overline{V} = V$
- $\overline{\mu_{1i}} = \mu_{1i}$ and $\overline{\gamma_{1i}} = \gamma_{1i}$, for all *i* = 1, 2, …, n.
- $\overline{\mu_{2ij}} = (\mu_{1i}.\mu_{1j}) - \mu_{2ij}$ and $\overline{\gamma_{2ij}} = (\gamma_{1i}.\gamma_{1j}) - \gamma_{2ij}$ for all *i*, *j* = 1, 2, …, n.

Example 3.2

Let $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{e_1, e_2\}$

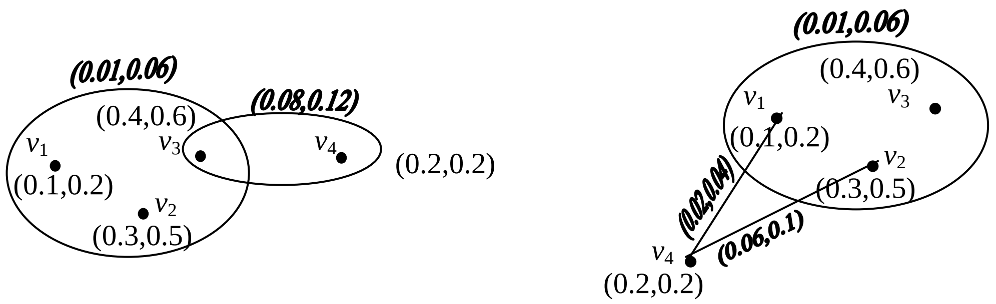

Fig 1: An Intuitionistic fuzzy hypergraph and its complement

Note 3.3

One can easily verify that $\overline{\overline{G}} = G$.

Proposition 3.4

(i) The complement of a complete μ - strong IFHG is a complete IFHG.

(ii) The complement of a complete γ - strong IFHG is a complete IFHG.

Proof

(i) Let $G = \langle V, E \rangle$ be a complete μ - strong IFHG. Therefore, $\mu_{2ij} = \mu_{1i}.\mu_{1j}$ and $\gamma_{2ij} < \gamma_{1i}.\gamma_{1j}$ for all *i* and *j*.

To prove that either (a) $\overline{\mu}_{2ij} > 0$ or $\overline{\gamma}_{2ij} > 0$, (b) $\overline{\mu}_{2ij} = 0$ or $\overline{\gamma}_{2ij} > 0$.

That is, $\overline{\mu}_{2ij} = (\mu_{1i}.\mu_{1j}) - \mu_{2ij}$

$= 0$ if $\mu_{2ij} > 0$

$= \mu_{1i}$ if $\mu_{2ij} = 0$ and

$\overline{\gamma}_{2ij} = (\gamma_{1i}.\gamma_{1j}) - \gamma_{2ij} > 0$ for every *i* and *j*, since *G* is a complete μ - strong IFHG. (ii) Proof of (ii) is the same as proof of (i).

Definition 3.5

Consider the two IFHGs $G_1 = \langle V_1, E_1 \rangle$ and $G_2 = \langle V_2, E_2 \rangle$. An isomorphism between two IFHGs G_1 and G_2 , denoted by $G_1 \cong G_2$, is a bijective map $h : V_1 \to V_2$ which satisfies $\mu_1(v_i) = \mu_1'(h(v_i))$; $\gamma_1(v_i) = \gamma_1'(h(v_i))$ and

$\mu_2(v_i,v_j)=\mu_2'(h(v_i),h(v_j))$; $\gamma_2(v_i,v_j)=\gamma_2'(h(v_i),h(v_j))$ for every $(v_l,v_j)\in V$.

Definition 3.6

An IFHG G is self-complementary if G is isomorphic to $\overline{G}$. Symbolically, $G_1\cong G_2$.

Theorem 3.7

Let $G=\langle V,E\rangle$ be a self-complementary IFHG. Then, there exists an isomorphism $h:V\to V$ such that $\overline{\mu_1}(v_i)=\mu_{1i}$; $\overline{\gamma_1}(v_i)=\gamma_{1i}$, for every $v_i\in V$ and $\overline{\mu_2}(h(v_i),h(v_j))=\mu_{2i}$; $\overline{\gamma_2}(h(v_i),h(v_j))=\gamma_{2i}$ for every $(v_l,v_j)\in V$.

Proof

By Definition 3.1, we have

$$\overline{\mu_2}(h(v_i),h(v_j))=(\overline{\mu_1}(h(v_i)).\overline{\mu_1}(h(v_j)))-\mu_{2ij}$$

$$\Rightarrow \mu_2(v_i,v_j)=\mu_{1i}.\mu_{1j}-\mu_{2ij}\Rightarrow \mu_{2ij}=\mu_{1i}.\mu_{1j}-\mu_{2ij}$$

$$\Rightarrow \sum_{i\neq j}\mu_{2ij}=\sum_{i\neq j}(\mu_{1i}.\mu_{1j})-\sum_{i\neq j}\mu_{2ij}\quad \Rightarrow 2\sum_{i\neq j}\mu_{2ij}=\sum_{i\neq j}(\mu_{1i}.\mu_{1j})$$

