Contra $\tilde{g}_\alpha$-WG Continuous Functions

G. Anitha
Research Scholar,
V.O.Chidambaram College,
Tuticorin,

M. Mariasingam
Department of Mathematics,
V.O.Chidambaram College,
Tuticorin,

ABSTRACT
In this paper, we introduce the new class of weaker form of contra $\tilde{g}_\alpha$ weakly generalized –continuous functions. Some characterization and several properties concerning contra $\tilde{g}_\alpha$ wg-continuity are obtained.

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1.INTRODUCTION
Levine[9] introduced generalized closed sets in topological space. Veera kumar[21] introduced and studied # generalized semi closed set, Naga veni.N[15] introduced weakly generalized closed sets in topological spaces. Rajesh and Lellis Thivagar[17] $\tilde{g}$ -closed set, Saied Jafari,Lellis Thivagar and Nirmala Rebecca Paul[18] introduced and studied $\tilde{g}_\alpha$ -closed set. The authors[11] introduced and studied the weaker form of $\tilde{g}_\alpha$ -WG closed set.

Dontchev [4] introduced and investigated a new notion of continuity is called contra-continuity. Jafari and Noiri [6],[7],[8] introduced new generalization of contra continuity called contra super continuity, contra-α-continuity and contra-pre continuity. The purpose of the present paper is to introduce and investigate some of the properties of contra $\tilde{g}_\alpha$ wg – continuous functions, contra $\tilde{g}_\alpha$ wg-irresolute functions and we obtain characterization of contra $\tilde{g}_\alpha$ wg-continuous function.

2.PRELIMINARIES
Throughout this paper (X, τ), (Y, σ) and (Z, η) will always denote topological spaces on which no separation axioms are assumed, unless otherwise mentioned. When A is a subset of (X, τ), cl(A) and int(A) denote the closure and interior of A respectively. We recall some known definitions needed in this paper.

Definition 2.1: Let (X, τ) be a topological space. A subset A of the space X is said to be
1. a semi-open set [10] if A $\subseteq$ cl(int(A))
2. a pre-open set [14] if A $\subseteq$ int(cl(A))
3. an α -open set [16] if A $\subseteq$ int(cl(int(A)))
4. a regular open[19] if A = int(cl(A))

The complements of the above sets are called their respective open sets.

Definition 2.2: Let (X, τ) be a topological space. A subset A $\subseteq$ X is said to be
1. a generalized closed set(g-closed)[11] if cl(A) $\subseteq$ U whenever A $\subseteq$ U, U is open in (X, τ).
2. a weakly generalized closed set(wg-closed)[15] if Cl(Int(A)) $\subseteq$ U whenever A $\subseteq$ U, U is open in (X, τ).
3.a w-closed set [17]if cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is semi-open in (X, τ).
4. a * g-closed set[20]if cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is w-open in (X, τ).
5. a # g-semi closed set(# gs-closed)[21]if scl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is * g-open in (X, τ).
6. a $\tilde{g}_\alpha$ -closed[18] if α cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is # gs-open in (X, τ).
7. a $\tilde{g}_\alpha$ -Weakly generalized closed set($\tilde{g}_\alpha$ wg-closed) [11] if Cl(Int(A)) $\subseteq$ U, whenever A $\subseteq$ U,U is $\tilde{g}_\alpha$ -open in (X, τ).

The complements of the above sets are called their respective open sets.

Definition 2.3: A function f: (X, τ) $\rightarrow (Y, \sigma)$ is called
1. $\tilde{g}_\alpha$ wg - continuous [12] if $f^{-1}(V)$ is $\tilde{g}_\alpha$ wg-closed in (X, τ) for every closed set V of (Y, σ).
2. $\tilde{g}_\alpha$ wg - irresolute [12] if $f^{-1}(v)$ is $\tilde{g}_\alpha$ wg-closed in (X, τ) for every $\tilde{g}_\alpha$ wg-closed set V in (Y, σ)
3. Contra-continuous [3] if $f^{-1}(v)$ is closed in (X, τ) for every open set V in (Y, σ).
4. RC-continuous [4] if $f^{-1}(V)$ is regular closed in (X, τ) for every open set V in (Y, σ).
5. Contra-α-continuous[6] if $f^{-1}(V)$ is α-closed in (X, τ) for every open set V in (Y, σ).
6. Contra-semi continuous[4] if $f^{-1}(V)$ is semi-closed in (X, τ) for every open set V in (Y, σ).
7. Contra-g-continuous[2] if $f^{-1}(V)$ is g-closed in (X, τ) for every open set V in (Y, σ).

Definition 2.4: A space(X, τ) is called
1. A $T_{1/2}$ space[9] if every g-closed set is closed.
2. A $T_{\tilde{g}_\alpha wg}$ -space[11] if every $\tilde{g}_\alpha$ wg-closed set is closed.
3. Urysohn space[22] if for each pair of distinct points x and y in X, there exists two open sets U and V in X such that x $\in$ U, y $\in$ V and cl(U) $\cap$ cl(V)= ϕ.

4. $\tilde{g}_\alpha$ wg-connected[13] if X cannot be written as the disjoint union of non empty $\tilde{g}_\alpha$ wg-open sets.

3.CONTRA $\tilde{g}_\alpha$ WG-CONTINUOUS FUNCTIONS.

Definition 3.1: A function f:(X, τ) $\rightarrow (Y, \sigma)$ is called contra $\tilde{g}_\alpha$ wg continuous if f^{-1}(V) is $\tilde{g}_\alpha$ wg closed in (X, τ) for each open set V in (Y, σ).

