# Fuzzy Transshipment Problem

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## ABSTRACT
The fuzzy transportation problem in which available commodity frequently moves from one source to another source or destination before reaching its actual destination is called a fuzzy transshipment problem. To solve the fuzzy transshipment problem by linear programming problem using Simplex-type Algorithm by [1]. The advantage of this algorithm is that it does not use any artificial variables and it also reduces the iterations to get an optimum solution.

## Mathematics Subject Classifications
90C08, 90C90.

## Keywords
Fuzzy transshipment problem, Arsham-Khan's Algorithm, Trapezoidal fuzzy numbers.

## 1. INTRODUCTION
In a fuzzy transportation problem shipment of commodity takes place among sources and destinations. But instead of direct shipments to destinations, the commodity can be transported to a particular destination through one or more intermediated or fuzzy transshipment points. Each of these points in turn supply to other points. Thus, when the shipments pass from destination to destination and from source to source, fuzzy transshipments exists here. Such a problem cannot be solved as such by the usual fuzzy transportation algorithm, but slight modification is required before applying to the fuzzy transshipment problem. Transportation problem is solved by a simplex type algorithm by [1] to get an optimum solution. Later it was developed by [2] to solve the fuzzy transportation problem using triangular fuzzy numbers. In this paper it is further developed to solve the fuzzy transshipment problem by simplex type algorithm using trapezoidal fuzzy number to get an optimum solution.

Fuzzy transshipment problem is solved by a simplex type algorithm by [1] in two phases. In the first phase uses a basic variable iteration to develop a basic variable set which may or may not be feasible. The second phase uses a feasibility iteration to develop a feasible and optimum solution. Both the phases are used by the Gauss-Jordan pivoting method. The advantage of this method by [1] avoids artificial variables required by simplex method.

## 2. PRELIMINARIES
In this section some basic definitions and arithmetic operations are reviewed by [6]:

**Definition 1**: A fuzzy number $\tilde{a}$ is a trapezoidal fuzzy number denoted by $(a_1,a_2,a_3,a_4)$ where $a_1,a_2,a_3$ and $a_4$ are real numbers and its membership function $\mu_{\tilde{a}}(x)$ is given below:

$$\mu_{\tilde{a}}(x) = \begin{cases} \mathbf{0} & \textbf{for } \mathbf{x \le a_1} \\ \frac{(\mathbf{x-a_1})}{(\mathbf{a_2-a_1})} & \textbf{for } \mathbf{a_1 \le x \le a_2} \\ \mathbf{1} & \textbf{for } \mathbf{a_2 \le x \le a_3} \\ \frac{\mathbf{a_4-x}}{\mathbf{a_4-a_3}} & \textbf{for } \mathbf{a_3 \le x \le a_4} \\ \mathbf{0} & \textbf{for } \mathbf{x \ge a_4} \end{cases}$$

**Definition 2:** A special version of the linear ranking functions was proposed by [3] and [4] is as follows:

$\mathcal{R}(\tilde{a}) = \frac{a^L+a^U}{2} + \frac{1}{2}(\beta - \alpha)$, where $\tilde{a} = (a^L, a^U, \alpha, \beta)$ be a trapezoidal number.

**Definition 3:** We next define arithmetic on trapezoidal fuzzy numbers. Let $\tilde{a} = (a^L, a^U, \alpha, \beta)$ and $\tilde{b} = (b^L, b^U, \gamma, \theta)$ be two trapezoidal fuzzy numbers.

Define, $x>0$, $x \in \mathbb{R}$; $x\tilde{a} = (xa^L, xa^U, x\alpha, x\beta)$,
$x<0$, $x \in \mathbb{R}$; $x\tilde{a} = (xa^L, xa^U, -x\beta, -x\alpha)$,
$\tilde{a} \oplus \tilde{b} = (a^L + b^L, a^U + b^U, \alpha+\gamma, \beta+\theta)$.
$\tilde{a} \otimes \tilde{b} = (a,b,c,d)$ where $a = \min(a^Lb^L, a^L\theta, b^L\beta, \beta\theta)$,
$b = \min(a^Ub^U, a^U\gamma, \alpha b^U, \alpha\gamma)$, $c = \max(a^Ub^U, a^U\gamma, \alpha b^U, \alpha\gamma)$,
$d = \max(a^Lb^L, a^L\theta, b^L\beta, \beta\theta)$.

