Mixed Convection Boundary Layer Flow past a Vertical Plate in Porous Medium with Viscous Dissipation and Variable Permeability

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ABSTRACT
Mixed convection boundary layer flow, over an isothermal vertical plate, of an incompressible and viscous fluid with variable thermal conductivity is studied in the present paper .The convective flow is taking place in such a porous medium whose permeability is assumed to be spatially variable. The mixed convective flow is due to the simultaneous effects of (1) free stream along the plate and (2) the buoyancy force caused by the variations in density due to temperature difference. The effects of viscous dissipation have been taken into account and the partial differential equations governing the boundary layer flow are converted into to a system of ordinary differential equations by using suitable similarity transformations. These equations are solved numerically and effects of various parameters such as $\frac{Gr}{R_e^2}$, $\frac{\sigma^2}{R_e}$ and Eckert number E_c etc. on the flow fields are investigated and presented graphically.

General Terms
u – Velocity in X-direction

v- Velocity in Y-direction

ν- Kinematic viscosity

ρ - Density of the fluid

g – Acceleration due to gravity

c_p = Specific heat at constant pressure

Pr – Prandtl Number

Gr – Thermal Grashof Number

U_∞ - Free stream velocity

T - Temperature of the fluid

T_w - Temperature of the fluid near the plate

T_∞ - Temperature of the fluid away from the plate

β – Coefficient of volume expansion

E_c - Eckert number

k - Variable permeability of the medium

σ^2 - Non dimensional permeability parameter

α(y)- Variable thermal conductivity

Keywords
Porous medium, variable permeability, thermal conductivity, heat transfer, viscous dissipation, , vertical plate.

1. INTRODUCTION
Free and forced convection flow and heat transfer problems in a fluid saturated porous medium have attracted the attention of many workers due to their immense practical importance as well as a number of engineering and geophysical applications such as, in petroleum reservoirs, geothermal reservoirs, industrial and agricultural water distribution, drying of porous solids and cooling processes of nuclear reactors, to name a few. It is reported in the literature that the study of free convection boundary layer flow was initiated by Polhausen[1]. Siegel [2] extended the previous investigations to deal with the problem of transient free convection flow about an isothermal vertical plate. Cheng and Minkowycz [3] gave a good account of free convection flow about a vertical plate embedded in porous medium with application to heat transfer using similarity transformations. The buoyancy driven heat and mass transfer problem in porous medium from a vertical plate was investigated by Bejan and Khair [4].

It is to be noted that all the aforementioned studies have been confined to flows involving such fluids and media that have constant properties. The physical properties of the fluids, mainly viscosity may change significantly with temperature. Gary et al. [5] have considered the effects of variation of viscosity on convective heat transfer. Kays and Crawferd[6] have described various relations between the physical properties of fluids and temperature. Most of the studies involving variable viscosity have considered the permeability and thermal conductivity as constants. Tierney [7] has shown that the porosity of the medium may change causing a change in permeability of the medium. Chandrasekhar et al. [8] took into account the variation of permeability in the study of mixed convection in the presence of horizontal impermeable surfaces in saturated porous media and have shown that the variability in porosity has significant effects on the velocity distribution and heat transfer. Chandrasekhar and Namboodiri [9] carried out the investigation for mixed convection about inclined surfaces in a saturated porous media incorporating the variation of permeability and thermal conductivity due to packing of particles. Singh and Sharma [10] have considered the porous medium with periodic permeability, while investigating the three dimensional free convection flow and heat transfer. Salem and Fathy[11], in their investigation relating to MHD heat and mass transfer flow near a stagnation point , have incorporated the temperature dependent variation of viscosity and thermal conductivity

Hassanien[12] considered the influence of variable permeability and thermal conductivity on the mixed convection flow from an impermeable vertical wedge wherein he obtained a non similarity solution for the case of variable surface heat flux. Ibrahim and Hassanien[13] and Massour and El-Shaer[14] have considered the effects of variable permeability and thermal conductivity under varying kinds of flow situations. Reddy and Reddy[15] studied the effects of variable viscosity and thermal conductivity on an electrically conducting fluid flow past a moving vertical plate. All these studies have generally neglected the viscous dissipation, although viscous dissipation in porous media is an interesting phenomenon and plays an important role in many physical processes .The effect of viscous dissipation has been incorporated in the modeling of flows in various ways by different workers. Nield[16] has discussed quite extensively the Brinkman equation and the concept and modeling of viscous dissipation in porous media .

Thus, the aim of the present paper is to investigate the effects of variable permeability and thermal conductivity on the mixed convection boundary layer flow adjacent to a vertical plate in porous medium taking into account viscous dissipation.