Example 3.2: Let X={a,b,c,d,e}=Y, τ = { ϕ,{a,b},{c,d},{a,b,c,d},X}, σ ={ ϕ,{b,c,d},{a,b,c,d,},{b,c,d,e},Y}. Define f: (X, τ) $\rightarrow (Y, \sigma)$ by f(a)=d, f(b)=c, f(c)=e, f(d)=a, f(e)=b. Then f is contra $\tilde{g}_\alpha$ wg continuous function.

Theorem 3.3: Every contra continuous is contra $\tilde{g}_\alpha$ wg continuous function but not conversely.

Proof: Let V be a open set in (Y, σ). Since f is contra continuous f^{-1}(V) is closed in (X, τ).By theorem 3.2 [11] f^{-1}(V) is $\tilde{g}_\alpha$ wg closed in (X, τ). Hence f is contra $\tilde{g}_\alpha$ wg continuous.

Example 3.4: Let X={a,b,c,d}=Y, τ ={ ϕ,X,{a},{b,c},{a,b,c}}, σ ={ ϕ,Y,{a},{a,c},{a,b,d}} f: (X, τ) $\rightarrow (Y, \sigma)$ Defined by f(a)=b, f(b)=c, f(c)=d, f(d)=a. f is contra $\tilde{g}_\alpha$ wg continuous but not contra continuous. Since {a,c} is open in (Y, σ) but f^{-1}({a,c})={b,c} not closed in (X, τ).

Theorem 3.5: Every contra α-continuous function is contra $\tilde{g}_\alpha$ wg continuous.

Proof: Let f: (X, τ) $\rightarrow (Y, \sigma)$ be a function and V be a open set in (Y, σ). Since f contra α-continuous then f^{-1}(V) is α-closed in (X, τ).By theorem 3.11[11] f^{-1}(V) is $\tilde{g}_\alpha$ wg closed in (X, τ).Hence f is contra $\tilde{g}_\alpha$ wg continuous.

But converse need not be true by Example: 3.4

Remark 3.6: The following examples show that contra $\tilde{g}_\alpha$ wg-continuous and contra semi continuous are independent concept

Example 3.7: Let X={a,b,c}=Y, τ ={ ϕ,X,{a},{b,c}}, σ ={ ϕ,Y,{a}}, The function f: (X, τ) $\rightarrow (Y, \sigma)$ defined by f(a)=c, f(b)=a, f(c)=b, f is contra $\tilde{g}_\alpha$ wg continuous but not Contra semi-continuous. Since {a} is open in (Y, σ) but f^{-1}({a}) = {c} is not in semi-closed in (X, τ).

Example 3.8: Let X={a,b,c}=Y, τ ={ ϕ,X,{a},{c},{a,c}}, σ ={ ϕ,Y,{a}}. f:(X, τ) $\rightarrow (Y, \sigma)$ defined by the identity map, f is contra semi-continuous but not contra $\tilde{g}_\alpha$ wg continuous. Since {a} is open in (Y, σ) but f^{-1}({a}) = {a} is not $\tilde{g}_\alpha$ wg closed in (X, τ).

Remark 3.9: The following examples show that contra $\tilde{g}_\alpha$ wg-continuous and contra g-continuous are independent concept.

Example 3.10: Let X={a,b,c}=Y, τ ={ ϕ,X,{a,c}}, σ ={ ϕ,Y,{a},{b,c}}, The function f: (X, τ) $\rightarrow (Y, \sigma)$ defined as identity map, f is contra $\tilde{g}_\alpha$ wg continuous but not Contra g-continuous. Since {a} is open in (Y, σ) but f^{-1}({a}) = {c} is not g-closed in (X, τ).

Example 3.11: Let X={a,b,c}=Y, τ ={ ϕ,X,{b,c},{c}}}, σ ={ ϕ,Y,{a,c}} f: (X, τ) $\rightarrow (Y, \sigma)$ defined by the identity map, f is contra g-continuous but not contra $\tilde{g}_\alpha$ wg continuous. Since {a,c} is open in (Y, σ) but f^{-1}({a,c}) = {a,c} is not in $\tilde{g}_\alpha$ wg closed in (X, τ).

From the above discussion and known results we have the following diagram (i)

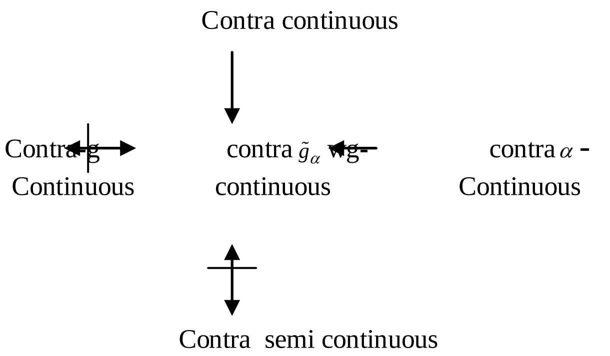

Diagram (i)

In this diagram,

"A → B" means A implies B but not conversely.
"A ↔ B" means A and B are independent each other.

Remark 3.12: Composition of two contra $\tilde{g}_\alpha$ wg-continuous function need not be contra $\tilde{g}_\alpha$ wg continuous.

