Fuzzy Petri Nets and Fuzzy Cognitive Maps

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ABSTRACT
Petri nets have been used to model many systems like computer systems, knowledge based system, manufacturing systems etc. But these Petri nets do not have sufficient power to represent and handle approximate and uncertain information. Hence Fuzzy Petri Net (FPN) are defined to model systems such as robot systems having lower level of operations. The study of uncertainty in the events and change of time can be represented in the FPN. In an another approach, Fuzzy Cognitive maps (FCM) links casual events values, goals and trends in a fuzzy feed back dynamical system. A Fuzzy Cognitive Maps lists the fuzzy rule or casual flow chart that relate events. This paper relate FCM and FPN. This paper show with an example that FCM concept easily carry through to FPN.

Keywords
Fuzzy Petri Nets, Fuzzy Cognitive Maps

1. INTRODUCTION
In classical logic[10], the conditions available for determination are either true or false. This imposes limitations on both decision making and human logic capabilities. Fuzzy logic uses the capabilities intervening between 0 and 1 and it relies on the studies of fuzzy set. Fuzzy sets deal with neighborhood of numbers. In the process of decision, the acceptability of the condition is active, when a certain interval is active. Fuzzy logic helps in some operations and elements which are basic in giving decision about the system. The functioning membership determines the resulting behavior of the system. These can be in triangle, trapezoid or any other styles such as hexagonal etc.,

Petri Net cannot deal with fuzzy events directly. These Petri Nets has to be modified according to system requirements. So Fuzzy Petri Net is helpful in the composition of properties by combining Petri Nets and fuzzy logic. FPN's as the subset of Petri Nets are used for the investigation of fuzzy system behavior. Petri Net's empower the performance of FPN in concurrent, synchronous, asynchronous, parallel, non deterministic and distributed processing. Thus, FPN's is an effective tool for the representation of uncertain knowledge about a system state.

Fuzzy Cognitive map[2] is a cognitive map within which the relations between the elements like concepts, events, project resources can be used to compute the "strength of impact" of these elements. The theory behind that computation is fuzzy logic. Fuzzy cognitive maps are signed fuzzy digraphs.

2. DIRECTED GRAPH
A directed graph (or digraph) is a triple $D = (V, E, \varphi)$ where V and E are finite sets and φ is a function with domain E and co-domain $V \times x\ V$. E is the set of edges of the digraph D and call V the set of vertices of D.

3. FUZZY COGNITIVE MAPS(FCM):
An FCM is a directed graph with concepts like policies, events etc. as nodes and causalities as edges. It represents causal relationship between concepts.

3.1 Formal Definition [10]:
Let R be the set of real numbers, N denote the set of natural numbers, K=[-1,1] and L = [0,1].

A fuzzy cognitive map FCM [11] is a 4-tuple (N,E,C,f) where

(1) $N = \{N_1, N_2, \ldots\ldots N_n\}$ is the set of n concepts forming of nodes of a graph.
(2) E: $(N_i, N_j) \to\to e_{ij}$ is a function of $N \times x\ N$ to K associating e_{ij} denoting a weight of directed edge from N_i to N_j if $i \neq\neq j$ and e_{ij} equals zero if i=j. Thus E: $(N \times N) = (e_{ij}) \in K^{n\times xn}$ is a connection matrix.
(3) C: $N_i \to\to C_i$ is a function that at each concept N_i associates the sequence of its activation degrees such as for $t \in\in\in N$, $C_i(t) \in\in L$ given its activation degree of the moment t. $C(0) \in\in L^n$ indicates the initial vector and specifies initial values of all concept nodes and $C(t) \in\in L^n$ is a state vector at certain iteration t.
(4) F: $R \to\to L$ is a transformation function, which includes recurring relationship on $t \geq\geq 0$ between C(t +1) and C(t).

Example 1
In Tamil Nadu (a southern state in India) in the last decade several new Art's and Science colleges have been approved and started. The resultant increase in the production of graduates in these years is disproportionate with the need of graduates. This has resulted in thousands of unemployed and under employed graduates. Using an expert's opinion this paper study the effect of such unemployed people on the society. An expert spells out the five major concepts relating to the unemployed graduates as

G1 – Frustration
G2 – Unemployment

G3 – Increase of educated criminals
G4 – Under employment
G5 – Taking up drugs etc.

