International Journal of Computer Applications (0975 – 8887)
Volume 51– No.11, August 2012

Reliability Modeling and Profit Analysis of a Repairable System of Non-identical Units with no Operation and Repair in Abnormal Weather

S.C Malik
Department of Statistics,
M.D. University, Rohtak-124001,
Haryana (India)

Savita Deswal
UGC Teacher Fellow
Department of Statistics,
M.D. University, Rohtak Haryana (India)

ABSTRACT

In this paper, a reliability model of a system of two non-identical units in which one main unit (called original) is initially operative and other is a substandard unit which is kept as spare in cold standby is examined probabilistically in detail under two weather conditions – normal and abnormal. Each unit has direct complete failure from normal mode. There is a single server who visits the system immediately whenever needed. The operation and repair of the units are not allowed in abnormal weather. However, operation and repair of the units are as usual in normal weather subject to the condition that duplicate unit will not work if main unit is available for working in the system. The distributions of failure time of the units and change of weather conditions follow negative exponential while that of repair time of the units are assumed as arbitrary with different probability density functions. All the random variables are mutually independent and uncorrelated. The expressions for some important measures of system effectiveness are derived in steady state using semi- Markov process and regenerative point technique. The graphical study of MTSF, availability and profit has also been made on the basis of numerical results obtained for a particular case. The results of the present paper has also been compared with the model proposed by Malik and Deswal [6].

KEYWORDS

Repairable system, Non-identical units, Weather conditions and Stochastic analysis

1. INTRODUCTION

In view of their frequent and vital use in modern industry, the repairable systems of two or more identical units have been investigated stochastically in detail by several engineers and researchers including Gopalan and Naidu [1] and Singh [2] under strict control of environment conditions such as pollution, moisture, voltage and temperature. But in case of high cost of identical units, the non-identical unit (may be a substandard unit) might be kept as spare in cold standby not only to improve the reliability of the system but also to maintain performance of the system in emergency. Each unit is capable of performing the same kind of functions but their degree of reliability and desirability may differ from unit to unit. Singh and Chander [3] and Chander et al. [4] discussed standby systems of non-identical units with different failure and repair policies. Also, some time it is very difficult to keep the environmental conditions under control which may fluctuate due changing climate and other natural catastrophic.

While considering this fact in mind, Malik and Barak [5] obtained reliability and economic measures of a single- unit system with no operation and repair activities in abnormal weather. Further, the cold standby systems of non-identical units under different weather conditions have not been studied so far by the researchers in the field of reliability. The application of the present work can be visualized in a system constituting of one unit as a power supply through electric transformer and other unit generator.

Hence, in the present paper, a system of two non-identical units – one is original (called main unit) and other is a substandard unit (called duplicate unit) has been analyzed probabilistically in detail under two weather conditions – normal and abnormal. For this purpose a reliability model is developed. The environmental conditions when satisfied to the system correspond to normal weather; otherwise, it is supposed that the system is working under abnormal weather. Initially, the system is operative with main unit and duplicate unit is kept a spare in cold standby. Both units have direct complete failure from normal mode. Each unit is capable of performing the same set of functions with different degree of reliability and desirability. There is a single server who visits the system immediately whenever needed to do repair of the failed unit. The operation and repair of the units are not allowed in abnormal weather. However, operation and repair of the units are as usual in normal weather subject to the condition that duplicate unit will not work if main unit is available for working in the system.

The distributions of failure time of the units and change of weather conditions follow negative exponential while that of repair times of the units are taken as arbitrary. All random variables are mutually independent and uncorrelated. The switch devices and repairs are perfect. The expressions for various measures of system effectiveness such as transition probabilities, mean sojourn times, mean time to system failure (MTSF), availability, busy period of the server and profit function in steady state are derived using semi-Markov process and regenerative point technique. The numerical results giving particular values to the parameters and various costs are obtained for MTSF, availability and profit to depict their graphical behavior. The MTSF and profit of the present model have also been compared with that of the model investigated by Malik and Deswal [6].