Example 3.13: Let X={a,b,c,d,e}=Y, Z={a,b,c,d} τ ={ ϕ,{a,b},{c,d}{a,b,c,d},X}, σ ={ ϕ,{b,c,d},{a,b,c,d},{b,c,d,e},Y}, η={ ϕ,{c,d},Z}.

Define f: (X, τ) $\rightarrow (Y, \sigma)$ by f(a)=d, f(b)=c,f(c)=e, f(d)=a, f(e)=b Define g: (Y, σ) $\rightarrow$(Z, η) by g(a)=a, g(b),g(c)=c,g(d)=d, g(e)=c. Function f and g are contra $\tilde{g}_\alpha$ wg continuous function. But their composition is not a contra $\tilde{g}_\alpha$ wg continuous function since the open set U={c,d} in (Z, η).

$(g \circ f)^{-1}$(U)={a,b,c} which is not in $\tilde{g}_\alpha$ wg-closed in (X, τ).

Lemma 3.14: [1] The following properties hold for the subset A,B of space X.

(i) x $\in$ Ker(A) iff A $\cap$ F $\neq \phi$, F $\in$ C(X,x)

(ii) A $\subseteq$ ker(A) and A=ker(A) if A is open in X.

(iii) If A $\subseteq$ B, then ker(A) $\subseteq$ ker(B)

Theorem 3.15: For the function f: $(X,\tau) \rightarrow (Y,\sigma)$, the following condition are equivalent

(i) f is contra $\tilde{g}_\alpha$ wg continuous

(ii) The inverse image of a closed set V of Y is $\tilde{g}_\alpha$ wg open in X.

(iii) For each $x \in X$ and $V \in C(Y,f(x))$, there exists $U \in \tilde{g}_\alpha wgO(X,x)$ such that $f(U) \subseteq V$

Proof:

(i)$\Rightarrow$(ii) obvious

(ii) $\Rightarrow$(iii) Let V be closed set of Y and let $f(x) \in V$ where $x \in X$. Then by (ii) $f^{-1}(V)$ is $\tilde{g}_\alpha$ wg open in X. Also $x \in f^{-1}(V)$. Take U= $f^{-1}(V)$ is $\tilde{g}_\alpha$ wg open set containing x and $f(U) \subseteq V$.

(iii) $\Rightarrow$(ii) Let v be any closed subset of Y. If $x \in f^{-1}(V)$ then $f(x) \in V$. By (iii) there exists a $\tilde{g}_\alpha$ wg open set U_x of X containing x such that $f(U_x) \subseteq V$. Then $f^{-1}(V) = \cup \{U_x / x \in f^{-1}(V)\}$.Hence $f^{-1}(V)$ is a $\tilde{g}_\alpha$ wg open in X.

Theorem 3.16: Let f: $X \rightarrow Y$ be a bijective function. Then following are equivalent

(i) f is contra $\tilde{g}_\alpha$ wg continuous.

(ii) $f(\tilde{g}_\alpha wgCl(A)) \subseteq ker(f(A))$ for every subset A of X

(iii) $\tilde{g}_\alpha wgCl(f^{-1}(B)) \subseteq f^{-1}(ker(B))$ for every subset B of Y.

Proof:

(i)$\Rightarrow$(ii) Let A be any subset of X. Suppose that $y \notin ker\ f(A)$, by lemma 3.14, there exist $F \in C(X,x)$, $f(A) \cap F = \phi$. $\Rightarrow A \cap f^{-1}(F) = \phi$ .Since $f^{-1}(F)$ is $\tilde{g}_\alpha$ wg-open by (i), $\tilde{g}_\alpha wgCl(A) \cap f^{-1}(F) = \phi \Rightarrow f(\tilde{g}_\alpha wgCl(A)) \cap F = \phi$ and $y \notin f(\tilde{g}_\alpha wgCl(A))$. Hence $f(\tilde{g}_\alpha wgCl(A) \subseteq ker(f(A))$.

(ii) $\Rightarrow$(iii) Let B be any subset of Y.by(ii) $f(\tilde{g}_\alpha wgCl(f^{-1}(B)) \subseteq ker(f(f^{-1}(B)) = ker(B)$. $f(\tilde{g}_\alpha wgCl(f^{-1}(B)) \subseteq ker(B)$ $\Rightarrow \tilde{g}_\alpha wgCl(f^{-1}(B)) \subseteq f^{-1}(ker(B))$.

(iii) $\Rightarrow$(i) Let V be open in Y. then by(iii) $\tilde{g}_\alpha wgCl(f^{-1}(V)) \subseteq f^{-1}(ker(V))= f^{-1}(V)$ by lemma3.14. But $f^{-1}(V) \subseteq \tilde{g}_\alpha wgCl(f^{-1}(V))$. So $f^{-1}(V) = \tilde{g}_\alpha wgCl(f^{-1}(V))$ which means $f^{-1}(V)$ is $\tilde{g}_\alpha$ wg closed in X. Hence f is contra $\tilde{g}_\alpha$ wg continuous.

Theorem 3.17: If a function f: $X \rightarrow Y$ is contra $\tilde{g}_\alpha$ wg continuous and X is $T_{\tilde{g}_\alpha wg}$ -space, then f is contra continuous.

Proof: Let V be a open set in Y. since f is $\tilde{g}_\alpha$ wg continuous, $f^{-1}(V)$ is $\tilde{g}_\alpha$ wg closed in X. since the space is $T_{\tilde{g}_\alpha wg}$ -space, $f^{-1}(V)$ is closed in X. Hence f is contra continuous function.