$$\Rightarrow \sum_{i\neq j}\mu_{2ij}=\frac{1}{2}\sum_{i\neq j}(\mu_{1i}.\mu_{1j}).$$

and

$$\overline{\gamma_2}(h(v_i),h(v_j))=(\overline{\gamma_1}(h(v_i)).\overline{\gamma_1}(h(v_j)))-\gamma_{2ij}$$

$$\Rightarrow \gamma_{2ij}=(\gamma_{1i}.\gamma_{1j})-\gamma_{2ij}\Rightarrow \sum_{i\neq j}\gamma_{2ij}=\sum_{i\neq j}(\gamma_{1i}.\gamma_{1j})-\sum_{i\neq j}\gamma_{2ij}$$

$$\Rightarrow 2\sum_{i\neq j}\gamma_{2ij}=\sum_{i\neq j}(\gamma_{1i}.\gamma_{1j})\Rightarrow \sum_{i\neq j}\gamma_{2ij}=\frac{1}{2}\sum_{i\neq j}(\gamma_{1i}.\gamma_{1j}).$$

Remark 3.8

The condition in the above theorem is not sufficient.

Theorem 3.9

If G is a strong IFHG, then $\overline{G}$ is also strong.

Proof

Let $uv\in E$. Then

$$\overline{\mu_2}(uv)=\mu_1(u).\,\mu_1(v)-\mu_2(uv)$$

$=\mu_1(u).\,\mu_1(v)-\mu_1(u).\,\mu_1(v)$, since G is strong

$=0$ and $\overline{\gamma_2}(uv)=\gamma_1(u).\gamma_1(v)-\gamma_2(uv)=\gamma_1(u).\gamma_1(v)-\gamma_1(u).\,\gamma_1(v)$, since G is strong

$=0.$

Let $uv\notin E$. Then $\overline{\mu_2}(uv)=(\mu_1(u).\,\mu_1(v))-\mu_2(uv)=\mu_1(u).\mu_1(v)$ and $\overline{\gamma_2}(uv)=\gamma_1(u).\,\gamma_1(v)-\gamma_2(uv)=\gamma_1(u).\,\gamma_1(v)$.

4. OPERATIONS ON INTUITIONISTIC FUZZY HYPERGRAPHS

Definition 4.1

Let $G_1=\langle V_1,E_1\rangle$ and $G_2=\langle V_2,E_2\rangle$ be two IFHGs with $V_1\cap V_2=\varphi$. Then the union of G_1 and G_2 is an IFHG $G=G_1\cup G_2=\langle V_1\cup V_2,E_1\cup E_2\rangle$ defined by

$$(\mu_1\cup\mu_1')(v)=\begin{cases}\mu_1(v) & if\quad v\in V_1-V_1\\ \mu_1'(v) & if\quad v\in V_2-V_1\end{cases}$$

$$(\gamma_1\cup\gamma_1')(v)=\begin{cases}\gamma_1(v) & if\quad v\in V_1-V_2\\ \gamma_1'(v) & if\quad v\in V_2-V_1\end{cases}$$

and $$(\mu_2\cup\mu_2')(v_iv_j)=\begin{cases}\mu_{2ij} & if\quad e_{ij}\in E_1-E_2\\ \mu_{2ij}' & if\quad e_{ij}\in E_2-E_1\end{cases}$$

$$(\gamma_2\cup\gamma_2')(v_iv_j)=\begin{cases}\gamma_{2ij} & if\quad e_{ij}\in E_1-E_2\\ \gamma_{2ij}' & if\quad e_{ij}\in E_2-E_1\end{cases}$$

where (μ_1,γ_1) and (μ_1',γ_1') refer the vertex membership and non-membership of G_1 and G_2 respectively; (μ_2,γ_2) and (μ_2',γ_2') refer the edge membership and non-membership of G_1 and G_2 respectively.

Definition 4.2

The join of two IFHGs G_1 and G_2 is an IFHG $G=G_1+G_2=\langle V_1\cup V_2,E_1\cup E_2\cup E'\rangle$ defined by

$$(\mu_1+\mu_1')(v)=(\mu_1\cup\mu_1')(v)\quad \text{if}\quad v\in V_1\cup V_2$$

$$(\gamma_1+\gamma_1')(v)=(\gamma_1\cup\gamma_1')(v)\quad \text{if}\quad v\in V_1\cup V_2$$

$$(\mu_2+\mu_2')(v_iv_j)=(\mu_2\cup\mu_2')(v_iv_j)\quad \text{if}\quad v_iv_j\in E_1\cup E_2$$

$$=(\mu_1(v_i).\mu_1'(v_j))\quad \text{if}\quad v_iv_j\in E'$$

and

$$(\gamma_2+\gamma_2')(v_iv_j)=(\gamma_2\cup\gamma_2')(v_iv_j)\quad \text{if}\quad v_iv_j\in E_1\cup E_2$$

$$=(\gamma_1(v_i).\gamma_1'(v_j))\ \text{if}\quad v_iv_j\in E'.$$

Definition 4.3

Let $G_1=\langle V_1,E_1\rangle$ and $G_2=\langle V_2,E_2\rangle$ be two IFHGs. Then the intersection of IFHGs G_1 and G_2 denoted by $G_1\cap G_2$ is an IFHG defined by $(\mu_1\cap\mu_1')(v)=\begin{cases}\mu_1(v) & \text{if}\quad v\in V_1 \text{ and } V_2\\ \mu_1'(v) & \text{if}\quad v\in V_2 \text{ and } V_1\end{cases}$