2. NOTATIONS

E : The set of regenerative states

MO/DO : Main/Duplicate unit is good and operative

$\overline{\text{MWO}}$ / $\overline{\text{DWO}}$: Main/Duplicate unit is good but waiting for operation due to abnormal weather

DCs : Duplicate unit is in cold standby mode

λ / λ1 : Constant failure rate of Original /Duplicate unit

β / β1 : Constant rate of change of weather from normal to abnormal/abnormal to normal weather

MFur/DFur : Main/duplicate unit failed and under repair

MFUR/DFUR : Main/duplicate unit failed and under repair continuously from previous state

MFwr/DFwr : Main/duplicate unit failed and waiting for repair

MFWR/DFWR : Main/duplicate unit failed and waiting for repair continuously from previous state

$\overline{\text{MFwr}}$ / $\overline{\text{DFwr}}$: Main/Duplicate unit failed and waiting for repair due to abnormal weather

$\overline{\text{MFWR}}$ / $\overline{\text{DFWR}}$: Main/Duplicate unit failed and waiting for repair continuously from previous state due to abnormal weather

g(t)/G(t) : pdf/cdf of repair time of Original unit

g1(t) / G1(t) : pdf/cdf of repair time of Duplicate unit

qij (t) / Qij (t) : pdf/cdf of passage time from regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in (0,t]

qij.kr (t) / Qij.kr (t) : pdf/cdf of direct transition time from Regenerative state i to a regenerative state j or to a failed state j visiting state k,r once in (0,t]

q ij.k,(r,s)n(t) /Qij.k,(r,s)n(t) : pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k once and n times states r and s.

Mi(t) : Probability that the system is up initially in regenerative state Si at time t without visiting to any other regenerative state

Wi(t) : Probability that the server is busy in state Si upto time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states

mij : The unconditional mean time taken by the system to transits from any regenerative state Si Є S when time is counted from epoch of entrance into that state Sj. Mathematically, it can be written as mij= ∫tQij(t)=-qij *'(0)

μ_i : The mean sojourn time in state Si this is given by μ_i =E(t)=∫P(T>t)dt=∑jmij,where T denotes the time to system failure

Ⓢ/©/©n : Symbol for Laplace Stieltjes Convolution / Laplace convolution / Laplace convolution n times

~ / * : Symbol for Laplace Steiltjes Transform (LST)/ Laplace Transform (LT)

' (desh) : Used to represent derivative

The following are the possible transition states of the system

S_0 = (MO, DCs), S_1 = (MFur, DO), S_2 = ($\overline{\text{MWO}}$, $\overline{\text{DCs}}$),

S_3=($\overline{\text{MFwr}}$, $\overline{\text{DWO}}$),

S_4 =(MFUR,DFwr),S_5=(MO,DFur),

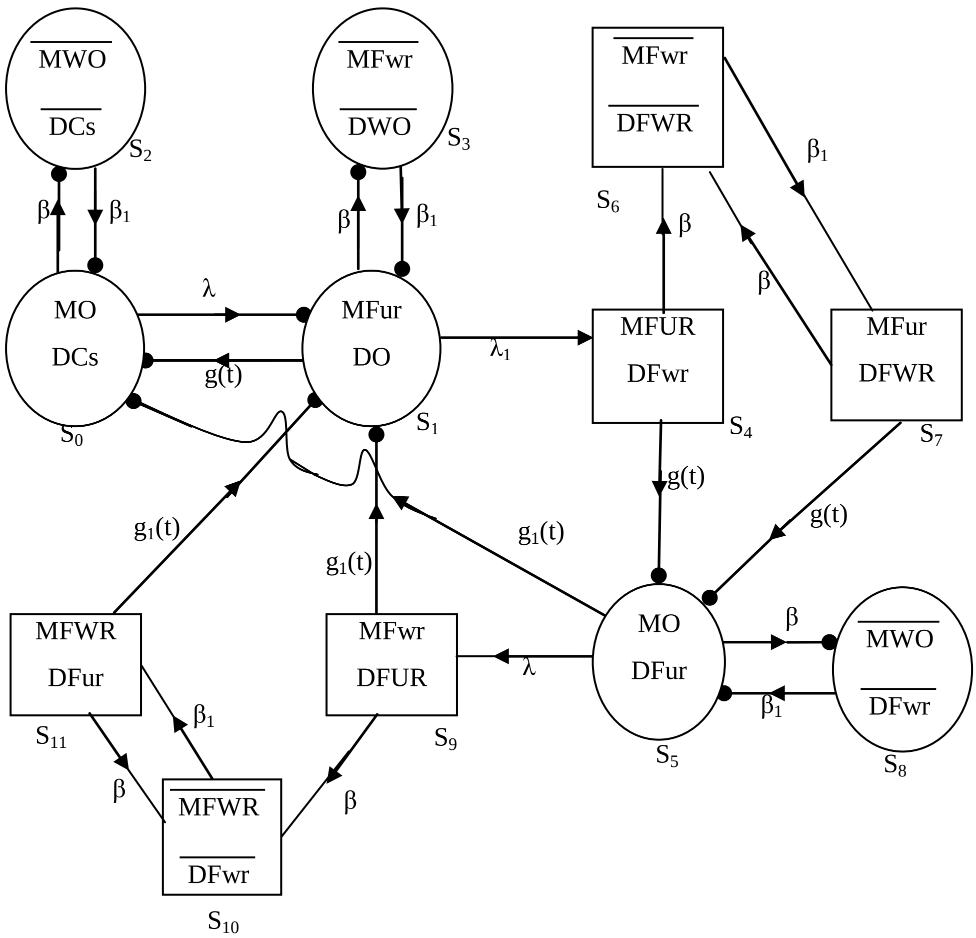
S_6=($\overline{\text{MFwr}}$, $\overline{\text{DFWR}}$), S_7=(MFur,DFWR),