## 3. FUZZY TRANSSHIPMENT MODEL
Let us consider a fuzzy transportation problem with p sources $S_i$ , i= 1 to p and destinations $D_j$ , j= 1 to q having fuzzy availability $\tilde{a_i}$ at initial source $S_i$ , i=1 to p and the fuzzy demand $\tilde{b_j}$ at initial destination $D_j$ , j=1 to q. Let the following type of transshipment be allowed:
1. From a source to any another source.
2. From a destination to another destination.
3. From a destination to any source.

The number of sources and destinations in the transportation problem are p and q respectively. In transshipment problem we have p+q sources and destinations.
Thus the fuzzy transshipment problem may be written as:

Minimize $\check{Z} = \sum_{i=1}^{p+q} \sum_{j=1, j\neq i}^{p+q} \widetilde{c_{ij}}\, \widetilde{x_{ji}}$

Subject to $\sum_{j=1, j\neq i}^{p+q} \widetilde{x_{ij}} - \sum_{j=1, j\neq i}^{p+q} \widetilde{x_{ji}} = \widetilde{a_i}$ ,
$\sum_{i=1, i\neq j}^{p+q} \widetilde{x_{ij}} - \sum_{i=1, i\neq j}^{p+q} \widetilde{x_{ji}} = \widetilde{b_j}$ ,

where $\widetilde{x_{ij}} \ge 0$, i,j = 1,2,3…p+q, j≠i

The above formulation is a fuzzy transshipment model is reduced to transportation form by [5] as:

Minimize $\check{Z} = \sum_{i=1}^{p+q} \sum_{j=1, j\neq i}^{p+q} \widetilde{c_{ij}}\, \widetilde{x_{ji}}$

Subject to $\sum_{j=1}^{p+q} \widetilde{x_{ij}} = \widetilde{a_i} + \tilde{P}$, i =1 to p
$\sum_{j=1}^{p+q} \widetilde{x_{ji}} = \tilde{P}$, i = p+1 to p+q

$\sum_{i=1}^{p+q} \widetilde{x_{ij}} = \tilde{P}$, j = 1 to p
$\sum_{i=1}^{p+q} \widetilde{x_{ij}} = \widetilde{b_j} + \tilde{P}$ , j = p+1 to p+q
where $\widetilde{x_{ij}} \geq 0$, i,j = 1,2,3…p+q, j≠i

The balanced fuzzy transportation problem with transshipment having p+q sources and p+q destinations. Add an amount of fuzzy buffer stock $\tilde{P} = \sum_{i=1}^{p} \widetilde{\alpha_i}(\text{or}\sum_{j=p+1}^{p+q} \widetilde{b_j})$ in the availability and demand corresponding to each source and destination nodes.

## 3.1 Algorithm to solve the Transshipment problem

**Step 1**: Find the total fuzzy availability $\sum_{i=1}^{p} \widetilde{\alpha_i}$ and the total fuzzy demand$\sum_{j=p+1}^{p+q} \widetilde{b_j}$. Let $\sum_{i=1}^{p} \widetilde{\alpha_i}$ = (a,b,c,d) and $\sum_{j=p+1}^{p+q} \widetilde{b_j}$ = (a',b',c',d'). Examine that the problem is balanced or not. i.e. $\sum_{i=1}^{p} \widetilde{\alpha_i} = \sum_{j=p+1}^{p+q} \widetilde{b_j}$ . If the problem is balanced then goes to step 2. The Fuzzy transportation problem table is shown in **Table 1**:

**Table 1. Fuzzy Transportation problem**

| $D_j \rightarrow$ $S_i \downarrow$ | $S_1$ | $S_2$ | … | $S_p$ | $D_1$ | … | $D_q$ | Availability |
|---|---|---|---|---|---|---|---|---|
| $S_1$ | $\tilde{0}$ | $\widetilde{c_{12}}$ | … | $\widetilde{c_{1p}}$ | $\widetilde{c_{1(p+1)}}$ | … | $\widetilde{c_{1(p+q)}}$ | $\widetilde{a_1}$ |
| $S_2$ | $\widetilde{c_{21}}$ | $\tilde{0}$ | … | $\widetilde{c_{2p}}$ | $\widetilde{c_{2(p+1)}}$ | … | $\widetilde{c_{2(p+q)}}$ | $\widetilde{a_2}$ |
| … | … | … | … | … | … | … | … | … |
| $S_p$ | $\widetilde{c_{p1}}$ | $\widetilde{c_{p2}}$ | … | $\tilde{0}$ | $\widetilde{c_{p(p+1)}}$ | … | $\widetilde{c_{p(p+q)}}$ | $\widetilde{a_p}$ |
| $D_1$ | … | … | … | … | … | … | … | - |
| … | … | … | … | … | … | … | … | - |
| $D_q$ | $\widetilde{c_{(p+q)1}}$ | $\widetilde{c_{(p+q)2}}$ | … | … | $\widetilde{c_{(p+q)p}}$ | … | $\tilde{0}$ | - |
| Demand | - | - | … | - | $\widetilde{b_{p+1}}$ | … | $\widetilde{b_{p+q}}$ | |

p+q = total number of sources = total number of destinations.

$\sum_{i=1}^{p+q} \widetilde{\alpha_i}$ = total fuzzy availability of the product corresponding to sources $S_i$, i=1 to p

$\sum_{j=p+1}^{p+q} \widetilde{b_j}$ = total fuzzy availability of the product corresponding to sources $D_i$, i=1to q.

$\widetilde{c_{ij}}$ = (0, 0, 0, 0)
= fuzzy transportation cost for unit quantity of the product from ith source to ith destination.

$\widetilde{x_{ij}}$ = ($a_{ij}$, $b_{ij}$, $c_{ij}$, $d_{ij}$)
= the fuzzy quantity of the product that should be transported from the ith source to jth destination to minimize the total fuzzy transportation cost.

**Step 2:** A row in the table will be needed for each supply point and transshipment point, and a column will be needed for each demand point and transshipment point. Add an amount of fuzzy buffer stock $\tilde{P} = \sum_{i=1}^{p} \widetilde{\alpha_i}(\text{or}\sum_{j=p+1}^{p+q} \widetilde{b_j})$ in the availability and demand corresponding to each source and destination. Balanced fuzzy transportation problem with transshipment after adding an amount of fuzzy buffer stock $\tilde{P} = \sum_{i=1}^{p} \widetilde{\alpha_i}(\text{or}\sum_{j=p+1}^{p+q} \widetilde{b_j})$ to each source and destination is shown in **Table 2:**

**Table 2. Balanced Fuzzy Transportation problem**

| $D_j \rightarrow$ $S_i \downarrow$ | $S_1$ | $S_2$ | … | $S_p$ | $D_1$ | … | $D_q$ | Availability |
|---|---|---|---|---|---|---|---|---|
| $S_1$ | $\tilde{0}$ | $\widetilde{c_{12}}$ | … | $\widetilde{c_{1p}}$ | $\widetilde{c_{1(p+1)}}$ | … | $\widetilde{c_{1(p+q)}}$ | $\widetilde{a_1} \oplus \tilde{P}$ |
| $S_2$ | $\widetilde{c_{21}}$ | $\tilde{0}$ | … | $\widetilde{c_{2p}}$ | $\widetilde{c_{2(p+1)}}$ | … | $\widetilde{c_{2(p+q)}}$ | $\widetilde{a_2} \oplus \tilde{P}$ |
| … | … | … | … | … | … | … | … | … |
| $S_p$ | $\widetilde{c_{p1}}$ | $\widetilde{c_{p2}}$ | … | $\tilde{0}$ | $\widetilde{c_{p(p+1)}}$ | … | $\widetilde{c_{p(p+q)}}$ | $\widetilde{a_P} \oplus \tilde{P}$ |
| $D_1$ | … | … | … | … | … | … | … | $\tilde{P}$ |
| … | … | … | … | … | … | … | … | - |
| $D_q$ | $\widetilde{c_{(p+q)1}}$ | $\widetilde{c_{(p+q)2}}$ | … | … | $\widetilde{c_{(p+q)p}}$ | … | $\tilde{0}$ | $\tilde{P}$ |
| Demand | - | - | … | - | $\widetilde{b_{p+1}} \oplus \tilde{P}$ | … | $\widetilde{b_{p+q}} \oplus \tilde{P}$ | |