Corollary 3.18: If X is $T_{\tilde{g}_\alpha wg}$ -space. Then for the function f: $X \rightarrow Y$, the following statement are equivalent.

(i) f is contra continuous

(ii) f is contra $\tilde{g}_\alpha$ wg continuous.

Proof: Obvious.

Theorem 3.19: Let f: $X \rightarrow Y$ be a function then following are equivalent:

(i) f is $\tilde{g}_\alpha$ wg continuous.

(ii) For each point $x \in X$ and each open set of Y with $f(x) \in V$, there exists a $\tilde{g}_\alpha$ wg-open set U of X such that $x \in U$, $f(U) \subseteq V$

(i)$\Rightarrow$(ii) Let $f(x) \in V$ then $x \in f^{-1}(V) \in \tilde{g}_\alpha wgO(X)$ since f is $\tilde{g}_\alpha$ wg continuous . Let $U=f^{-1}(V)$, then $x \in U$ and $f(U) \subseteq V$

(ii) $\Rightarrow$(i) Let V be any open set of Y and $x \in f^{-1}(V)$ then $f(x) \in V$. By hypothesis, there exists a $\tilde{g}_\alpha$ wg-open set U_x of X such that $x \in U_x \subseteq f^{-1}(V)$ and $f^{-1}(V) = \cup \{U_x\}$. Then $f^{-1}(V)$ is $\tilde{g}_\alpha$ wg-open in X.

Theorem 3.20: If a function f: $X \rightarrow Y$ is a contra $\tilde{g}_\alpha$ wg continuous and Y is regular, then f is $\tilde{g}_\alpha$ wg continuous.

Proof: Let x be a arbitrary point of X and V be an open set of Y containing f(x). since Y is regular, there exists an open set W in Y such that $f(x) \in W$ and $cl(W) \subseteq V$. since f is contra $\tilde{g}_\alpha$ wg continuous by theorem 3.15, there exist a $\tilde{g}_\alpha$ wg open set U of X with $x \in U$ such that $f(U) \subseteq cl(W)$. Then $f(U) \subseteq cl(W) \subseteq V$. $f(U) \subseteq V$. Hence f is $\tilde{g}_\alpha$ wg continuous.

Definition 3.21: A space X is said to be $\tilde{g}_\alpha$ wg-T_2-space if for each pair of distinct points x and y in X, there exists $\tilde{g}_\alpha$ wg-open sets U and V containing x and y respectively such that $U \cap V = \phi$.

Definition 3.22: A function f: $X \rightarrow Y$ is $\tilde{g}_\alpha$ wg-open if the image of each open set in X is $\tilde{g}_\alpha$ wg-open set in Y.

Definition 3.23: A function f: $X \rightarrow Y$ is $(\tilde{g}_\alpha wg)^*$-open if the image of each $\tilde{g}_\alpha$ wg-open set in X is a $\tilde{g}_\alpha$ wg-open in Y.

Definition 3.24: A function f: $X \rightarrow Y$ is Strongly $\tilde{g}_\alpha$ wg-continuous if $f^{-1}(V)$ is closed in X for every $\tilde{g}_\alpha$ wg-closed set V in Y.

Definition 3.25: A function f: $X \rightarrow Y$ is Perfectly $\tilde{g}_\alpha$ wg-continuous if $f^{-1}(V)$ is clopen in X for every $\tilde{g}_\alpha$ wg-closed set V in Y.

Definition 3.26: $\tilde{g}_\alpha$ wg-Hausdorff

A topological space (X,τ) is said to be $\tilde{g}_\alpha$ wg-Hausdorff if for each pair of distinct points x and y in X, there exist $\tilde{g}_\alpha$ wg-open subsets U and V of X containing x and y respectively such that $U \cap V = \phi$.

Definition 3.27: A topological space (X, τ) is said to be $\tilde{g}_\alpha$ wg-ultra-Hausdorff if for each pair of distinct points x and y in X, there exist $\tilde{g}_\alpha$ wg-clopen subsets U and V of X containing x and y respective such that U $\cap$ V= ϕ .

Definition 3.28:[5] A space (X, τ) is locally indiscrete space every open subset of X is closed

Theorem 3.29: If a function f: (X, τ) $\to (Y,\sigma)$ is injective, contra $\tilde{g}_\alpha$ wg continuous and Y is Uryshon space then X is $\tilde{g}_\alpha$ wg-T_2.

Proof: Let x, y $\in$ X with x $\neq$ y then f(x) $\neq$ f(y). Since Y is Uryshon space, there exist open sets U and V in Y such that f(x) $\in$ U, f(y) $\in$ V and cl(U) $\cap$ cl(V)= ϕ . Since f is contra $\tilde{g}_\alpha$ wg continuous, by theorem 3.15, there exists $\tilde{g}_\alpha$ wg-open sets A and B in X such that x $\in$ A and y $\in$ B and f(A) $\subseteq$ cl(U), f(B) $\subseteq$ cl(V). Then f(A) $\cap$ f(B)= ϕ .so f(A $\cap$ B)= ϕ which implies A $\cap$ B= ϕ and hence X is $\tilde{g}_\alpha$ wg-T_2.

Theorem 3.30: If f: (X, τ) $\to (Y,\sigma)$ is a surjective ($\tilde{g}_\alpha$ wg)*-open function g: (Y, σ) $\to$ (Z, η) is a function such that g $\circ$ f : (X, τ) $\to$ (Z, η) is a contra $\tilde{g}_\alpha$ wg-continuous, then g is contra $\tilde{g}_\alpha$ wg-continuous.