$$(\gamma_1\cap\gamma_1')(v)=\begin{cases}\gamma_1(v) & \text{if}\quad v\in V_1 \text{ and } V_2\\ \gamma_1'(v) & \text{if}\quad v\in V_2 \text{ and } V_1\end{cases}$$

and

$$(\mu_2 \cap \mu_2')(v_i v_j) = \begin{cases} \mu_{2ij} & \text{if} \quad e_{ij} \in E_1 \text{ and } E_2 \\ \mu_{2ij}' & \text{if} \quad e_{ij} \in E_1 \text{ and } E_2 \end{cases}$$

$$(\gamma_2 \cap \gamma_2')(v_i v_j) = \begin{cases} \gamma_{2ij} & \text{if} \quad e_{ij} \in E_1 \text{ and } E_2 \\ \gamma_{2ij}' & \text{if} \quad e_{ij} \in E_1 \text{ and } E_2 \end{cases}$$

Definition 4.4

Let $G_1 = \langle V_1, E_1 \rangle$ and $G_2 = \langle V_2, E_2 \rangle$ be two IFHGs. Then the ringsum of IFHGs G_1 and G_2 is an IFHG defined by $G_1 \oplus G_2 = (G_1 \cup G_2) - (G_1 \cap G_2)$.

Example 4.5

Let $V_1 = \{v_1, v_2, v_3, v_4\}$ and $V_2 = \{u_1, u_2\}$ such that $V_1 \cap V_2 = \varphi$

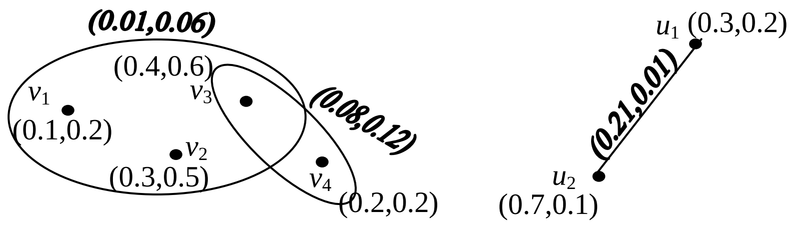

Fig 2: G_1 and G_2

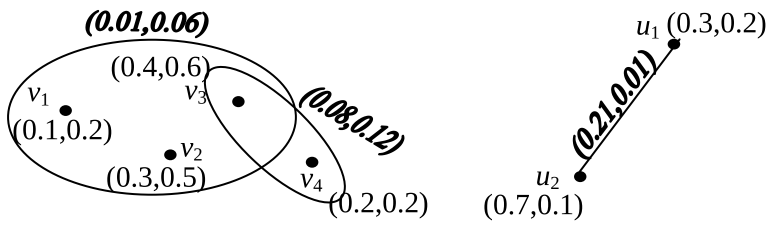

Fig 3: $G_1 \cup G_2$

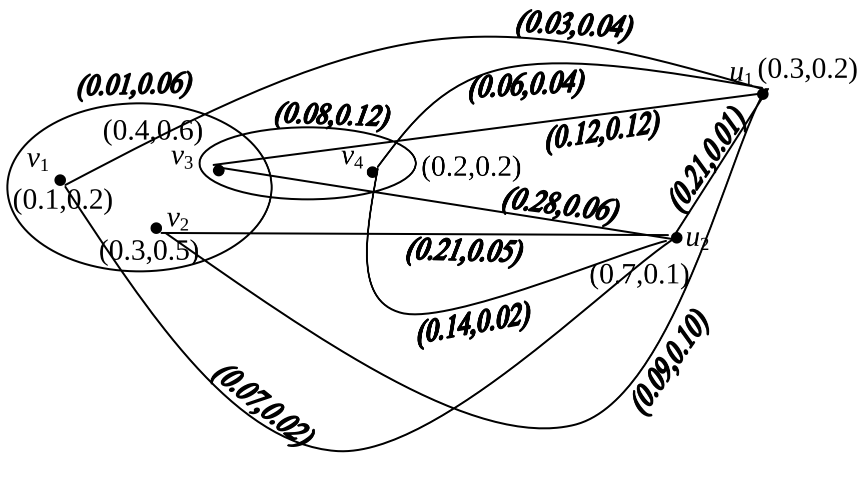

Fig 4: $G_1 + G_2$

Example 4.6

For the operation intersection and ringsum we consider $V_1=\{v_1,v_2,v_3\}$ and $V_2=\{v_1,v_2,v_3\}$ and $V_1 \cap V_2 \neq \varphi$

Theorem 4.7

Let $G_1=\langle V_1,E_1\rangle$ and $G_2=\langle V_2,E_2\rangle$ be two IFHGs. Then (i) $\overline{G_1+G_2}\cong\overline{G_1}\cup\overline{G_2}$ (ii) $\overline{G_1\cup G_2}\cong\overline{G_1}+\overline{G_2}$