$\tilde{P}$ = ($p_1$, $p_2$, $p_3$, $p_4$) is the fuzzy buffer stock.

**Step 3:** Using Arsham-Khan's Simplex Tableau [1] to get an optimum basic feasible solution for the given fuzzy transshipment problem.

## 3.2 Numerical Example

Consider the following fuzzy transshipment problem involving two sources and two destinations. The fuzzy availability of the sources S1 and S2 are (3,6,2,4) and (13,28,8,26) respectively. The fuzzy demand values of destinations D1 and D2 are (12.3,21.2,6.5,19.5) and (3.7,12.8,3.5,10.5) respectively. The fuzzy transportation cost for unit quantity of the product between different sources and destinations are summarized as in Table 3. Find the fuzzy optimal shipping plan for this fuzzy transportation problem with transshipment.

**Step 1:** Total fuzzy availability = (16,34,10,30) and total fuzzy demand = (16,34,10,30). So it is a balanced fuzzy transportation problem, if not balanced it.

**Step 2:** The balanced fuzzy transportation problem with transshipment having four sources and four destinations after adding an amount of fuzzy buffer stock $\tilde{P}$ = (16,34,10,30) to each source and destination nodes in Table 4.

**Step 3:** After Row – Column Reduction (From each row subtract the smallest cost and subtract the smallest cost from each column) we get the same Table 4.

**Step 4:** Construct Arsham- Khan's Simplex Tableau as shown in **Table 5, Table 6.** Right hand side values are positive as shown in **Table 8**; therefore we get an optimum solution. Hence Arsham-Khan's simplex table by [1] is complete.

**Table 3. Summary of fuzzy transportation cost for unit quantity of the product between different sources and destinations**

| | $S_1$ | $S_2$ | $D_1$ | $D_2$ | Availability |
|---|---|---|---|---|---|
| $S_1$ | (0,0,0,0) | (.5,1,1,2) | (1,4,1,3) | (2,5,1,3) | (3,6,2,4) |
| $S_2$ | (.5,1,1,2) | (0,0,0,0) | (.2,2.8,.5,2.5) | (2,5,1,3) | (13,28,8,26) |
| $D_1$ | (1,4,1,3) | (.2,2.8,.5,2.5) | (0,0,0,0) | (.5,1,1,2) | - |
| $D_2$ | (2,5,1,3) | (2,5,1,3) | (.5,1,1,2) | (0,0,0,0) | - |
| **Demand** | - | - | (12.3,21.2,6.5,19,5) | (3.7,12.8,3.5,10.5) | |

**Table 4. Balanced fuzzy transshipment problem**

| | $S_1$ | $S_2$ | $D_1$ | $D_2$ | Availability |
|---|---|---|---|---|---|
| $S_1$ | (0,0,0,0) | (.5,1,1,2) | (1,4,1,3) | (2,5,1,3) | (19,40,12,34) |
| $S_2$ | (.5,1,1,2) | (0,0,0,0) | (.2,2.8,.5,2.5) | (2,5,1,3) | (29,62,18,56) |
| $D_1$ | (1,4,1,3) | (.2,2.8,.5,2.5) | (0,0,0,0) | (.5,1,1,2) | (16,34,10,30) |
| $D_2$ | (2,5,1,3) | (2,5,1,3) | (.5,1,1,2) | (0,0,0,0) | (16,34,10,30) |
| **Demand** | (16,34,10,30) | (16,34,10,30) | (28.3,55.2,16.5,49.5) | (19.7,46.8,13.5,40.5) | |