S_8=($\overline{\text{MWO}}$, $\overline{\text{DFwr}}$),S_9=(MFwr,DFUR)

,S_{10} =($\overline{\text{MFWR}}$, $\overline{\text{DFwr}}$), S_{11}=(MFWR,DFur)

The states S_0, S_1,S_2,S_3,S_5,S_8 are regenerative while the states S_4,S_6,S_7,S_9,S_{10},S_{11} are non -regenerative as shown in figure1.

State Transition Diagram



○ Up-State

□ Failed-State

• Regenerative point

Fig. 1

3. RELIABILITY INDICES

3.1 Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements

$$p_{ij}=Q_{ij}(\infty)=\int q_{ij}(t)\,dt \text{ as}$$

$p_{01}=\dfrac{\lambda}{\beta+\lambda}$, $p_{02}=\dfrac{\beta}{\beta+\lambda}$, $p_{10}=g^*(\beta+\lambda_1)$,

$p_{13}=\dfrac{\beta}{\beta+\lambda_1}(1-g^*(\beta+\lambda_1))$, $p_{14}=\dfrac{\lambda_1}{\beta+\lambda_1}(1-g^*(\beta+\lambda_1))$,

$p_{20}=1, p_{31}=1, p_{45}=g^*(\beta)$, $p_{46}=1-g^*(\beta), p_{50}=g_1^*(\beta+\lambda)$,

$p_{58}=\dfrac{\beta}{\beta+\lambda}(1-g_1^*(\beta+\lambda)), p_{59}=\dfrac{\lambda}{\beta+\lambda}(1-g_1^*(\beta+\lambda))$,

$p_{67}=1, p_{75}=g^*(\beta), p_{76}=1-g^*(\beta), p_{85}=1, p_{91}=g_1^*(\beta)$,

$p_{9,10}=1-g_1^*(\beta), p_{10,11}=1, p_{11,1}=g_1^*(\beta), p_{11,10}=1-g_1^*(\beta)$ (1)

It can be easily verified that

$p_{01}+p_{02}=p_{10}+p_{13}+p_{14}=p_{20}=p_{31}=p_{45}+p_{46}=p_{50}+p_{58}+p_{59}=1$,

$p_{67}=p_{75}+p_{76}=p_{85}=p_{91}+p_{9,10}=p_{10,11}=p_{11,1}+p_{11,10}=1$ (2)

The mean sojourn times (μ_i) in the state S_i are

$\mu_0 = \dfrac{1}{\beta+\lambda}$, $\mu_1 = \dfrac{1}{\beta+\lambda_1}(1-g^*(\beta+\lambda_1))$, $\mu_2 = \dfrac{1}{\beta_1}$, $\mu_3 = \dfrac{1}{\beta_1}$, $\mu_4 = \dfrac{1}{\beta}(1-g^*(\beta))$

