Structural Properties of Torus-Butterfly Interconnection Network

Latifah
STMIK Jakarta STI&K
Jl.BRI Radio Dalam No. 17
Jakarta13470
Indonesia

Ernastuti
Gunadarma University
Jl. Margonda Raya no. 100
Depok,
Indonesia

Djati Kerami
University of Indonesia
Depok
Indonesia

ABSTRACT
This paper introduced new interconnection network named as Torus-Butterfly. The network is generated by a product of network from Torus and Enhanced Butterfly interconnection networks which is suitable for parallel computers. We have analyzed and proved that the structural properties such as network diameter and node degree of the Torus-Butterfly interconnection networks is more scalable than other interconnection networks. In addition to them, the network cost is presented. The result is also more scalable.

General Terms
Design, performance

Keywords:
Torus Network, Enhanced Butterfly Network, Cartesian product network. Cayley Graph.

1. INTRODUCTION
Current computer interconnection networks have been widely applied in various areas, such as parallel computing system, multiprocessor systems, and workstation networks [1]. According to Zhang [2] model (topology) of interconnection networks is an important part for parallel processing or distributed system. Zhang [3] states that a good model of the interconnection network must have the symmetry properties, measured (scalable), has a small diameter, and also has a constant and a limited degree [4, 5]. In practice it is more desirable that the network model has a high connectivity and a smaller diameter. Connectedness is widely used to measure the fault-tolerance capacity of the network, whereas the diameter showed the efficiency of routing (sending data) [6]. To evaluate an interconnection network model, the researchers compared the interconnection network models through an analysis of various parameters, such as, for structural properties: degree and diameter.

In recent years, there is research on a class graph as a model of interconnection networks called the Calley graph [2]. This is due to Cayley graphs have many desirable properties as good as the network model that has been mentioned, which has the properties of symmetry, small diameter, connectedness and high fault tolerance [2]. Cayley graph also has a regular character, ie at each node has a constant degree [7].
Some models of interconnection networks that are Cayley graph is a hypercube, butterfly, mesh and torus [3]. Hypercube or n-cube, is a network model that has characteristics of small diameter and potentially in the interconnection network model, however hypercube have limitations on the degree that are not constant, but evolve according to its size. In order to have constant degree, it has been designed a network model called Butterfly. Some research developed Wrap around Butterfly [8]. It has been also developed a network model called Enhanced Butterfly [4]. Enhanced Butterfly network model has the properties of small diameter, constant degree and also of symmetry. Other models of interconnection networks that are Cayley graph is called Torus, which is widely used in parallel computing system [9]. In recent years, demand for high speed and high throughput computing machines has led to the development of a new interconnection network model with a lot number of processors [10, 11]. For some models it is a modified version of existing models or to combine two proposed models to benefit from the properties owned by both models [10]. There are many methods for combining two existing models, one of them is the multiplication (product) such as the Cartesian product [12, 3]. Research has been done using a method of product of two network models such as star-cube [13], hyper-butterfly [14], the Torus embedded Hypercube [15] and Scalable twisted Hypercube [16], but the results of these studies indicate the degree of hypercube is not constant, and consequently a large enough network cost. This situation is precisely the wish to avoid. In this paper, we introduced another product network of Torus and Enhanced Butterfly topology, named as Torus-Butterfly interconnection network. The advantages of Torus and Enhanced Butterfly are used for this product network.

The Idea of this research is to design the interconnection network model called Torus-Butterfly, which is the Cartesian product of the interconnection network model Torus and Enhanced Butterfly. Then analyze the properties of models of Torus-Butterfly interconnection network through structural parameter such as degree and diameter and evaluate the network cost, symetry and regularity.

2. THEORETICAL REVIEW
It is known that the structure of the model of a network can be described as a connected graph G = (V, E), with V the set of vertices is the set of processors and E is the set of edge or link in the network [8]. An edge is an ordered pair of $x, y = \{x, y\}$ of distinct vertices in G. The set of vertices V in a graph G is denoted as V_G and the set of edge E in a graph G is denoted by E_G.

Definition : A graph G = (V, E) is called connected if for any two nodes of a graph G there is always the path that connects the second node [17].

Definition : The degree of a node $x \in V_G$, denoted by deg (x), is a connected arc from x to nodes $y \in V_G$, $y \neq x$ [18].

Definition: The diameter of the connected graph G (V_G, E_G) is the maximum distance of all pairs of vertices [19].

In interconnection network model, diameter can be interpreted as the maximum number of links that should be traced when sending a message to any processor along the shortest path [20]. The smaller the diameter the shorter the time the network to send messages from one processor to another processor [21]. Diameter is needed to speed the time required at the time of one processor sends a message to another processor, the smaller the diameter the faster time of delivery of the message. The smaller diameter will also be beneficial to the reduction of network cost and will improve the performance of the processor.

As already mentioned in the introduction, one of the models that are widely used interconnection networks are Cayley graphs, this is caused by the Cayley graph has the properties finite, connect, undirected and symmetry [22]. Because of the interconnection network model is considered as a graph then the term interconnection network model and a graph can be replace each other. Here is the definition of the Cayley graph.