The directed graph where G1, …, G5 are taken as the nodes and causalities as edges as given by an expert is given in the following Figure 1:

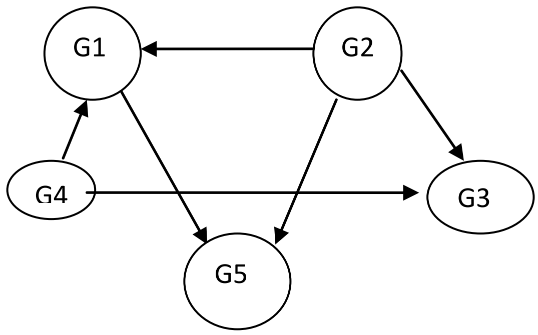


Fig. 1

Generally, due to unemployment lot of unwanted actions are being dealt with by unemployment graduates. The above illustration shows how FCM is prescribed by directed graph. In this FCM, if increase (or decrease) in are concept leads to increase or decrease another, the value given is 1, otherwise it is -1. If there is no relation, then give the value 0.

4. IMPLEMENTATION OF FCM IN A PROCEESS CONTROL PROBLEM:

Fuzzy Cognitive Map (FCM) Theory uses a symbolic representation for the description and modeling of the system. It utilizes concepts to illustrate different aspects in the behavior of the system and these concepts interact each other showing dynamics of the system. In the related things FCMs have been used model and support plan control [2], to represent failure models the supervisor of control systems[3].

This can be achieved by considering that the firing of a transition is a continuous process: the tokens progressively disappear from the input places and appear in the output places. It is the basic idea of fuzzy firings in a safe Petri Nets[7].

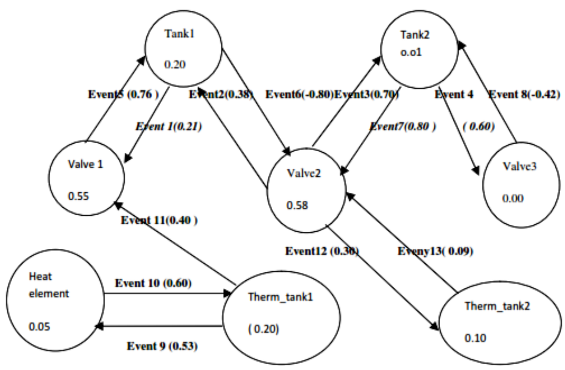
4.1 Local control

Very often, Programmable Logic Controllers are supplemented by simple regulation functions. However these functions cannot be called in the body of a sequential control program and no formal model authorizes the description of a continuous control leading a system from one stat to another on within a sequence description.

A FCM will be constructed which will model and control the whole system. In order to determine the concepts of the FCM that describe the system, the variables of the system must be taken into account, such as the height of the liquid in each tank or the temperature. Then concepts are assigned for the system's elements that affect the variables such as the state of the valves.

Each event has a weight which ranges between [1,1] and in this case it was determined arbitrarily and then has changed during the training period of the FCM. Ach concept has a value which ranges in the interval [0,1] and it is obtained after thresholding the real value of the concept. It is apparent that an interface is needed which will transform the real measures of the system to their representative values in the FCM and vice versa. It should be mentioned that the transformation from the real values of the physical measurement to the values of the concept needs investigation and must take into consideration the actual mechanisms depicted in the FCM.

Using these concepts the constructed FCM as follows



4.2 Supervisory control:

Various applications can be imagined at this level. The first one concerns diagnosis. Typically, diagnosis involves ill-known information. The propositions which have to be handled are hypotheses .Fuzzy sets are a convenient tool to establish a tradeoff between these two situations. In continuous industrial processes threshold values are fuzzified , in discrete event systems, events have also to be "fuzzified" which leads to another application of Fuzzy Petri Nets.[4]

Example 2:

The given example of a system [8] in which two identical tanks are considered. This description is being used later for the convenience of FPN's. Each tank has an inlet valve and a outlet valve. The outlet valve of the first tank is the inlet valve of the second. The control objective of the system is to keep the amount of liquid, in both tanks, between some limits, an upper H_{max} and a low limit H_{min} , and furthermore, the temperature of the liquid in both tanks must be kept between maximum value T_{max} and a minimum value T_{min} . The target is keeping these variables in the middle of their range of values.

$$H^1_{min} \leq H^1 \leq H^1_{max}$$

$$H^2_{min} \leq H^2 \leq H^2_{max}$$

$$T^1_{min} \leq T^1 \leq T^1_{max}$$

$$T^2_{min} \leq T^2 \leq T^2_{max} \quad \text{--------------(1)}$$

The temperature of the liquid in tank1 is increased through a heating element. A thermostat continuously senses the temperature of the liquid in tank 1 and turns the heater on and off. The temperature of the liquid in the tank 2 is measured through a thermometer; when the temperature of the liquid decreases, this causes the valve 2 to open, so hot liquid comes into tank 2.