Proof: Let V be any closed subset of (Z, η). Since g $\circ$ f is a contra $\tilde{g}_\alpha$ wg-continuous, then $(g \circ f)^{-1}_{(V)}$ =

$f^{-1}(g^{-1}(V))$ is $\tilde{g}_\alpha$ wg-open in (X, τ) ,since f is surjective and ($\tilde{g}_\alpha$ wg)*-open then $f(f^{-1}(g^{-1}(V)))$=

$g^{-1}(V)$ is $\tilde{g}_\alpha$ wg-open in (Y, σ). Hence g is contra $\tilde{g}_\alpha$ wg-continuous.

Theorem 3.31: If f: (X, τ) $\to (Y,\sigma)$ is a surjective ($\tilde{g}_\alpha$ wg)*-open and $\tilde{g}_\alpha$ wg –irresolute, g: (Y, σ) $\to$ (Z, η) be any function then g $\circ$ f : (X, τ) $\to$ (Z, η) is a contra $\tilde{g}_\alpha$ wg-continuous, iff g is contra $\tilde{g}_\alpha$ wg-continuous.

Proof: Suppose g $\circ$ f is a contra $\tilde{g}_\alpha$ wg-continuous, Let V be any closed subset of (Z, η). Then $(g \circ f)^{-1}_{(V)}$ = $f^{-1}(g^{-1}(V))$ is $\tilde{g}_\alpha$ wg-open in (X, τ), since f is surjective and ($\tilde{g}_\alpha$ wg)*-irresolute, then

$f((g \circ f)^{-1}(V))$= $f(f^{-1}(g^{-1}(V)))$=$g^{-1}(V)$ is $\tilde{g}_\alpha$ wg-open in (Y, σ).Hence g is contra $\tilde{g}_\alpha$ wg-continuous function.

Conversely, suppose g is contra $\tilde{g}_\alpha$ wg-continuous function. Let V be any closed subset of (Z, η). then

$g^{-1}(V)$ is $\tilde{g}_\alpha$ wg-open in (Y, σ). since f is surjective and ($\tilde{g}_\alpha$ wg)*-irresolute, then $f^{-1}(g^{-1}(V))$ is $\tilde{g}_\alpha$ wg-open in (X, τ). $(g \circ f)^{-1}_{(V)}$ is $\tilde{g}_\alpha$ wg -open in (X, τ). Hence g $\circ$ f is contra $\tilde{g}_\alpha$ wg-continuous.

Theorem 3.32: If f: (X, τ) $\to (Y,\sigma)$ is strongly $\tilde{g}_\alpha$ wg continuous function g: (Y, σ) $\to$ (Z, η) is a contra $\tilde{g}_\alpha$ wg – continuous function then g $\circ$ f : (X, τ) $\to$ (Z, η) is a contra continuous.

Proof: Let V be any closed subset of (Z, η). Since g is contra $\tilde{g}_\alpha$ wg-continuous function then $g^{-1}(V)$ is $\tilde{g}_\alpha$ wg-closed in (Y, σ). Since f is strongly $\tilde{g}_\alpha$ wg continuous function, then $f^{-1}(g^{-1}(V))=(g \circ f)^{-1}_{(V)}$ is closed in (X, τ). Hence g $\circ$ f is contra continuous.

Theorem 3.33: If a function f: (X, τ) $\to (Y,\sigma)$ is a g-continuous function and g: (Y, σ) $\to$ (Z, η) is contra $\tilde{g}_\alpha$ wg continuous function and the (Y, σ) be $T_{1/2}$ space, then g $\circ$ f: (X, τ) $\to$ (Z, η) of contra $\tilde{g}_\alpha$ wg continuous function.

Proof: Let V be any closed subset of (Z, η). Since g is g-continuous function then $g^{-1}(V)$ is g-closed in (Y, σ). Since (Y, σ) be $T_{1/2}$ space, then $g^{-1}(V)$ is closed in (Y, σ). Since f is contra $\tilde{g}_\alpha$ wg continuous function, then $f^{-1}(g^{-1}(V))=(g \circ f)^{-1}_{(V)}$ is $\tilde{g}_\alpha$ wg -open in (X, τ). Hence g $\circ$ f is contra $\tilde{g}_\alpha$ wg-continuous.

Theorem 3.34: Every Rc-continuous function is a contra $\tilde{g}_\alpha$ wg-continuous.

Proof: Let V be a open set in Y. since f is RC- continuous, $f^{-1}(V)$ regular closed in X, by proposition 1.4[12] $f^{-1}(V)$ is $\tilde{g}_\alpha$ wg-closed.. Hence f is contra $\tilde{g}_\alpha$ wg -continuous.

Theorem 3.35: Let f: X $\to$ Y be a function and g: X $\to$ X $\times$ Y be the graph function given by

g(x) = (x,f(x)) for every x $\in$ X. If g is contra $\tilde{g}_\alpha$ wg-continuous then f is contra $\tilde{g}_\alpha$ wg-continuous.

Proof: Let V be a closed subset of Y. Then X $\times$ V is a closed subset of X $\times$ Y. Since g is contra $\tilde{g}_\alpha$ wg-continuous, $g^{-1}(X \times V)$ is $\tilde{g}_\alpha$ wg-open subset of X. Also $g^{-1}(X \times V) = f^{-1}(V)$. Hence f is contra $\tilde{g}_\alpha$ wg-continuous.