Definition : Suppose H is a group and $S \subset H$ forming a set of H such that $S = S^{-1}$. A Cayley graph of H against S is undirected graph Cay (H, S) where the set of vertices is H and the edge connecting g to gs for every selection $g \in H$ and $s \in S$. A Cayley graph Cay (H, S) is a regular graph $|S|$ of order $|H|$ [23].

Now we give definition of Butterfly network.

Definition: Suppose $d \in N$ (N = set of positive integers). Butterfly interconnection network model of dimension d is denoted by B(d) is a graph with vertex set $V = [d+1] \times [2]^d$ and the set of edge $E = E_1 \cup E_2$ with $E_1 = \{\{(i, \alpha), (i+1, \alpha)\} / i \in [d], \alpha \in [2]^d\}$ and $E_2 = \{(i, \alpha), (i+1, \beta)\} / i \in [d], \alpha, \beta \in [2]^d\}$, α, β differ only at position i}. The set of vertices $\{(i, \alpha) / \alpha \in [2]^d$ is said to form the i-th level of the Butterfly [24].

Butterfly interconnection network model of dimension n has n levels and $N = 2^n$ input and output. Input / output is set in the 2^n columns are labeled from 0 to 2^n-1 (in binary). Each level is numbered 0, 1, 2, ..., n from top to bottom [22]. The Butterfly interconnection network model only node to node in a neighboring row. Edge between the vertices in the same column is called a straight edge and the edge between nodes in different columns are called cross sections [8]. In the graph B (d), when level d is replaced with level 0, then we said a Wrap around Butterfly dimension d or WB (d) [25]. At Wrap around Butterfly (WB) dimension $n \geq 3$ when we added a particular edge on the graph is called Enhanced Butterfly [4].

Definition: Torus (m, l) = T (m, l) is a graph that contains (m, l) mesh with wrap around sides in rows and columns [26].

Definition : Given two graphs $G = (V_1, E_1)$, and $H = (V_2, E_2)$, Cartesian product operation defined G and H is denoted by $G \times H$ is the graph $T(V, E)$, with V and E as follows

1)V={(a,b)/a∈V_1, y∈V_2}.

2)For any x =(a,b) and y = (c,d) in V, (x, y) is and edge in E if and only if (a, c) is an edge in E_1 and b = d or (b, d) is an edge in E_2 and a = c [27].

Proposition 2.1: Suppose that two graph $G = (V_1, E_1)$ and $H = (V_2, E_2)$. The Cartesian product $G \times H$ is a connected graph which has a size $V_1 V_2$, degrees = degrees G + degree H and diameter = diameter G + diameter H [13, 12].

Proposition 2.2: Suppose that Cay (S, G) and Cay (S ', G') are two Cayley graph, then the Cartesian product Cay (S, G) × Cay (S ', G') = H is also a Cayley graph [3].

Proposition 2.3: The degree of interconnection network model Enhanced Butterfly dimension n denoted as EB(n), $n \geq 3$, is 5 and the diameter is n [4].

Proposition 2.4: Torus Interconnection Model denoted as T (m, l) has degrees 4 and the diameter = $\max\{\lfloor m/2 \rfloor, \lfloor l/2 \rfloor\}$ [26].

From the above definitions and propositions we have following definition for the new interconnection network named Torus-Butterfly.

Definition: If $G = (V_1, E_1)$ is the Torus interconnection network model of size ml and $H = (V_2, E_2)$ is the Enhanced Butterfly interconnection network model dimension n, then the Torus-Butterfly interconnection network model, denoted as TB (m, l, n), is the Cartesian product of Torus and Enhanced Butterfly, with m and l is the size of Torus interconnection network model and n is the dimension of the Enhanced Butterfly interconnection network model. This is true for $n \geq 3$, $m \geq 2$ and $l \geq 2$.

3. RESULTS AND DISCUSSION

3.1 Node degree

We have the following Lemma:

Lemma 1: The degree of each node in the Torus -Butterfly interconnection network model is 9.

Proof: By proposition 2.1 every node in the Torus-Butterfly interconnection network model = TB (m, l, n) has degree 5 + 4 = 9, thus the degree = 9.

3.2 Diameter and Network Cost

We have the following Lemma for Diameter of Torus-Butterfly Interconnections model:

Lemma 2: The diameter of the interconnection network Torus-Butterfly TB (m, l, n) is $= \max\{\lfloor m/2 \rfloor, \lfloor l/2 \rfloor\} + n$.

Proof: by proposition 2.1, then the diameter Torus-Butterfly is diamater Torus + Diameter Enhanced Butterfly = $\max\{\lfloor m/2 \rfloor, \lfloor l/2 \rfloor\} + n$.

From the above degree and diameter of Torus-Butterfly interconnections network formula, we have the following network cost:

Network Cost of Torus-Butterfly interconnection network model is $9(\max\{\lfloor m/2 \rfloor, \lfloor l/2 \rfloor\} + n)$.

3.3 Symetri and Regularity.

Torus interconnection network is a Cayley graph and Enhanced Butterfly is also a Cayley graph, hence from proposition 2. 2 the new Torus-Butterfly interconnection network is Cayley graph. It follows that this new interconnection network is symetri and regular.