Fig. 2 The initial FCM, with the first values for the concept

Figure 1 shows the FCM that is used to describe and control the system, with the initial value of each concept and the interconnections between concepts. The values of concepts correspond to the real measurements of the physical magnitude. The values of the events have been determined after observation of the changes in the real experimental system and then training the FCM according to the Differential Hebbian learning method[1].

The model of the above problem using Fuzzy Petri Nets in order to automata this system since Petri Nets tools are available.

5. FUZZY PETRI NET(FPN)

A fuzzy petri net model[FPN] [4,5] is a petri net having places and transitions and with each transition a rule is associated with a certain factor value 0 and 1. Input functions and Output functions are defined similar to Petri nets. But in a addition, associate functions, $\mu, \alpha\ and\ \beta$ are also defined giving output values as [0,1]. The formal definition is given below, Ref [9].

5.1 Formal Definition:

As with [6 and its refs] FPN can be defined as an 8 – tuple . FPN = { P,T,D,I,O, μ , αα,β} where

P = {P_1,P_2,…..P_n} is a finite set of places, T={t_1,t_2,…..t_n] is a finite set of transitions,

D = { d_1,d_2,…d_n} is a finite set of propositions. P∩T∩ $D = \emptyset$
$|P| = |D|$
I:P ×T→→ $\{0,1\}$ is the input function, a mapping from places to transitions.

O: T××P→→{0,1} is the output function, a mapping from transition to places.

μ: T→→ [0.1] is an association function, a mapping from transitions to [0,1] i.e., certain factor

α α:P→→ [0,1] is an association function, a mapping from places to [0,1] i.e., the truth degree

β: P→→D, is an association function, a mapping from places to propositions.

Example 3:

There was an antecedent condition that " temperature is hot" and its consequent was "humidity is low".

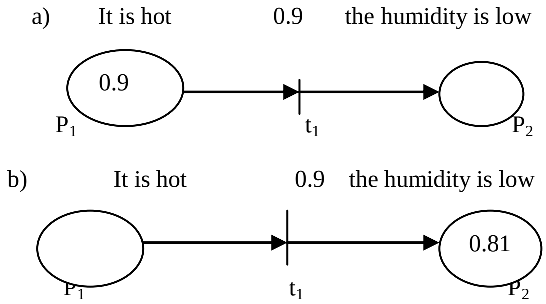


Fig. 3 Firing of Fuzzy Petri nets

a) Before firing b) After firing

Where P = { P_1 , P_2}, T= { t_1}, D={ It is hot , the humidity is low}, T(t_1)={P_1},O(t_1)={P_2}, f(t_1)=0.9, α(P_1) = 0.9,

α (P_2) = 0.81,. β (P_1)= it is hot, β(P_2) = the humidity is low .

IF….Then rule in that example was expressed as If β_1 THEN β_2 (CF), where β_1 was antecedent qualification, β_2 was consequent result, and CF was the "Certainty Factor" , a larger CF value indicated a higher certainty of the rule. In the above example "temperature is hot" with the membership degree of 0.9 and CF for the truthiness of the result is 0.9, so the membership degree of the consequent that "humidity is low" was 0.81.