, $\mu_5 = \dfrac{1}{\beta+\lambda}(1-g_1^*(\beta+\lambda))$,

$\mu_6 = \dfrac{1}{\beta_1}$, $\mu_7 = \dfrac{1}{\beta}(1-g^*(\beta))$, $\mu_8 = \dfrac{1}{\beta_1}$,

$\mu_9 = \dfrac{1}{\beta}(1-g_1^*(\beta))$, $\mu_{10} = \dfrac{1}{\beta_1}$, $\mu_{11} = \dfrac{1}{\beta}(1-g_1^*(\beta))$

(3)

it can be observed that

$m_{01}+m_{02}=\mu_0$, $m_{10}+m_{13}+m_{14}=\mu_1$, $m_{20}=\mu_2$, $m_{31}=\mu_3$, $m_{45}+m_{46}=\mu_4$, $m_{50}+m_{58}+m_{59}=\mu_5$, $m_{67}=\mu_6$, $m_{75}+m_{76}=\mu_7$, $m_{85}=\mu_8$, $m_{91}+m_{9,10}=\mu_9$, $m_{10,11}=\mu_{10}$, $m_{11,1}+m_{11,10}=\mu_{11}$ (4)

and

$\mu_1' = m_{10}+m_{13}+m_{15.4}+m_{15.4,(6,7)}^n$, $\mu_5' = m_{50}+m_{51.9}+m_{51.9,(10,11)}^n+m_{58}$ (5)

3.2 Reliability and Mean Time to System Failure (MTSF)

Let $\phi_i(t)$ be the cdf of first passage time from regenerative state i to a failed state.

Regarding failed state as absorbing state, we have following recursive relations for $\phi_i(t)$:

$\phi_0(t) = Q_{01}(t)Ⓢ\phi_1(t) + Q_{02}(t)Ⓢ\phi_2(t)$

$\phi_1(t) = Q_{10}(t)Ⓢ\phi_0(t)+Q_{13}(t)Ⓢ\phi_1(t)+Q_{14}(t)$

$\phi_2(t) = Q_{20}(t)Ⓢ\phi_0(t)$, $\phi_3(t) = Q_{31}(t)Ⓢ\Phi\phi_1(t)$

(6)

Taking LST of above relation (6) and solving for $\tilde{\phi}_0(s)$

We have

$$R^*(s) = \frac{1-\tilde{\phi}_0(s)}{s}$$

(7)

The reliability of the system model can be obtained by taking Laplace inverse transform of (7).

The mean time to system failure (MTSF) is given by

$$\text{MTSF} = \lim_{s\to o}\frac{1-\tilde{\phi}_0(s)}{s} = \frac{N_1}{D_1}$$

(8)

where

$N_1 = p_{01}(p_{13}\mu_3+\mu_1)+(1-p_{13})(\mu_0+p_{02}\mu_2)$

$D_1 = p_{01}p_{14}$

3.3 Steady State Availability

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state i at t = 0.The recursive relations for $A_i(t)$ are given as

$A_0(t) = M_0(t) + q_{01}(t) © A_1(t) + q_{02}(t) © A_2(t)$

$A_1(t) = M_1(t) + q_{10}(t)©A_0(t) + q_{13}(t) © A_3(t) + (q_{15.4}(t)+q_{15.4,(6,7)}^n(t)) ©A_5(t)$

$A_2(t) = q_{20}(t) © A_0(t)$, $A_3(t) = q_{31}(t)©A_1(t)$

$A_5(t) = M_5(t)+q_{50}(t)©A_0(t)+(q_{51.9}(t)+q_{51.9,(10,11)}^n(t))©A_1(t)+q_{58}(t) ©A_8(t)$

$A_8(t) = q_{85}(t)©A_5(t)$

(10)

where $M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state, we have

$M_0(t)=e^{-(\beta+\lambda)t}$, $M_1(t)=e^{-(\beta+\lambda_1)t}\overline{G(t)}$, $M_5(t)=e^{-(\beta+\lambda)t}\overline{G_1(t)}$ (11)

Taking LT of above relations (10) and solving for $A_0^*(s)$. The steady state availability is given by