2. MATHEMATICAL FORMULATION

Let us consider a steady two dimensional flow of a viscous, incompressible electrically conducting fluid along a vertical plate in a porous medium. x direction is taken along the leading edge of the plate and y is normal to it, i.e., the plate starts at x=0 and extends parallel to the x axis and is of semi infinite length. The plate temperature is uniformly maintained at T_w and the temperature of the fluid far away from the plate is T_∞ . T_w is higher than the temperature T_∞. A steady flow parallel to the plate with free stream velocity U_∞ is assumed to take place. The mixed convective flow is the result of the simultaneous effects of (1) the free stream along the plate and (2) the buoyancy force caused by the variations in density due to temperature difference. Let u and v be the velocity components in the boundary layer region along the x and y- axes respectively. The permeability of the porous medium and the thermal conductivity of the fluid are assumed to be variable. Permeability and thermal conductivity both are assumed to be exponential functions of distance as proposed by Chandrasekhar[9]. Then under the Boussinesq and the usual boundary layer approximations, the governing equations for the Darcy type flow, following Nield and Bejan [18] and Schlichting[19] are given by:

$$\frac{\partial u}{\partial x}+\frac{\partial \mathrm{v}}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+\mathrm{v}\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta\left(T-T_\infty\right)+\frac{\nu}{k_0}U_\infty-\frac{\nu}{k}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+\mathrm{v}\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha(y)\frac{\partial T}{\partial y}\right)+\frac{\nu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\nu}{kc_p}u^2 \tag{3}$$

Boundary conditions

$$\begin{aligned} &u=0,\ \mathrm{v}=0,\ T=T_w, &&\text{at} \quad y=0 \\ &u=U_\infty, T=T_\infty, &&\text{at} \quad y\to\infty \end{aligned} \tag{4}$$

We introduce the following non-dimensional variables:

$$\eta=y\left(\frac{U_\infty}{\nu x}\right)^{\frac{1}{2}}$$

$\psi=\sqrt{(\nu U_\infty x)}\,f(\eta)$, such that $u=\frac{\partial\psi}{\partial y}$, $\mathrm{v}=-\frac{\partial\psi}{\partial x}$.

Now, we have $u=U_\infty f'(\eta)$, $\mathrm{v}=\frac{1}{2}\sqrt{\frac{U_\infty\nu}{x}}\left(\eta f'-f\right)$

And these values of the velocity components satisfy the equation (1).

Further we take

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}$$

Following Chandrasekhar et al. as stated above, the variations in the permeability and thermal conductivity have been taken as given below-

$$\begin{aligned} &k(\eta)=k_0(1+d^*e^{-\eta}) \\ &\alpha(\eta)=\alpha_0[\varepsilon_0(1+de^{-\eta})+b\{1-\varepsilon_0(1+de^{-\eta})\}] \end{aligned} \tag{5}$$

where d^* and d are constants, α_0 , k_0 and ε_0 are the values of the diffusivity, permeability and porosity respectively at the edge of the boundary layer, b being the ratio of the thermal conductivity of the solid to that of the fluid.

Now using above non-dimensional variables equations (2) to (4) become

$$f'''+\frac{1}{2}ff''+\frac{Gr}{R_e^2}\theta+\frac{\sigma^2}{R_e}\left(1-\frac{f'}{(1+d^*e^{-\eta})}\right)=0 \tag{6}$$

$$\begin{aligned} &\frac{1}{P_r}[\varepsilon_0(1+de^{-\eta})+b\{1-\varepsilon_0(1+de^{-\eta})\}]\theta''+\varepsilon_0 d(b-1)e^{-\eta}\theta' \\ &+\frac{1}{2}f\theta'+E_c f''+E_c\frac{\sigma^2}{R_e}\left(\frac{f'^2}{(1+d^*e^{-\eta})}\right)=0 \end{aligned} \tag{7}$$

where,

$$\begin{aligned} &Gr=\frac{g\beta(T_w-T_\infty)x^3}{\nu^2} &&P_r=\frac{\nu}{\alpha} \\ &E_c=\frac{U_\infty^{\ 2}}{c_p(T_w-T_\infty)} &&R_e=\frac{U_\infty x}{\nu} \\ &\sigma^2=\frac{x^2}{k_0} \end{aligned} \tag{8}$$

The transformed boundary conditions are given by-

$$\left.\begin{aligned} &f=0 \quad f'=0 \quad \theta=1 \quad \varphi=1 \quad \text{at} \quad \eta=0 \\ &f'=1 \quad \theta=0 \quad \varphi=0 \quad \text{at} \quad \eta\to\infty \end{aligned}\right\} \tag{9}$$