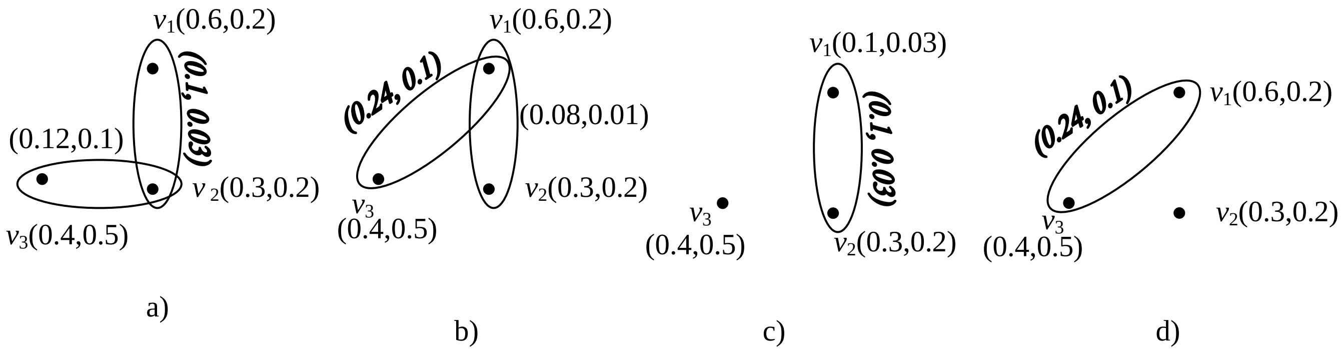

Fig 5: a) G_1 **b)** G_2 **c)** $G_1\cap G_2$ **d)** $G_1\oplus G_2$

Proof

Consider the identity map $I:V_1\cup V_2\to V_1\cup V_2$ .To prove (i), it is enough to prove

a) (i) $(\overline{\mu_1+\mu_1'})(v_i)=(\overline{\mu_1}\cup\overline{\mu_1'})(v_i)$

(ii) $(\overline{\gamma_1+\gamma_1'})(v_i)=(\overline{\gamma_1}\cup\overline{\gamma_1'})(v_i)$

(b) (i) $(\overline{\mu_2+\mu_2'})(v_i,v_j)=(\overline{\mu_2}\cup\overline{\mu_2'})(v_i,v_j)$

(ii) $(\overline{\gamma_2+\gamma_2'})(v_i,v_j)=(\overline{\gamma_2}\cup\overline{\gamma_2'})(v_i,v_j)$.

(a) (i) $(\overline{\mu_1+\mu_1'})(v_i)=(\mu_1+\mu_1')(v_i)$, by Definition 4.1

$$=\begin{cases}\mu_1(v_i) & \text{if } v_i\in V_1\\ \mu_1'(v_i) & \text{if } v_i\in V_2\end{cases}$$

$$=\begin{cases}\overline{\mu_1}(v_i) & \text{if } v_i\in V_1\\ \overline{\mu_1'}(v_i) & \text{if } v_i\in V_2\end{cases} = (\overline{\mu_1}\cup\overline{\mu_1'})(v_i).$$

(ii) $(\overline{\gamma_1+\gamma_1'})(v_i)=(\gamma_1+\gamma_1')(v_i)$, by Definition 4.1

$$=\begin{cases}\gamma_1(v_i) & \text{if } v_i\in V_1\\ \gamma_1'(v_i) & \text{if } v_i\in V_2\end{cases} = \begin{cases}\overline{\gamma_1}(v_i) & \text{if } v_i\in V_1\\ \overline{\gamma_1'}(v_i) & \text{if } v_i\in V_2\end{cases} = (\overline{\gamma_1}\cup\overline{\gamma_1'})(v_i).$$

(b) (i) $(\overline{\mu_2+\mu_2'})(v_i,v_j)=(\mu_1+\mu_1')(v_i).(\mu_1+\mu_1')(v_j)-(\mu_2+\mu_2')(v_i,v_j)$

$$=\begin{cases}(\mu_1\cup\mu_1')(v_i).(\mu_1\cup\mu_1')(v_j)-(\mu_2\cup\mu_2')(v_i,v_j) & \text{if } (v_i,v_j)\in E_1\cup E_2\\ (\mu_1\cup\mu_1')(v_i).(\mu_1\cup\mu_1')(v_j))-\mu_1(v_i).\mu_1'(v_j) & \text{if } (v_i,v_j)\in E'.\end{cases}$$

$$=\begin{cases}\mu_1(v_i).\mu_1(v_j)-\mu_2(v_i,v_j) & \text{if } (v_i,v_j)\in E_1\\ \mu_1'(v_i).\mu_1'(v_j))-\mu_2'(v_i,v_j) & \text{if } (v_i,v_j)\in E_2\\ \mu_1(v_i).\mu_1'(v_j)-\mu_1(v_i).\mu_1'(v_j) & \text{if } (v_i,v_j)\in E'\end{cases}$$

$$=\begin{cases}\overline{\mu_2}(v_i,v_j) & \text{if } (v_i,v_j)\in E_1\\ \overline{\mu_2'}(v_i,v_j) & \text{if } (v_i,v_j)\in E_2\\ 0 & \text{if } (v_i,v_j)\in E'\end{cases} = (\overline{\mu_2}\cup\overline{\mu_2'})(v_i,v_j).$$