Table 1 and figure 1 gives the comparison of degree for various same processors of Hyper-Butterfly and Torus embedded Hypercube networks along with Torus-Butterfly network.

Table 1. Degree Comparison of three models of interconnections network

Network Type No. of processor	HB (*k, n*)	TH(*16,16,k*)	TB(*m,l,n*)
512	7	5	9
1024	8	6	9
2048	9	7	9
4096	10	8	9
8192	11	9	9
16384	12 [14]	10[5]	9
32768	13	11	9

Remark: HB = Hyper-Buttterfly, TH =Torus Torus Embedded and TB= Torus Butterfly.

In table 1 it is seen that Torus-Butterfly Interconnection model has a constant degree,so it is regular whereas the Hyper-Butterfly and Torus-embedded-Hypercube has linier degree depends on the size of processor.

Table 2 and figure 2 gives the comparison of diameter for various same processors of Hyper-Butterfly and Torus embedded Hypercube networks along with Torus-Butterfly network.

Table 3 and figure 3 gives the comparison of network costs for various same processors of Hyper-Butterfly and Torus embedded Hypercube networks along with Torus-Butterfly network.

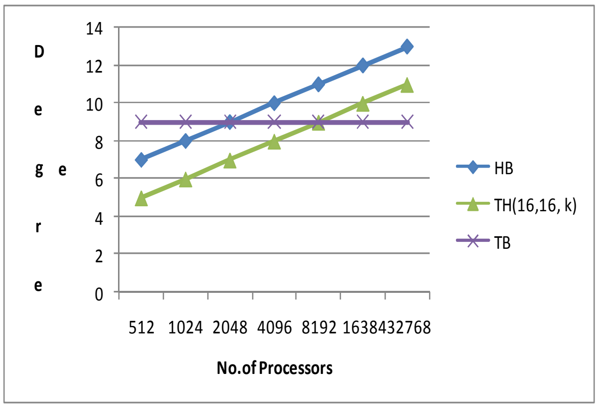

Fig 1: Comparison of the degree of 3 models interconnection networks.

Table 2. Diameter comparison of three models of interconnection network

Network Type No. of processor	HB (*k, n*)	TH(*16,16,k*)	TB(*m,l,n*)
512	7	17	5
1024	8	18	6
2048	9	19	8
4096	10	20	8
8192	11	21	12
16384	12[14]	22[5]	12
32768	15	23	20

In table 2 it is seen that Torus-Butterfly Interconnection model has lower diameter than Torus-embedded-Hypercube and has lower diameter than Hyper-Butterfly for number of processor 512 till 4096, except for the number of processor 16384 and 32768.

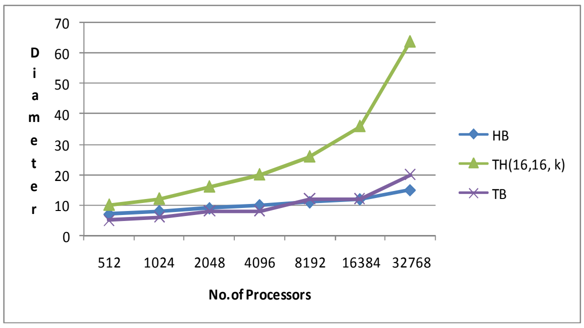

fig 2: Comparison of the diameter of 3 models interconnection networks

4. CONCLUSION

From the above evaluation model of Torus-Butterfly interconnection network and the three comparison table of degree, diameter and visible network cost, Torus Butterfly interconnection network has better properties than the model Torus embedded Hypercube interconnection networks, and Hyper Butterfly. This new interconnection network also has symetry and regular properties. Hence this Torus-Butterfly Interconnection network can be used as an alternative model for interconnection network.

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Table 3.Network cost comparison of three models of interconnection network

Network Type No. of processor	HB (*k, n*)	TH(*16,16,k*)	TB(*m,l,n*)
512	49	136	45
1024	64	144	54
2048	81	152	72
4096	100	160	72
8192	121	168	108
16384	144[14]	176[5]	108
32768	195	184	180

In table 3 it is seen that Torus-Butterfly Interconnection model has lower network cost than Torus-embedded-Hypercube and Hyper-Butterfly for all the same number of processor.

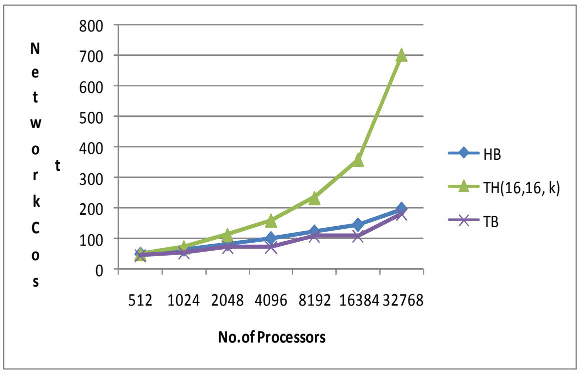

Fig 3: Comparison of the network cost of 3 models interconnection networks

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