**Table 5. Iterations I**

| | Var | $\widetilde{x_{11}}$ | $\widetilde{x_{12}}$ | $\widetilde{x_{13}}$ | $\widetilde{x_{14}}$ | $\widetilde{x_{21}}$ | $\widetilde{x_{22}}$ | $\widetilde{x_{23}}$ | $\widetilde{x_{24}}$ | $\widetilde{x_{31}}$ | $\widetilde{x_{32}}$ | $\widetilde{x_{33}}$ | $\widetilde{x_{34}}$ | $\widetilde{x_{41}}$ | $\widetilde{x_{42}}$ | $\widetilde{x_{43}}$ | $\widetilde{x_{44}}$ | RHS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $S_1$ | ? | (1,1,1,1) | (1,1,1,1) | (1,1,1,1) | (1,1,1,1) | | | | | | | | | | | | | (19,40,12,34) |
| $S_2$ | ? | | | | | (1,1,1,1) | (1,1,1,1) | (1,1,1,1) | (1,1,1,1) | | | | | | | | | (29,62,18,56) |
| $D_1$ | ? | | | | | | | | | (1,1,1,1) | (1,1,1,1) | (1,1,1,1) | (1,1,1,1) | | | | | (16,34,10,30) |
| $D_2$ | ? | | | | | | | | | | | | | (1,1,1,1) | (1,1,1,1) | (1,1,1,1) | (1,1,1,1) | (16,34,10,30) |
| $S_1$ | ? | (1,1,1,1) | | | | (1,1,1,1) | | | | (1,1,1,1) | | | | (1,1,1,1) | | | | (16,34,10,30) |
| $S_2$ | ? | | (1,1,1,1) | | | | (1,1,1,1) | | | | (1,1,1,1) | | | | (1,1,1,1) | | | (16,34,10,30) |
| $D_1$ | ? | | | (1,1,1,1) | | | | (1,1,1,1) | | | | (1,1,1,1) | | | | (1,1,1,1,) | | (28.3,55.2,16.5,49.5) |
| $D_2$ | ? | | | | (1,1,1,1) | | | | (1,1,1,1) | | | | (1,1,1,1) | | | | (1,1,1,1) | (19.7,46.8,13.5,40.5) |
| | | (0,0,0,0) | (0.5,1,1,2) | (1,4,1,3) | (2,5,1,3) | (0.5,1,1,2) | (0,0,0,0) | (0.2,2.8,0.5,2.5) | (2,5,1,3) | (1,4,1,3) | (0.2,2.8,0.5,2.5) | (0,0,0,0) | (0.5,1,1,2) | (2,5,1,3) | (2,5,1,3) | (0.5,1,1,2) | (0,0,0,0) | |
| $S_1$ | $\widetilde{x_{11}}$ | | (1,1,1,1) | (1,1,1,) | (1,1,1,1) | | | | | | | | | | | | | (19,40,12,34) |
| $S_2$ | ? | | (-1,-1,-1,-1) | | | (1,1,1,1) | (0,0,0,0) | (1,1,1,1) | (1,1,1,1) | | (-1,-1,-1,-1) | | | | (-1,-1,-1,-1) | | | (-5,46,48,66) |