(b) (ii) $(\overline{\gamma_2+\gamma_2'})(v_i,v_j)=(\gamma_1+\gamma_1')(v_i).(\gamma_1+\gamma_1')(v_j)-(\gamma_2+\gamma_2')(v_i,v_j)$

$$=\begin{cases}(\gamma_1\cup\gamma_1')(v_i).(\gamma_1\cup\gamma_1')(v_j)-(\gamma_2\cup\gamma_2')(v_i,v_j) & \text{if } (v_i,v_j)\in E_1\cup E_2\\ (\gamma_1\cup\gamma_1')(v_i).(\gamma_1\cup\gamma_1')(v_j)-\gamma_1(v_i).\gamma_1'(v_j) & \text{if } (v_i,v_j)\in E'.\end{cases}$$

$$=\begin{cases}\gamma_1(v_i).\gamma_1(v_j)-\gamma_2(v_i,v_j) & \text{if } (v_i,v_j)\in E_1\\ \gamma_1'(v_i).\gamma_1'(v_j)-\gamma_2'(v_i,v_j) & \text{if } (v_i,v_j)\in E_2\\ \gamma_1(v_i).\gamma_1'(v_j)-\gamma_1(v_i).\gamma_1'(v_j) & \text{if } (v_i,v_j)\in E'\end{cases}$$

$$=\begin{cases}\overline{\gamma_2}(v_i,v_j) & \text{if } (v_i,v_j)\in E_1\\ \overline{\gamma_2'}(v_i,v_j) & \text{if } (v_i,v_j)\in E_2\\ 0 & \text{if } (v_i,v_j)\in E'\end{cases}$$

$= (\overline{\gamma_2}\cup\overline{\gamma_2'})(v_i,v_j)$.

To prove (ii), it is enough to prove that

(a) (i) $\overline{(\mu_1 \cup \mu_1')}(v_i) = (\overline{\mu_1} + \overline{\mu_1'})(v_i)$

(ii) $\overline{(\gamma_1 \cup \gamma_1')}(v_i) = (\overline{\gamma_1} + \overline{\gamma_1'})(v_i)$

(b) (i) $\overline{(\mu_2 \cup \mu_2')}(v_i, v_j) = (\overline{\mu_2} + \overline{\mu_2'})(v_i, v_j)$

(ii) $\overline{(\gamma_2 \cup \gamma_2')}(v_i, v_j) = (\overline{\gamma_2} + \overline{\gamma_2'})(v_i, v_j)$.

Consider the identity map : $I : V_1 \cup V_2 \to V_1 \cup V_2$.

(a) (i) $\overline{(\mu_1 \cup \mu_1')}(v_i) = (\mu_1 \cup \mu_1')(v_i)$

$$= \begin{cases} \mu_1(v_i) & \text{if } v_i \in V_1 \\ \mu_1'(v_i) & \text{if } v_i \in V_2 \end{cases} = \begin{cases} \overline{\mu_1}(v_i) & \text{if } v_i \in V_1 \\ \overline{\mu_1'}(v_i) & \text{if } v_i \in V_2 \end{cases}$$

$= (\overline{\mu_1} \cup \overline{\mu_1'})(v_i) = (\overline{\mu_1} + \overline{\mu_1'})(v_i)$.

(a)(ii) $\overline{(\gamma_1 \cup \gamma_1')}(v_i) = (\gamma_1 \cup \gamma_1')(v_i)$

$$= \begin{cases} \gamma_1(v_i) & \text{if } v_i \in V_1 \\ \gamma_1'(v_i) & \text{if } v_i \in V_2 \end{cases} = \begin{cases} \overline{\gamma_1}(v_i) & \text{if } v_i \in V_1 \\ \overline{\gamma_1'}(v_i) & \text{if } v_i \in V_2 \end{cases}$$

$= (\overline{\gamma_1} \cup \overline{\gamma_1'})(v_i) = (\overline{\gamma_1} + \overline{\gamma_1'})(v_i)$.

(b)(i)

$\overline{(\mu_2 \cup \mu_2')}(v_i, v_j) = (\mu_1 \cup \mu_1')(v_i).(\mu_1 \cup \mu_1')(v_j) - (\mu_2 \cup \mu_2')(v_i, v_j)$

$$= \begin{cases} \mu_1(v_i).\mu_1(v_j) - \mu_2(v_i, v_j) & \text{if } (v_i, v_j) \in E_1 \\ \mu_1'(v_i).\mu_1'(v_j) - \mu_2'(v_i, v_j) & \text{if } (v_i, v_j) \in E_2 \\ \mu_1(v_i).\mu_1'(v_j) - 0 & \text{if } v_i \in V_1, v_j \in V_2 \end{cases}$$

$$= \begin{cases} \overline{\mu_2}(v_i, v_j) & \text{if } (v_i, v_j) \in E_1 \\ \overline{\mu_2'}(v_i, v_j) & \text{if } (v_i, v_j) \in E_2 \\ \mu_1(v_i).\mu_1'(v_j) & \text{if } v_i \in V_1, v_j \in V_2 \end{cases}$$

$$= \begin{cases} (\overline{\mu_2} \cup \overline{\mu_2'})(v_i, v_j) & \text{if } (v_i, v_j) \in E_1 \text{ or } E_2 \\ \mu_1(v_i).\mu_1'(v_j) & \text{if } (v_i, v_j) \in E' \end{cases}$$