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_2}$$ (12)

where

$N_2 = \mu_0((1-p_{13})(1-p_{58})-p_{14}p_{59})+\mu_1 p_{01}(1-p_{58})+p_{01}p_{14}\mu_5$

and

$D_2 = (1-p_{58})(p_{01}(\mu'_1+p_{13}\mu_3)+m_{01}p_{10})+p_{14}(p_{01}(\mu'_5+p_{58}\mu_8)+m_{01}p_{50})+(m_{02}+p_{02}\mu_2)((1-p_{58})(1-p_{13})-p_{14}p_{59})$

(13)

3.4 Busy period analysis for server

Let $B_i(t)$ be the probability that the server is busy in repairing the unit at an instant

't' given that the system entered regenerative state i at t=0.The recursive relations for $B_i(t)$ are as follows:

$B_0(t)=q_{01}(t)©B_1(t)+q_{02}(t)©B_2(t)$

$B_1(t)=W_1(t)+q_{10}(t)©B_0(t)+q_{13}(t)©B_3(t)+(q_{15.4}(t)$

$+q_{15.4,(6,7)}^n(t))©B_5(t)$

$B_2(t)=q_{20}(t)©B_0(t), B_3(t)=q_{31}(t)©B_1(t)$

$B_5(t)=W_5(t)+q_{50}(t)©B_0(t)+(q_{51.9}(t)+q_{51.9,(10,11)}^n(t))$

$©B_1(t)+q_{58}(t)©B_8(t)$

$B_8(t)=q_{85}(t)©B_5(t)$ (13)

where $W_i(t)$ be the probability that the server is busy in state S_i due to failure upto time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states .

so,

$W_1(t)=e^{-(\beta+\lambda_1)t}\overline{G(t)}+(\lambda_1 e^{-(\beta+\lambda_1)t}©1)\overline{G(t)}$, $W_5(t)=e^{-(\beta+\lambda)t}\overline{G_1(t)}+(\lambda e^{-(\beta+\lambda)t}©1)\overline{G_1(t)}$ (14)

Taking LT of above relations (13) . And, solving for $B_0^*(s)$, the time for which server is busy due to repair is given by

$$B_0^*(\infty)=\lim_{s\to 0} s\, B_0^*(s)=\frac{N_3}{D_2}$$

where

$N_3=p_{01}(W_1^*(0)(1-p_{58})+p_{14}W_5^*(0))$

and D_2 is already mentioned.

3.5 Expected number of visits by the server

Let $N_i(t)$ be the expected number of visits by the server in (0,t] given that the system entered the regenerative state i at t=0. The recursive relations for $N_i(t)$ are given as :

$N_0(t) = Q_{01}(t)$ Ⓢ$[1+N_1(t)] + Q_{02}(t)$ Ⓢ$N_2(t)$

$N_1(t)=Q_{10}(t)$Ⓢ$N_0(t)+Q_{13}(t)$

Ⓢ$N_3(t)+(Q_{15.4}(t)+Q_{15.4,(6,7)}^n(t))$Ⓢ$N_5(t)$

$N_2(t)=Q_{20}(t)$Ⓢ$N_0(t)$, $N_3(t)=Q_{31}(t)$Ⓢ$N_1(t)$

$N_5(t)=Q_{50}(t)$Ⓢ$N_0(t)+(Q_{51.9}(t)+Q_{51.9,(10,11)}^n(t))$Ⓢ$N_5(t)+Q_{58}(t)$Ⓢ$N_8(t)$

$N_8(t)=Q_{85}(t)$Ⓢ$N_5(t)$ (15)

Taking LST of relations (15) and solving for $\tilde{N}_0(s)$.

The expected number of visits per unit time by the server is given by

$$N_0(\infty)=\lim_{s\to 0} s\tilde{N}_0(s)=\frac{N_4}{D_2}$$

where

$N_4= p_{01}((1-p_{13})(1-p_{58})-p_{14}p_{59})$

and D_2 is already specified.

3.6 Profit Analysis

The profit incurred to the system model in steady state can be obtained as

$P_i=K_0A_0-K_1B_0-K_2N_0$

where

K_0=Revenue per unit up-time of the system

K_1=Cost per unit for which server is busy

K_2= Cost per unit visit by the server and A_0,B_0,N_0 are already defined.