| $D_1$ | ? | | | | | | | | | (1,1,1,1) | (1,1,1,1) | (1,1,1,1) | (1,1,1,1) | | | | | (16,34,10,30) |
| $D_2$ | ? | | | | | | | | | | | | | (1,1,1,1) | (1,1,1,1) | (1,1,1,1) | (1,1,1,1) | (16,34,10,30) |
| $S_2$ | $\widetilde{x_{22}}$ | | (1,1,1,1) | | | | (1,1,1,1) | | | | (1,1,1,1) | | | | (1,1,1,1) | | | (16,34,10,30) |
| $D_1$ | ? | | | (1,1,1,1) | | | | (1,1,1,1) | | | | (1,1,1,1) | | | | (1,1,1,1) | | (28.3,55.2,16.5,49.5) |
| $D_2$ | ? | | | | (1,1,1,1) | | | | (1,1,1,1) | | | | (1,1,1,1) | | | | (1,1,1,1) | (19.7,46.8,13.5,40.5) |
| | | | (0.5,1,1,2) | (1,4,1,3) | (2,5,1,3) | (0.5,1,1,2) | (0,0,0,0) | (0.2,2.8,0.5,2.5) | (2,5,1,3) | (1,4,1,3) | (0.2,2.8,0.5,2.5) | (0,0,0,0) | (0.5,1,1,2) | (2,5,1,3) | (2,5,1,3) | (0.5,1,1,2) | (0,0,0,0) | |
| $S_1$ | $\widetilde{x_{11}}$ | | (1,1,1,1) | (1,1,1,1) | (1,1,1,1) | | | | | | | | | | | | | (19,40,12,34) |
| $S_2$ | ? | | (-1,-1,-1,-1) | | | (1,1,1,1) | | (1,1,1,1) | (1,1,1,1) | | (-1,-1,-1,-1) | | | | (-1,-1,-1,-1) | | | (-5,46,48,66) |
| $D_1$ | $\widetilde{x_{33}}$ | | | | | | | | | (1,1,1,1) | (1,1,1,1) | | (1,1,1,1) | | | | | (16,34,10,30) |
| $D_2$ | $\widetilde{x_{44}}$ | | | | | | | | | | | | | (1,1,1,1) | (1,1,1,1) | (1,1,1,1) | (1,1,1,1) | (16,34,10,30) |
| $S_2$ | $\widetilde{x_{22}}$ | | (1,1,1,1) | | | | | | | | (1,1,1,1) | | | | (1,1,1,1) | | | (16,34,10,30) |
| $D_1$ | ? | | | (1,1,1,1) | | | | (1,1,1,1) | | (-1,-1,-1,-1) | (-1,-1,-1,-1) | | (-1,-1,-1,-1) | | | (1,1,1,1) | | (-5.7,39.2,46.5,59.5) |
| $D_2$ | ? | | | | (1,1,1,1) | | | | (1,1,1,1) | | | | (1,1,1,1) | (-1,-1,-1,-1) | (-1,-1,-1,-1) | (-1,-1,-1,-1) | (0,0,0,0) | (-14.3,30.8,43.5,50.5) |
| | | | (0.5,1,1,2) | (1,4,1,3) | (2,5,1,3) | (0.5,1,1,2) | | (0.2,2.8,0.5,2.5) | (2,5,1,3) | (1,4,1,3) | (0.2,2.8,0.5,2.5) | | (0.5,1,1,2) | (2,5,1,3) | (2,5,1,3) | (0.5,1,1,2) | (0,0,0,0) | |