$= (\overline{\mu_2} + \overline{\mu_2'})(v_i, v_j)$

(b)(ii) $\overline{(\gamma_2 \cup \gamma_2')}(v_i, v_j) = (\gamma_1 \cup \gamma_1')(v_i).(\gamma_1 \cup \gamma_1')(v_j) - (\gamma_2 \cup \gamma_2')(v_i, v_j)$

$$= \begin{cases} \gamma_1(v_i).\gamma_1(v_j) - \gamma_2(v_i, v_j) & \text{if } (v_i, v_j) \in E_1 \\ \gamma_1'(v_i).\gamma_1'(v_j) - \gamma_2'(v_i, v_j) & \text{if } (v_i, v_j) \in E_2 \\ \gamma_1(v_i).\gamma_1'(v_j) - 0 & \text{if } v_i \in V_1, v_j \in V_2 \end{cases}$$

$$= \begin{cases} \overline{\gamma_2}(v_i, v_j) & \text{if } (v_i, v_j) \in E_1 \\ \overline{\gamma_2'}(v_i, v_j) & \text{if } (v_i, v_j) \in E_2 \\ \gamma_1(v_i).\gamma_1'(v_j) & \text{if } v_i \in V_1, v_j \in V_2 \end{cases}$$

$$= \begin{cases} (\overline{\gamma_2} \cup \overline{\gamma_2'})(v_i, v_j) & \text{if } (v_i, v_j) \in E_1 \text{ or } E_2 \\ \gamma_1(v_i).\gamma_1'(v_j) & \text{if } (v_i, v_j) \in E' \end{cases}$$

$= (\overline{\gamma_2} + \overline{\gamma_2'})(v_i, v_j)$.

Definition 4.8

Let $G = G_1 \times G_2 = \langle V, E'' \rangle$ be the Cartesian product of two graphs G_1 and G_2 where $V = V_1 \times V_2$ and

$$E'' = \{(u, u_2)(u, v_2) : u \in V_1, u_2 v_2 \in E_2\} \cup \{(u_1, w)(v_1, w) : w \in V_2, u_1 v_1 \in E_1\}.$$

Then, the Cartesian product of IFHGs G_1 and G_2 is an IFHG defined by $G = G_1 \times G_2 = \langle V, E'' \rangle$ where

(i)

$(\mu_1 \times \mu_1')(u_1, u_2) = \mu_1(u_1).\mu_1'(u_2)$ for every $(u_1, u_2) \in V$ and

$(\gamma_1 \times \gamma_1')(u_1, u_2) = \gamma_1(u_1).\gamma_1'(u_2)$ for every $(u_1, u_2) \in V$

(ii)

$(\mu_2 \times \mu_2')(u, u_2)(u, v_2) = \mu_1(u).\mu_2(u_2 v_2)$
for every $u \in V_1$, and $u_2 v_2 \in E_2$

$(\gamma_2 \times \gamma_2')(u, u_2)(u, v_2) = \gamma_1(u).\gamma_2(u_2 v_2)$
for every $u \in V_1$, and $u_2 v_2 \in E_2$ and

$(\mu_2 \times \mu_2')(u_1, w)(v_1, w) = \mu_1(w).\mu_2(u_1 v_1)$
for every $w \in V_2$, and $u_1 v_1 \in E_1$

$(\gamma_2 \times \gamma_2')(u_1, w)(v_1, w) = \gamma_1(w).\gamma_2(u_1 v_1)$
for every $w \in V_2$, and $u_1 v_1 \in E_1$.

Example 4.9

Let $V_1 = \{v_1, v_2, v_3, v_4\}$ and $V_2 = \{u_1, u_2\}$ such that $V_1 \cap V_2 = \varphi$

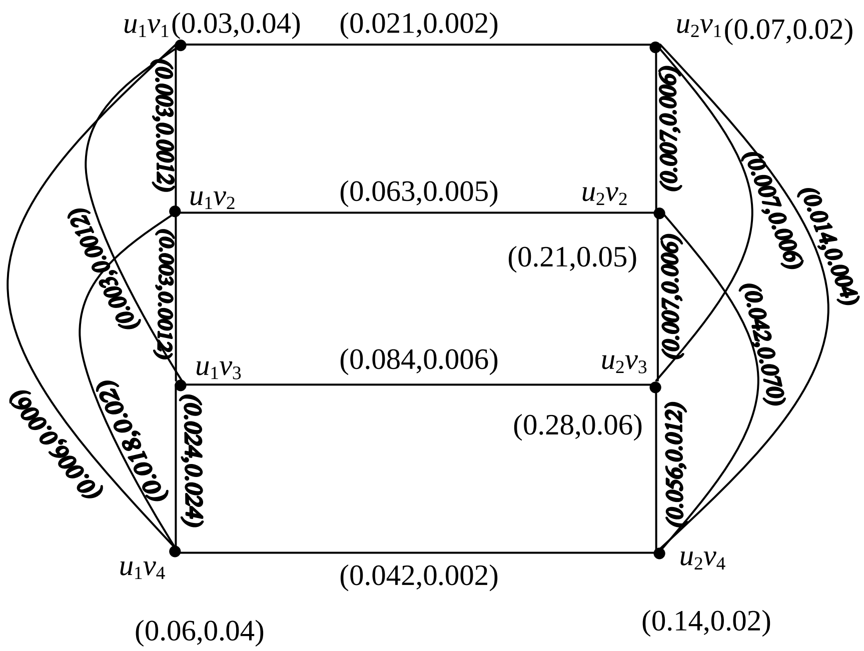

Fig 6: $G_1 \times G_2$

Definition 4.10

Let $G = G_1 \circ G_2 = (V_1 \times V_2, E)$ be the composition of two graphs G_1 and G_2, where