4. CONCLUSION

Giving some particular values to the parameters and various costs, the numerical results for MTSF, availability and profit function are obtained to depict their graphical behavior with respect to normal weather rate (β_1) keeping fixed values of other parameters as shown in figures 2, 3 and 4 respectively. From figure 2, it is observed that MTSF declines with the increase of normal weather rate (β_1) and failure rates (λ, λ_1) of the units. But MTSF increases with increase of abnormal weather rate (β) and repair rate (α) of the main unit. Figures 3 and 4 show that availability and profit of the system model go on increasing with increase of normal weather rate (β_1) and repair rates (α and α_1) of the units. However, there is a downward trend in the values of these measures as and when values of abnormal weather rate (β) and failure rates (λ and λ_1) increase. On the basis of the results obtained for a particular case, it is concluded that a cold standby system of non-identical units will be more profitable if it is allowed to operate under controlled weather i.e. normal weather. If we compare the MTSF and Profit of the present model with that of the model Malik and Deswal [2012], it is found that MTSF of both the models is same however, the present model is less profitable. Hence, we conclude that priority to the operation and repair of original unit should be given over the duplicate unit in order to improve the profit of the model.

5. ACKNOWLEDGEMENT

The author Ms. Savita Deswal is grateful to the DHE, Haryana and UGC, New Delhi for awarding Teacher Fellowship to carry out this research work. The authors are also thankful to the reviewers for making suggestions to improve the importance of the paper.

6. REFERENCES

[1] Gopalan, M.N and Naidu, R.S (1982): Stochastic Behavior of a Two -Unit Repairable System subject to Inspection. Microelectron. Reliab. , Vol. 22, pp. 717-722.

[2] Singh, S.K (1989): Profit evaluation of Two -Unit Cold Standby System with Random Appearance and Disappearance Time of the Service Facility. Microelectron. Reliab. , Vol. 29, pp. 705-709.

[3] Singh, Mukender and Chander, S. (2005): Stochastic Analysis of Reliability Models of an Electric Transformer and Generator with Priority and Replacement. Journal of Decision and Mathematical Sciences, Vol. 10 (1-3), pp. 79-100.

[4] Chander, S.; Singh, Mukender and Kumari, Meena (2007): Cost Benefit Analysis of a Stochastic Analysis of an Electric Transformer and Generator. Journal of Pure and Applied Mathematika Sciences, Vol. LXVI, No. 1-2, pp. 13-25.

[5] Malik, S.C. amd Barak, M.S (2009): Reliability and Economic Analysis of a System Operating under Different Weather Conditions. Journal of Proc. Mat. Acad. Sci, India, Sect. A, Vol. 79, pt. II, pp205-213.

[6] Malik, S.C. amd Deswal, Savita (2012): Stochastic Analysis of a Repairable System of Non-Identical Units with Priority for Operation and Repair Subject to Weather Conditions. International Journal of Computer Applications, Vol. 49(14),pp.33-41.