**Table 6 Iterations II**

| | Var | $\widetilde{x_{11}}$ | $\widetilde{x_{12}}$ | $\widetilde{x_{13}}$ | $\widetilde{x_{14}}$ | $\widetilde{x_{21}}$ | $\widetilde{x_{22}}$ | $\widetilde{x_{23}}$ | $\widetilde{x_{24}}$ | $\widetilde{x_{31}}$ | $\widetilde{x_{32}}$ | $\widetilde{x_{33}}$ | $\widetilde{x_{34}}$ | $\widetilde{x_{41}}$ | $\widetilde{x_{42}}$ | $\widetilde{x_{43}}$ | $\widetilde{x_{44}}$ | RHS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| S1 | $\widetilde{x_{11}}$ | | (1,1,1,1) | (1,1,1,1) | (1,1,1,1) | | | | | | | | | | | | | (19,40,12,34) |
| S2 | $\widetilde{x_{21}}$ | | (-1,-1,-1,-1) | | | (1,1,1,1) | | (1,1,1,1) | (1,1,1,1) | | (-1,-1,-1,-1) | | | | (-1,-1,-1,-1) | | | (-5,46,48,66) |
| D1 | $\widetilde{x_{33}}$ | | | | (-1,-1,-1,-1) | | | | (-1,-1,-1,-1) | (1,1,1,1) | (1,1,1,1) | | | (1,1,1,1) | (1,1,1,1) | (1,1,1,1) | | (-14.8,48.3,60.5,73.5) |
| D2 | $\widetilde{x_{44}}$ | | | | | | | | | | | | | (1,1,1,1) | (1,1,1,1) | (1,1,1,1) | | (16,34,10,30) |
| S2 | $\widetilde{x_{22}}$ | | (1,1,1,1) | | | | | | | | (1,1,1,1) | | | | | | | (16,34,10,30) |
| D1 | ? | | | (1,1,1,1) | (1,1,1,1) | | | (1,1,1,1) | (1,1,1,1) | (-1,-1,-1,-1) | (-1,-1,-1,-1) | | | (-1,-1,-1,-1) | (-1,-1,-1,-1) | (0,0,0,0) | | (-20,70,90,110) |
| D2 | $\widetilde{x_{34}}$ | | | | (1,1,1,1) | | | | (1,1,1,1) | | | | | (-1,-1,-1,-1) | (-1,-1,-1,-1) | (-1,-1,-1,-1) | | (-14.3,30.8,43.5,50.5) |
| | | | (0.5,1,1,2) | (1,4,1,3) | (2,5,1,3) | (0.5,1,1,2) | | (0.2,2.8,0.5,2.5) | (2,5,1,3) | (1,4,1,3) | (0.2,2.8,0.5,2.5) | | | (2,5,1,3) | (2,5,1,3) | (0.5,1,1,2) | | |
| S1 | $\widetilde{x_{11}}$ | | (1,1,1,1) | (1,1,1,) | (1,1,1,1) | | | | | | | | | | | | | (19,40,12,34) |
| S2 | $\widetilde{x_{21}}$ | | (-1,-1,-1,-1) | (-1,-1,-1,-1) | (-1,-1,-1,-1) | | | | | (1,1,1,1) | | | | (1,1,1,1) | | (0,0,0,0) | | (-75,66,158,156) |
| D1 | $\widetilde{x_{33}}$ | | | | (-1,-1,-1,-1) | | | | (-1,-1,-1,-1) | (1,1,1,1) | (1,1,1,1) | | | (1,1,1,1) | (1,1,1,1) | (1,1,1,1) | | (-14.8,48.3,60.5,73.5) |
| D2 | $\widetilde{x_{44}}$ | | | | | | | | | | | | | (1,1,1,1) | (1,1,1,1) | (1,1,1,1) | | (16,34,10,30) |
| S2 | $\widetilde{x_{22}}$ | | (1,1,1,1) | | | | | | | | (1,1,1,1) | | | | (1,1,1,1) | | | (16,34,10,30) |
| D1 | $\widetilde{x_{23}}$ | | | (1,1,1,1) | (1,1,1,1) | | | | (1,1,1,1) | (-1,-1,-1,-1) | (-1,-1,-1,-1) | | | (-1,-1,-1,-1) | (-1,-1,-1,-1) | (0,0,0,0) | | (-20,70,90,110) |
| D2 | $\widetilde{x_{34}}$ | | | | (1,1,1,1) | | | | (1,1,1,1) | | | | | (-1,-1,-1,-1) | (-1,-1,-1,-1) | (-1,-1,-1,-1) | | (-14.3,30.8,43.5,50.5) |
| | | | (0.5,1,1,2) | (1,4,1,3) | (2,5,1,3) | | | | (2,5,1,3) | (1,4,1,3) | (0.2,2.8,0.5,2.5) | | | (2,5,1,3) | (2,5,1,3) | (0.5,1,1,2) | | |