$E = \{(u,u_2)(u,v_2) : u \in V_1, u_2v_2 \in E_2\} \cup$
$\{(u_1,w)(v_1,w) : w \in V_2, u_1v_1 \in E_1\} \cup$
$\{(u_1,u_2)(v_1,v_2) : u_1v_1 \in E_1, u_2 \neq v_2\}$.

Then the composition of IFHGs G_1 and G_2, denoted by $G = G_1 \circ G_2$, is an IFHG defined by

(i) $(\mu_1 \circ \mu_1')(u_1,u_2) = \mu_1(u_1).\mu_1'(u_2)$ for every $(u_1,u_2) \in V_1 \times V_2$
and

$(\gamma_1 \circ \gamma_1')(u_1,u_2) = \gamma_1(u_1).\gamma_1'(u_2)$ for every $(u_1,u_2) \in V_1 \times V_2$

(ii) $(\mu_2 \circ \mu_2')(u,u_2)(u,v_2) = \mu_1(u).\mu_2(u_2v_2)$ for every $u \in V_1$, and $u_2v_2 \in E_2$

$(\gamma_2 \circ \gamma_2')(u,u_2)(u,v_2) = \gamma_1(u).\gamma_2(u_2v_2)$ for every $u \in V_1$, and $u_2v_2 \in E_2$

and $(\mu_2 \circ \mu_2')(u_1,w)(v_1,w) = \mu_1(w).\mu_2(u_1v_1)$ for every $w \in V_2$, and $u_1v_1 \in E_1$

$(\gamma_2 \circ \gamma_2')(u_1,w)(v_1,w) = \gamma_1(w).\gamma_2(u_1v_1)$ for every $w \in V_2$, and $u_1v_1 \in E_1$

$(\mu_2 \circ \mu_2')(u_1,u_2)(v_1,v_2) = \mu_1'(u_2).\mu_1'(v_2).\mu_2(u_1,v_1)$ for every $(u_1,u_2)(v_1,v_2) \in E - E''$
and

$(\gamma_2 \circ \gamma_2')(u_1,u_2)(v_1,v_2) = \gamma_1'(u_2).\gamma_1'(v_2).\gamma_2(u_1,v_1)$ for every $(u_1,u_2),\ (v_1,v_2) \in E - E''$

where

$E'' = \{(u,u_2)(u,v_2) : u \in V_1, \text{ for every } u_2v_2 \in E_2\} \cup$
$\{(u_1,w)(v_1,w) : w \in V_2, \text{ for every } u_1v_1 \in E_1\}$.

Theorem 4.9

Let $G_1 = \langle V_1, E_1 \rangle$ and $G_2 = \langle V_2, E_2 \rangle$ be two strong IFHGs. Then, $G_1 \circ G_2$ is a strong IFHG.

Proof

Let $G_1 \circ G_2 = G = \langle V, E \rangle$ where $V = V_1 \times V_2$ and

$E = \{(u,u_2)(u,v_2) : u \in V_1, u_2v_2 \in E_2\} \cup$
$\{(u_1,w)(v_1,w) : w \in V_2, u_1v_1 \in E_1\} \cup$
$\{(u_1,u_2)(v_1,v_2) : u_1v_1 \in E_1, u_2 \neq v_2\}$.

(i) $\mu_2(u,u_2)(u,v_2) = \mu_1(u).\mu_2(u_2v_2)$
$= \mu_1(u).(\mu_1'(u_2).\mu_1'(v_2))$, since G_2 is strong

$= \mu_1(u).\mu_1'(u_2).\mu_1(u).\mu_1'(v_2)$

$= (\mu_1 \circ \mu_1')(u,u_2).(\mu_1 \circ \mu_1')(u,v_2)$.

$\gamma_2(u,u_2)(u,v_2) = \gamma_1(u).\gamma_2(u_2 v_2)$

$= \gamma_1(u).(\gamma_1'(u_2).\gamma_1'(v_2))$, since G_2 is strong

$= \gamma_1(u).\gamma_1'(u_2).\gamma_1(u).\gamma_1'(v_2)$

$= (\gamma_1 \circ \gamma_1')(u,u_2).(\gamma_1 \circ \gamma_1')(u,v_2)$.

(ii) $\mu_2((u_1,w)(v_1,w)) = \mu_1'(w).\mu_2(u_1,v_1)$

$= \mu_1'(w).(\mu_1(u_1).\mu_1(v_1))$, since G_1 is strong

$= \mu_1'(w).\mu_1(v_1).\mu_1'(w).\mu_1(v_1)$

$= (\mu_1 \circ \mu_1')(u_1,w).(\mu_1 \circ \mu_1')(v_1,w)$.