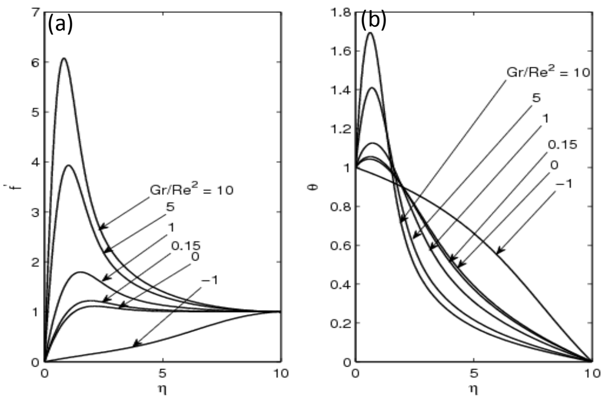
The physical problem is now mathematically represented by the equations (6) and (7) and these equations involve four parameters $\frac{Gr}{R_e^{\ 2}},\frac{\sigma^2}{R_e},P_r$ and E_c. From the expressions of Gr and R_e given above in the equation (8), it is clear that Gr represents the non-dimensional buoyancy parameter and, therefore, gives a measure of the free convection and R_e , the Reynolds number, gives the forced convection. Thus, the value of the ratio $\frac{Gr}{R_e^{\ 2}}$ represents the relative importance of forced and free convection in determining the over all flow. In the similar fashion, the parameter $\frac{\sigma^2}{R_e}$, being a ratio of non-dimensional permeability parameter and the Reynolds number represents the relative importance of Darcy and general viscous drag.

The Eckert number E_c is due to the viscous dissipation.

3. RESULT AND DISCUSSION

The equations (6) and (7) together with the boundary conditions (8) are solved numerically using BVP 4C method. The results depicting the nature of effects of various parameters on the profiles of velocity f' and temperature θ are displayed with the help of graphical illustrations.

In figure 1, the effects of the parameter $\frac{Gr}{R_e^2}$ on the velocity and temperature distribution are shown.



a = 0.1; σ^2/Re = 1; d_1 = 3.0; b = 2;
ε_0 =0.4; d = 1.5; Pr =0.7; Ec=0.5

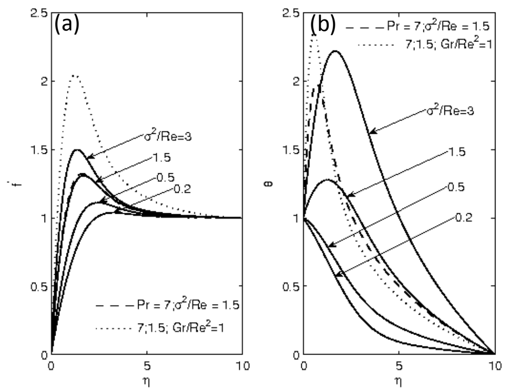
Figure 1

From the figure, it is observed that $\frac{Gr}{R_e^2}$ has very significant effects on the velocity as well as on the temperature profiles. As we increase the value of $\frac{Gr}{R_e^2}$, we find from the figure 1(a) that the velocity in the boundary layer increases rapidly near the plate and then gradually decreases and smoothly mixes with the uniform free stream. We have considered the effect of this parameter by taking its value negative also, which means the opposing flow, because $\frac{Gr}{R_e^2} < 0$ means $T_w < T_\infty$, ($\frac{Gr}{R_e^2} > 0$ means aiding flow); its effect is depicted by taking $\frac{Gr}{R_e^2}$ =-1.

From the figure 1(b), we find that the effects of the parameter $\frac{Gr}{R_e^2}$ on the temperature is almost similar, i.e., the temperature increases very rapidly close to the plate if we increase the value of $\frac{Gr}{R_e^2}$ and then the temperature profile asymptotically merges with the ambient fluid . A noteworthy aspect of this profile is that, from and after a certain point on η - axis, the value of non-dimensional temperature is greater for smaller values of $\frac{Gr}{R_e^2}$.

The effects of the parameter $\frac{\sigma^2}{R_e}$ on the velocity and temperature profiles are shown in the figures 2(a) & 2(b) respectively.