**Table 7. Iterations III**

| | Var | $\widetilde{x_{11}}$ | $\widetilde{x_{12}}$ | $\widetilde{x_{13}}$ | $\widetilde{x_{14}}$ | $\widetilde{x_{21}}$ | $\widetilde{x_{22}}$ | $\widetilde{x_{23}}$ | $\widetilde{x_{24}}$ | $\widetilde{x_{31}}$ | $\widetilde{x_{32}}$ | $\widetilde{x_{33}}$ | $\widetilde{x_{34}}$ | $\widetilde{x_{41}}$ | $\widetilde{x_{42}}$ | $\widetilde{x_{43}}$ | $\widetilde{x_{4}}$ | RHS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| S1 | $\widetilde{x_{11}}$ | | (1,1,1,1) | (0,0,0,0) | (0,0,0,0) | | | | | (1,1,1,1) | | | | (1,1,1,1) | | | | (-56,106,170,190) |
| S2 | $\widetilde{x_{21}}$ | | (-1,-1,-1,-1) | (1,1,1,1) | (1,1,1,1) | | | | | (1,1,1,1) | | | | (-1,-1,-1,-1) | | (0,0,0,0) | | (-66,75,156,158) |
| D1 | $\widetilde{x_{33}}$ | | | | (-1,-1,-1,-1) | | | | (-1,-1,-1,-1) | (1,1,1,1) | (1,1,1,1) | | | (1,1,1,1) | (1,1,1,1) | (1,1,1,1) | | (-14.8,48.3,60.5,73.5) |
| D2 | $\widetilde{x_{44}}$ | | | | | | | | | | | | | (1,1,1,1) | (1,1,1,1) | (1,1,1,1) | | (16,34,10,30) |
| S2 | $\widetilde{x_{22}}$ | | (1,1,1,1) | (-1,-1,-1,-1) | (-1,-1,-1,-1) | | | | | (1,1,1,1) | (1,1,1,1) | | | (1,1,1,1) | (1,1,1,1) | (1,1,1,1) | | (-59,100,168,186) |

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **D** 1 | $\widetilde{x_{23}}$ | | | (1,1, 1,1) | (1,1, 1,1) | | | | (1,1, 1,1) | (-1, -1, -1, -1) | (-1, -1, -1,-1) | | | (-1, -1, -1, -1) | (-1, -1, -1, -1) | (0,0,0, 0) | | (-20, 70,90, 110) |
| **D** 2 | $\widetilde{x_{34}}$ | | | | (1,1,1, 1) | | | | (1,1,1, 1) | | | | | (-1, -1, -1, -1) | (-1, -1, -1, -1) | (-1, -1, -1, -1) | | (-14.3, 30.8, 43.5, 50.5) |
| | | | (0.5,1, 1, 2) | (1,4,1, 3) | (2,5,1, 3) | | | | (2,5,1, 3) | (1,4,1, 3) | (0.2,2.8,0.5, 2.5) | | | (2,5,1, 3) | (2,5,1, 3) | (0.5,1, 1, 2) | | |

**Table 8. The optimum Fuzzy Transshipment Table**

| | $S_1$ | $S_2$ | $D_1$ | $D_2$ | Availability |
|---|---|---|---|---|---|
| $S_1$ | (0,0,0,0) [-56,106,170,190] | (0.5,1,1,2) [-66,75,156,158] | (1,4,1,3) | (2,5,1,3) | (19,40,12,34) |
| $S_2$ | (0.5,1,1,2) | (0,0,0,0) [-59,100,168,186] | (0.2,2.8,0.5,2.5) [-20,70,90,110] | (2,5,1,3) | (29,62,18,56) |
| $D_1$ | (1,4,1,3) | (0.2,2.8,0.5,2.5) | (0,0,0,0) [-14.8,48.3,60.5,73.5] | (0.5,1,1,2) [-14.3,30.8,43.5,52.5] | (16,34,10,30) |
| $D_2$ | (2,5,1,3) | (2,5,1,3) | (0.5,1,1,2) | (0,0,0,0) [16,34,10,30] | (16,34,10,30) |
| **Demand** | (16,34,10,30) | (16,34,10,30) | (28.3,55.2,16.5,49.5) | (19.7,46.8,13.5,40.5) | |

It is shown in **Table 8** that the optimum fuzzy transportation cost is [-111.6, 140.8, 451.5, 617.25]

Defuzzified fuzzy transportation cost is 56.0375.

## 4. CONCLUSION

The proposed algorithm yields an optimum solution of the fuzzy transshipment problem. The main advantage of the proposed method is solved manually, without using artificial variables which is required by the simplex method also it reduces number of iterations.

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