Theorem 3.36: A function f: X $\to$ Y is contra $\tilde{g}_\alpha$ wg-continuous, injection and Y is urysohn. Then the space X is $\tilde{g}_\alpha$ wg-Hausdorff.

Let x_1 and x_2 be two distinct points of X. Suppose $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since f is injective $x_1 \neq x_2$ then $y_1 \neq y_2$. since Y is urysohn, there exist open sets O_{y_1} and O_{y_2} containing y_1 and y_2 respectively in Y, such that cl(O_{y_1}) $\cap$ cl(O_{y_2})= ϕ . Since f contra $\tilde{g}_\alpha$ wg-continuous by theorem 3.15, there exists $\tilde{g}_\alpha$ wg-open sets $U_{x_1} \in \tilde{g}_\alpha$ wgO(X,x_1) and $U_{x_2} \in \tilde{g}_\alpha$ wgO(X,x_2) such that f(U_{x_1}) $\subseteq$ cl(O_{y_1}) and f(U_{x_2}) $\subseteq$ cl(O_{y_2}) since cl(O_{y_1}) $\cap$ cl(O_{y_2})= ϕ , then $U_{x_1} \cap U_{x_2} = \phi$ .Hence X is $\tilde{g}_\alpha$ wg-Hausdorff

Theorem 3.37: If f: (X, τ) $\rightarrow (Y,\sigma)$ is a contra $\tilde{g}_\alpha$ wg-continuous, injective and Y is $\tilde{g}_\alpha$ wg-ultra-Hausdorff, then the topological space (X, τ) is $\tilde{g}_\alpha$ wg-Hausdorff.

Proof: Let x_1 and x_2 be two distinct points of X. Since f is injective f(x_1) $\neq$ f(x_2) and since f is $\tilde{g}_\alpha$ wg-ultra-Hausdorff, there exist clopen sets U and V in Y such that f(x_1) $\in$ U and f(x_2) $\in$ V where U $\cap$ V= ϕ . $X_1 \in f^{-1}$(U) and $X_2 \in f^{-1}$(V) where f^{-1}(U) $\cap$ f^{-1}(V)= ϕ .Hence X is $\tilde{g}_\alpha$ wg-Hausdorff.

Theorem 3.38: If a function f: (X, τ) $\rightarrow (Y,\sigma)$ is continuous and (X, τ) is locally indiscrete space, then f is contra $\tilde{g}_\alpha$ wg-continuous.

Proof: Let U be any open set in (Y, σ). Since f is continuous then f^{-1}(U) is open in (X, τ). Since (X, τ) is locally indiscrete space then f^{-1}(U) is closed in (X, τ) by theorem 3.2 [11] f^{-1}(U) $\tilde{g}_\alpha$ wg-closed in X. Hence f is contra $\tilde{g}_\alpha$ wg-continuous.

Theorem 3.39: If f: (X, τ) $\rightarrow (Y,\sigma)$ is surjective, contra $\tilde{g}_\alpha$ wg-continuous and X is $\tilde{g}_\alpha$ wg-connected then Y is connected.

Proof: Assume that Y is not connected. Then Y=A $\cup$ B where A and B are non empty open sets in Y such that A $\cap$ B= ϕ . Set U=Y\A and V=Y\B. Then U and V are non empty closed set in Y. Since f is surjective and contra $\tilde{g}_\alpha$ wg-continuous, then non empty f^{-1}(U) and f^{-1}(V) are $\tilde{g}_\alpha$ wg open set in (X, τ).Now f^{-1}(U) $\cap$ f^{-1}(V)=X which is contradiction to the fact X is $\tilde{g}_\alpha$ wg-connected and so Y is connected.

Theorem 3.40: If f: X $\rightarrow$ Y and g: X $\rightarrow$ Y is contra $\tilde{g}_\alpha$ wg-continuous and Y is Urysohn then K={x $\in$ X,f(x)=g(x)} is $\tilde{g}_\alpha$ wg closed in X.

Proof: Let x $\in$ X-K. then f(x) $\neq$ g(x) . Since Y is Urysohn, there exist open sets U and V such that f(x) $\in$ U, g(x) $\in$ V cl(U) $\cap$ cl(V)= ϕ . Since f and g are contra $\tilde{g}_\alpha$ wg-continuous f^{-1}(Cl(U)) $\in$ $\tilde{g}_\alpha$ wgO(X) and g^{-1}(Cl(U)) $\in$ $\tilde{g}_\alpha$ wgO(X).Let A= f^{-1}(Cl(U)), B= g^{-1}(Cl(U)) then A nd B contains x. C=A $\cap$ B, then C is $\tilde{g}_\alpha$ wgO(X). Hence f(C) $\cap$ g(C)= ϕ and x $\notin$ $\tilde{g}_\alpha$ wgCL(K). thus K is $\tilde{g}_\alpha$ wg closed in X.

Theorem 3.41: Let (X, τ) be a $\tilde{g}_\alpha$ wg-connected space and (Y, σ) be any topological space. f: X $\rightarrow$ Y is surjective and contra $\tilde{g}_\alpha$ wg-continuous then Y is not a discrete space.