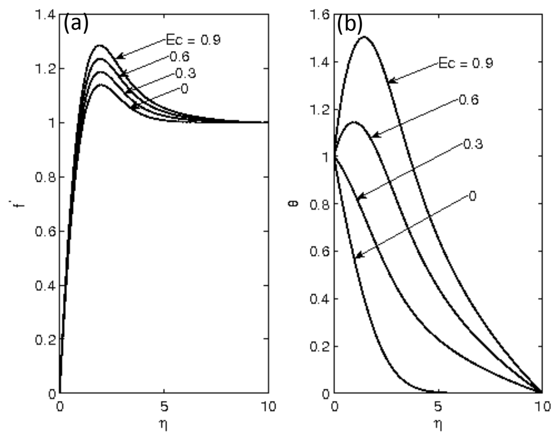
We observe that as the value of this parameter is increased, the non-dimensional velocity increases near the plate and then it steadily acquires the free stream velocity value. We have also investigated the effect on liquids (water) by taking the value of the Prandtl number Pr =7.0. We have taken $\frac{Gr}{R_e^2}$ =0.15, 1.0 and $\frac{\sigma^2}{R_e}$ =1.5. $\frac{Gr}{R_e^2}$ > 0.75 represents forced convection, and in the case of $\frac{Gr}{R_e^2}$ =1.0, a significant increase in the velocity is observed.



a = 0.1;Gr/Re^2 = 0.15; d_1 = 3.0; b = 2;
ε_0 =0.4; d = 1.5; Pr =0.7; Ec=0.5

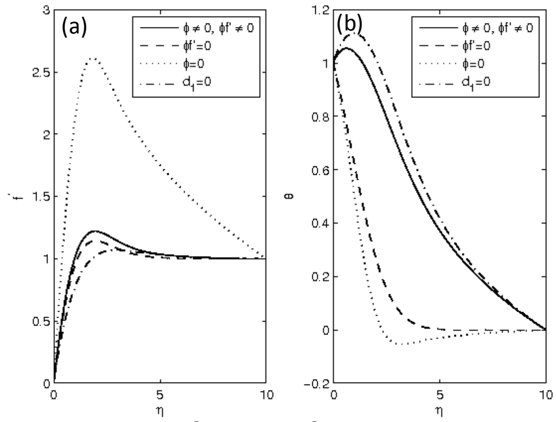
Figure 2

The parameter $\frac{\sigma^2}{R_e}$ has an increasing effect on the temperature profiles. Again, we have differentiated the case of higher Pr fluids by plotting the effects by dashed lines in the figure 2 and it is observed that in the case of mixed convection, the increase in temperature is more than it is in the case of forced convection.



a = 0.1;Gr/Re^2 = 0.15; σ^2/Re = 1; d_1 = 3.0; b = 2;
ε_0 =0.4; d = 1.5; Pr =0.7

Figure 3



a = 0.1;Gr/Re^2 = 0.15; σ^2/Re = 1; d_1 = 3.0; b = 2;
ε_0 =0.4; d = 1.5; Pr =0.7; Ec=0.5

Figure 4

The effects of Eckert number E_c on the flow and heat transfer are shown in figures 3(a) and 3(b) respectively. From both the figures, it is observed that an increase in E_c has the uniform effect of increasing the velocity and temperature profiles alike.

Figures 4(a) and 4(b) depict the effects of the permeability parameter on the velocity and temperature profiles. For the sake of discussing the various cases, we have made use of following notation-

$\phi = \dfrac{f'}{(1+d^{*}e^{-\eta})}$, and as a result the equations (6) and (7) respectively become

$$f''' + \frac{1}{2} ff'' + \frac{Gr}{R_e^{2}}\theta + \frac{\sigma}{R^{2}_{e}}(1-\phi) = 0$$

$$\frac{1}{P_r}[\varepsilon_0(1+de^{-\eta}) + b\{1-\varepsilon_0(1+de^{-\eta})\}]\theta'' + \varepsilon_0 d(b-1)e^{-\eta}\theta'$$
$$+\frac{1}{2} f\theta' + E_c f'' + E_c \frac{\sigma}{R^{2}_{e}}(\phi f') = 0$$

Various cases corresponding to $\phi \neq 0, \phi f' \neq 0, \phi f' = 0, \phi = 0$ and $d_1(= d^{*}) = 0$ have been plotted in figures 4(a) and 4(b). It is noteworthy that $d_1 = 0$ corresponds to constant permeability case and in this case we find the standard velocity profile i.e., velocity, satisfying no slip condition at the plate, steadily increases and attains the free stream velocity. Also, it is observed that in this case the velocity is less than what it is when permeability of the medium varies. But it has an increasing effect on the temperature profile. $\phi f' = 0$ corresponds to the case when the modification suggested by Nield in modeling the dissipation is not taken into account and the effect of the absence of this modification is to decrease both the velocity as well as the temperature. ϕ =0 corresponds to the case when there is no Darcy resistance in the boundary layer as well as no Nield correction term in the expression for viscous dissipation. This has the effect of significantly increasing the velocity but it has the opposite effect on the temperature profile.

4. CONCLUSION

From the foregoing discussions, it is clear that the variations in the thermal conductivity and medium permeability have significant effects on the velocity and temperature profiles. The findings may prove to be important from the point of view of having greater insight and understanding of various physical processes.

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