$\gamma_1((u_1,w)(v_1,w)) = \gamma_1'(w).\gamma_2(u_1,v_1)$

$= \gamma_1'(w).(\gamma_1(u_1).\gamma_1(v_1))$, since G_1 is strong

$= \gamma_1'(w).\gamma_1(v_1).\gamma_1'(w).\gamma_1(v_1) = (\gamma_1 \circ \gamma_1')(u_1,w).(\gamma_1 \circ \gamma_1')(v_1,w)$

(iii) $\mu_2(u_1,u_2)(v_1,v_2) = \mu_2(u_1,v_1).\mu_1'(u_2).\mu_1'(v_2)$

$= (\mu_1(u_1).\mu_1(v_1)).\mu_1'(u_2).\mu_1'(v_2)$, G_1 is strong

$= \mu_1(u_1).\mu_1'(u_2).\mu_1(v_1).\mu_1'(v_2)$

$= (\mu_1 \circ \mu_1')(u_1,u_2).(\mu_1 \circ \mu_1')(v_1,v_2)$.

$\gamma_2(u_1,u_2)(v_1,v_2) = \gamma_2(u_1,v_1).\gamma_1'(u_2).\gamma_1'(v_2)$

$= (\gamma_1(u_1).\gamma_1(v_1)).\gamma_1'(u_2).\gamma_1'(v_2)$, G_1 is strong

$= (\gamma_1(u_1).\gamma_1'(u_2)).\gamma_1(v_1).\gamma_1'(v_2)$

$= (\gamma_1 \circ \gamma_1')(u_1,u_2).(\gamma_1 \circ \gamma_1')(v_1,v_2)$.

From (i), (ii), (iii), G is a strong IFHG.

Example 4.11

Let Let $V_1 = \{v_1,v_2,v_3,v_4\}$ and $V_2 = \{u_1,u_2\}$ such that $V_1 \cap V_2 = \varphi$.

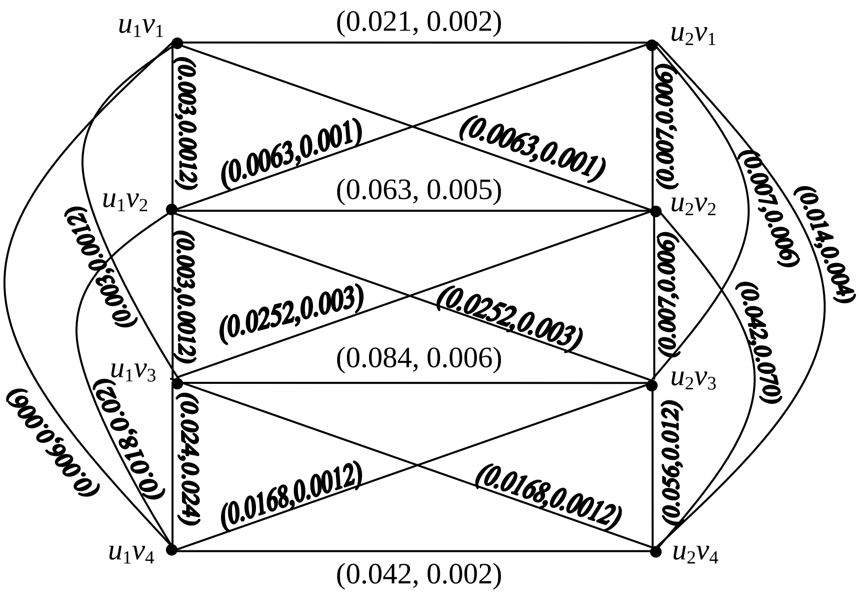

Fig 7: $G_1 \circ G_2$

5. CONCLUSION

In this paper, operations like complement, join, union, intersection, ringsum, Cartesian product, composition on IFHGs are defined. Currently, the authors are working on existing clustering techniques. Further, it is proposed to apply the properties of IFHGs to develop a new clustering algorithm and the same may be checked with a numerical dataset .

6. REFERENCES

[1] K.T.Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20, (1986), 87-96.

[2] K.T.Atanassov, More on intuitionistic fuzzy sets, Fuzzy Sets and Systems 33, (1989), 37-46.

[3] K.T.Atanassov, New operations defined over the intuitionistic fuzzy sets, Fuzzy Sets and Systems 61, (1994), 137-142.

[4] K.T.Atanassov, Remarks on the intuitionistic fuzzy sets-III, Fuzzy Sets and Systems 75, (1995), 401-402.

[5] S.Thilagavathi, R.Parvathi and M.G.Karunambigai, Operations on Intuitionistic fuzzy graphs II, Developments in fuzzy sets, intuitionistic fuzzy sets, generalized nets and related topics 1, (2008), 319-331.

[6] R.Parvathi, M.G.Karunambigai and K.T.Atanassov Operations on Intuitionistic fuzzy graphs, Fuzzy Systems, 2009, Fuzz-IEEE 2009. IEEE international conference 1396-1401.

[7] S.Thilagavathi, R.Parvathi and M.G.Karunambigai, Intuitionistic fuzzy hypergraphs, J.Cybernetics and Information Technologies 9(2), (2009), 46-53, Sofia.

[8] M.Akram and W.A.Dudek Intuitionistic fuzzy hypergraphs with applications, Information Sciences, 2012