Proof: If possible, let Y be a discrete space. Let A be any proper non empty subset of Y. Then A is both open and closed in (Y, σ). Since f is contra $\tilde{g}_\alpha$ wg-continuous, f^{-1}(A) is $\tilde{g}_\alpha$ wg-closed and $\tilde{g}_\alpha$ wg-open in (X, τ). Since X is $\tilde{g}_\alpha$ wg-connected by theorem 4.2[13], the only subset of X which are both $\tilde{g}_\alpha$ wg-open and $\tilde{g}_\alpha$ wg-closed are the sets X and ϕ . Then f^{-1}(A) is either X or ϕ .If f^{-1}(A) = ϕ , it contradicts to the fact A $\neq$ ϕ and f is surjective if f^{-1}(A) = X then f fails to be a map. Hence Y is not a discrete space.

Theorem 3.42: Let f: X $\rightarrow$ Y be a surjective, closed and contra $\tilde{g}_\alpha$ wg-continuous. If X is $\tilde{g}_\alpha$ wg-space, then Y is locally indiscrete.

Proof: Let V be any open set in Y. Since f is contra $\tilde{g}_\alpha$ wg-continuous then f^{-1}(V) is $\tilde{g}_\alpha$ wg-closed in X. Since X is $\tilde{g}_\alpha$ wg-space, f^{-1}(V) is closed in X.f is closed and surjective f(f^{-1}(V))=V is closed in Y. Hence Y is locally indiscrete

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4. CONTRA $\tilde{g}_\alpha$ WG-IRRESOLUTE FUNCTIONS

Definition 4.1: A function f: (X, τ) $\rightarrow (Y,\sigma)$ is called contra $\tilde{g}_\alpha$ wg-irresolute, if f^{-1}(V) is $\tilde{g}_\alpha$ wg-closed in (X, τ) for each $\tilde{g}_\alpha$ wg-open in (Y, σ).

Example 4.2: Let X={a,b,c} = Y, τ ={ ϕ ,{a},{b,c}X}, σ = { ϕ ,{b},{c},{a,b},{b,c}},Y}.A function f: (X, τ) $\rightarrow (Y,\sigma)$ is defined by f(a)=b, f(b)=c, f(c)=a. The function f is contra $\tilde{g}_\alpha$ wg-irresolute function.

Remark 4.3: Contra $\tilde{g}_\alpha$ wg-irresolute and $\tilde{g}_\alpha$ wg-irresolute are independent as seen from the following examples.

Example 4.4: Let X = {a,b,c} = Y, τ ={ ϕ ,{a,c},X}, σ = { ϕ ,{c},{a,c},{b,c},Y}.A function f: (X, τ) $\rightarrow (Y,\sigma)$ is defined as identity map is $\tilde{g}_\alpha$ wg-irresolute but not a contra $\tilde{g}_\alpha$ wg-irresolute.

Example 4.5: Let X = {a,b,c} =Y, τ ={ ϕ ,{a},{c}, {a,c}, X}, σ = { ϕ ,{c},{a,c},{b,c},Y}.A function f: (X, τ) $\rightarrow (Y,\sigma)$ is defined by f(a) = b, f(b) = c, f(c) = a. f is a contra $\tilde{g}_\alpha$ wg-irresolute but not a $\tilde{g}_\alpha$ wg-irresolute.

Remark 4.6: $\tilde{g}_\alpha$ wg continuous and contra $\tilde{g}_\alpha$ wg continuous are independent concept.

Example 4.7: Let X={a,b,c}=Y, τ ={ ϕ ,X,{a},{c}, {a,c}}, σ ={ ϕ ,Y,{a}}, The function f: (X, τ) $\rightarrow (Y,\sigma)$ defined by f(a)=b, f(b)=c, f(c)=a, f is $\tilde{g}_\alpha$ wg continuous but not Contra $\tilde{g}_\alpha$ wg continuous.

Example 4.8: Let X={a,b,c}=Y, τ ={ ϕ ,X,{a,b}}, σ ={ ϕ ,Y,{b,c},{c}}, f: (X, τ) $\rightarrow (Y,\sigma)$ defined by the identity map, f is contra $\tilde{g}_\alpha$ wg continuous but not $\tilde{g}_\alpha$ wg continuous.

Theorem 4.9: Every contra $\tilde{g}_\alpha$ wg-irresolute function is contra $\tilde{g}_\alpha$ wg-continuous but not conversely.

Proof: Let V be any open set in (Y, σ), by theorem V is $\tilde{g}_\alpha$ wg-open in (Y, σ). Since f in contra $\tilde{g}_\alpha$ wg irresolute, then f $^{-1}$(V) is $\tilde{g}_\alpha$ wg-closed in (X, τ). Hence f is contra $\tilde{g}_\alpha$ wg-continuous.

Example 4.10: Let X = {a,b,c} = Y, τ ={ ϕ ,{a,c},X}, σ = { ϕ ,{a},{b,c},Y}.A function f: (X, τ) $\rightarrow (Y,\sigma)$ is defined as identity map is contra $\tilde{g}_\alpha$ wg-continuous but not contra $\tilde{g}_\alpha$ wg-irresolute function.

Theorem 4.11: Let f: (X, τ) $\rightarrow (Y,\sigma)$ is a contra $\tilde{g}_\alpha$ wg-irresolute and g: (Y, σ) $\rightarrow$ (Z, η) is a $\tilde{g}_\alpha$ wg – irresolute functions then g $\circ$ f: (X, τ) $\rightarrow$ (Z, η) of contra $\tilde{g}_\alpha$ wg irresolute.