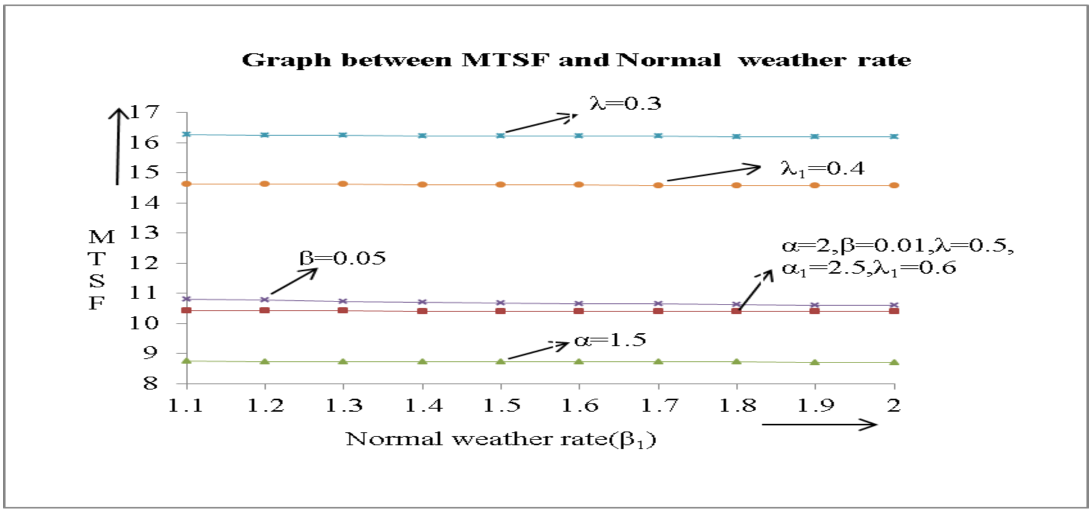


Fig.2

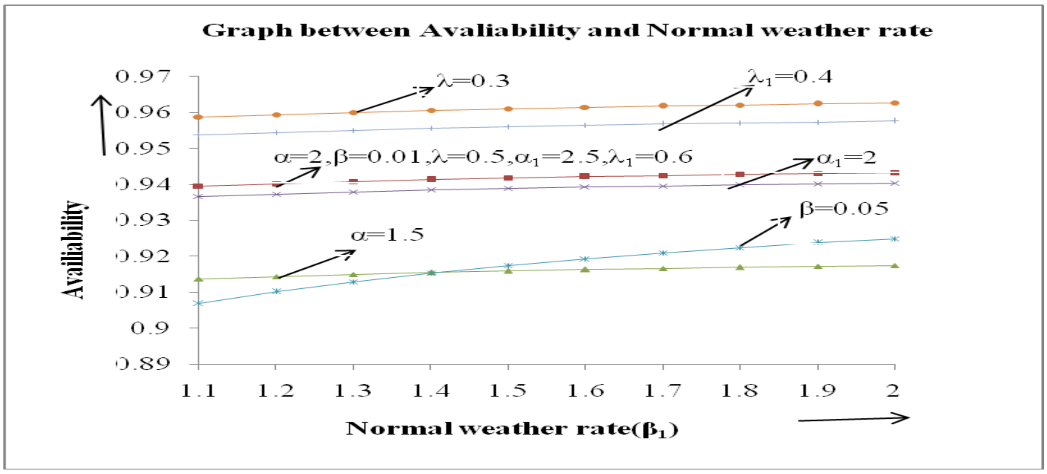


Fig.3

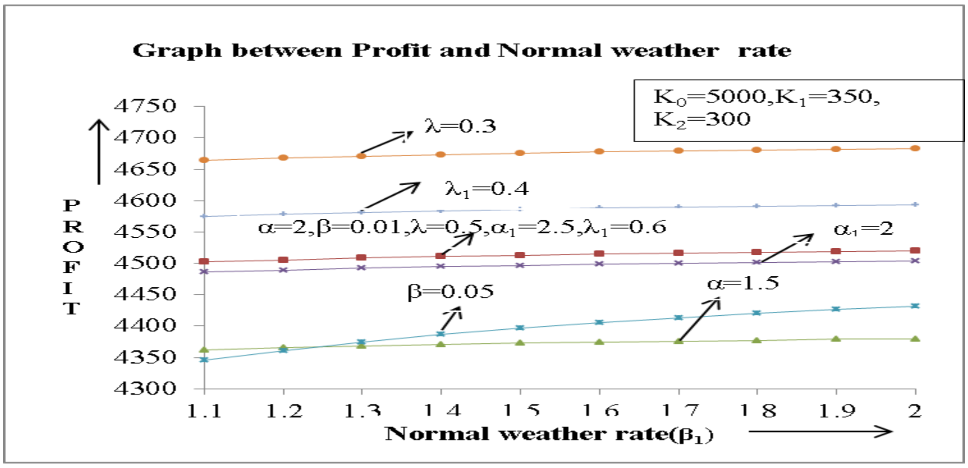

Graph between Profit and Normal weather rate
K0=5000,K1=350,
K2=300
λ=0.3
λ1=0.4
α=2,β=0.01,λ=0.5,α1=2.5,λ1=0.6
α1=2
α=1.5
β=0.05
PROFIT
4750
4700
4650
4600
4550
4500
4450
4400
4350
4300
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
Normal weather rate(β1)

Fig.4