Proof: Let V be $\tilde{g}_\alpha$ wg-open in (Z, η). Since g is $\tilde{g}_\alpha$ wg-irresolute then g $^{-1}$(V) is $\tilde{g}_\alpha$ wg-open in (Y, σ). since f is contra $\tilde{g}_\alpha$ wg-irresolute then f $^{-1}$(g $^{-1}$(V)) = (g $\circ$ f) $^{-1}$(V) is $\tilde{g}_\alpha$ wg-closed in (X, τ). Hence g $\circ$ f is contra $\tilde{g}_\alpha$ wg-irresolute.

Theorem 4.12: Let f: (X, τ) $\rightarrow (Y,\sigma)$ is a contra $\tilde{g}_\alpha$ wg-irresolute and g: (Y, σ) $\rightarrow$ (Z, η) is a $\tilde{g}_\alpha$ wg – continuous functions then g $\circ$ f: (X, τ) $\rightarrow$ (Z, η) of contra $\tilde{g}_\alpha$ wg -continuous.

Proof: Let V be open set in (Z, η). Since g is $\tilde{g}_\alpha$ wg-continuous then g $^{-1}$(V) is $\tilde{g}_\alpha$ wg-open in (Y, σ).since f is contra $\tilde{g}_\alpha$ wg-irresolute then
f $^{-1}$(g $^{-1}$(V)) = (g $\circ$ f) $^{-1}$(V) is $\tilde{g}_\alpha$ wg-closed in (X, τ). Hence g $\circ$ f is contra $\tilde{g}_\alpha$ wg-continuous.

Definition 4.13: A function f: (X, τ) $\rightarrow (Y,\sigma)$ is called perfectly contra $\tilde{g}_\alpha$ wg-irresolute if f $^{-1}$(V) is $\tilde{g}_\alpha$ wg-clopen in X for each $\tilde{g}_\alpha$ wg-open set V in Y.

Example 4.14: Let X={a,b,c} = Y, τ ={ ϕ ,{a},{b,c}X}, σ ={ ϕ ,{c},{a,c},{b,c}},Y}.A function f: (X, τ) $\rightarrow (Y,\sigma)$ is defined by identity map. The function f is perfectly contra $\tilde{g}_\alpha$ wg-irresolute function.

Remark 4.15: Every perfectly contra $\tilde{g}_\alpha$ wg irresolute map is contra $\tilde{g}_\alpha$ wg-irresolute.

Follwing example shows contra $\tilde{g}_\alpha$ wg-irresolute map need not be a perfectly contra $\tilde{g}_\alpha$ wg-irresolute.

Example 4.16: Let X = {a,b,c} =Y, τ ={ ϕ ,{a},{c}, {a,c}, X}, σ = { ϕ ,{c},{a,c},{b,c},Y}.A function f: (X, τ) $\rightarrow (Y,\sigma)$ is defined by f(a) = b, f(b) = c, f(c) = a. f is a contra $\tilde{g}_\alpha$ wg-irresolute but not a $\tilde{g}_\alpha$ wg-irresolute. Let V={a} is $\tilde{g}_\alpha$ wg –closed in (Y, σ) the set f $^{-1}$(V)={c} not a $\tilde{g}_\alpha$ wg-closed in (X, τ).

Remark: 4.17: Every perfectly contra $\tilde{g}_\alpha$ wg irresolute map is $\tilde{g}_\alpha$ wg-irresolute

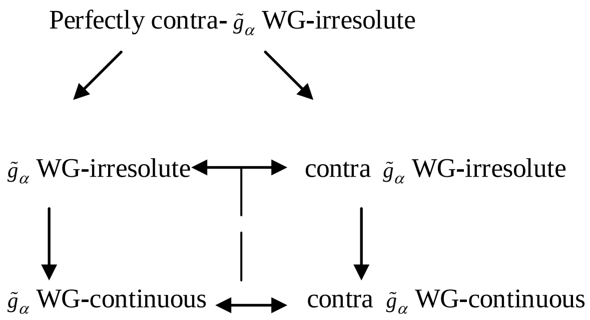
Follwing example shows contra $\tilde{g}_\alpha$ wg-irresolute map need not be a perfectly contra $\tilde{g}_\alpha$ wg-irresolute.

Example 4.18: Let X={a,b,c} = Y, τ ={ ϕ ,{a,c}X}, σ = { ϕ ,{a},{c},{a,c},Y}.A function f: (X, τ) $\rightarrow (Y,\sigma)$ is defined by the identity map. The function f is $\tilde{g}_\alpha$ wg-irresolute but not a perfectly contra $\tilde{g}_\alpha$ wg-irresolute. since f $^{-1}$({a,c})={a,c} is a $\tilde{g}_\alpha$ wg open in (Y, σ) but not a $\tilde{g}_\alpha$ wg clopen set in(X, τ).

Theorem 4.19: A function f: (X, τ) $\rightarrow (Y,\sigma)$ is perfectly contra $\tilde{g}_\alpha$ wg-irresolute iff f is contra $\tilde{g}_\alpha$ wg-irresolute and $\tilde{g}_\alpha$ wg-irresolute.

Proof: It directly follows from the definitions.

From the above discussion and known results we have the following diagram (ii)



Diagram(ii)

In this diagram,

"A ⟶ B" means A implies B but not conversely.
"A ⟷ B" means A and B are independent each other.

5. CONCLUSION

In this paper, we have introduced contra $\tilde{g}_\alpha$ wg-continuous and contra $\tilde{g}_\alpha$ wg-irresolute functions and established their relationships with some other functions. Some of their properties are established.

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