6. APPLICATION OF FUZZY LOGICAL PETRI NETS FROM THE ABOVE CONCEPT OF FCM

Table 1: Rules Set and Corresponding transitions

No.	Rules	Transitions
1	IF P_1 is low THEN connect to P_3	t_1
2	IF P_1 is high THEN relates to P_4	t_2
3	IF P_2 is low THEN connect toP_4	t_3
4	IF P_2 is high THEN connect t P_5	t_4
5	IF any change in P_3 THEN P_1 also changes	t_5
6	IF P_4 changes THEN P_1 may decrease or may not	t_6
7	IF P_4 changes THEN P_2 may increase or not	$\mathbf{t_7}$
8	IF P_5 changes THEN P_2 may decrease r not	$\mathbf{t_8}$
9	IF P_6 is low THEN P_8	$\mathbf{t_9}$
10	IF P_8 THEN P_6 increases	$\mathbf{t_{10}}$
11	IF P_6 increases THEN P_3	$\mathbf{t_{11}}$
12	IF P_7 decreases THEN P_4	$\mathbf{t_{12}}$
13	IF P_7 increases THEN P_2 increases	$\mathbf{t_{13}}$

Table 2 Propositions set and with correspondence places

No.	Propositions	Places
1.	The amount of liquid in tank1 which is dependent on valve 1 and valve2	P_1
2.	The amount of liquid in tank2 which is related to valve 1 and valve2	P_2
3.	The state of the valve1 is open, closed or partially open	P_3
4.	The state of the valve2	P_4
5.	The state of the valve3	P_5
6.	The temperature of the liquid in tank1	P_6
7.	The temperature of the liquid in tank2	P_7
8.	Describes the operation of the heating element which increases the temperature of the liquid in tank1	P_8

7. ALGORITHM FOR THE ACTION PERFORMED:

Step 1: IF the height of the liquid in tank1 is low **THEN** P_3 is open
Step 2: IF the height of the liquid in tank1 is High **THEN** P_4 is open
Step 3 : IF the height of the liquid in tank2 is low **THEN** P_4 is open
Step 4: IF the height of the liquid in tank2 is high **THEN** P_5 is open
Step 5 : IF any change in P_3 **Then** P_1 also changes
Step 6: IF P_1 increases **THEN** Go to P_3.
Step 7: **IF** P_2 decreases **THEN** Go to P_4
Step 8: IF P_2 increases **THEN** Go to P_5
Step 9: IF P_6 is low **THEN** P_8 is open
Step 10: IF P_8 increases **THEN** P_6 also increases
Step 11: IF P_6 is high **THEN** P_3 is open
Step 12: IF P_7 is low **THEN** P_4 is open
Step 13: IF P_4 is open **THEN** P_7 is increased.

By using this algorithm, the value of each concept is defined by the result of taking all the causual event weights pointing into this concept and multiplying each weight by C_i the value of the concept that causes the event, according to equation :

$$A_i = f(\sum_{\substack{j=1 \\ j\neq 1}}^{n} A_{j\,W_{ji}}) + A_i^{old}$$

where A_i is the activation level of concept C_i at time t+1, A_j is the activation level of concept C_j at time t, A_i^{old} is the activation level of concept C_i at time t, and W_{ji} is theC_i at time t +1, A_j is the activation level of concept C_j at time t, A_i^{old} is the activation level of concept C_i at time t, and W_{ji} is the weight of the interconnection between C_j and C_i, and f is a threshold function. Then the sigmoid function is applied on the result of calculation and it is transformed in the interval between 0.00 and 1.00. As running step or running cycle of the FPN is defined the time unit during which the values of the concepts are calculated and change.

Table 3 The values of FPN concepts for fixed event values

	Tank1	Tank2	Valve1	Valve2	Valve3	Heat	Therm_tank1	Therm_tank2
1	0.20	0.01	0.55	0.58	0.00	0.05	0.20	0.10
2	0.49	0.61	0.53	0.53	0.50	0.53	0.51	0.51
3	0.50	0.55	0.58	0.68	0.59	0.57	0.58	0.51
4	0.47	0.57	0.58	0.67	0.58	0.58	0.58	0.52
5	0.48	0.57	0.58	0.68	0.59	0.58	0.59	0.52
6	0.48	0.57	0.58	0.68	0.59	0.58	0.59	0.52

In this problem a model for a process control problem has been proposed, this model could be enhanced if a two-level structure model is considered. In the lower level of the structure will lie the FPN that it has just constructed and it will reflect the model of the process during normal operation conditions.

The FPN on the upper level will consist of concepts that may represent the irregular operation of some elements of the system, failure mode variables, failure effects variables, failure cause variables, severity of the effect or design variables. In this example, the FPN will describe the failure state of the valves, possible malfunction in the heating element, , leaks in the tanks or other alarm schemes. Moreover, this FPN will include concepts for determination of a specific operation of the system. As an example1, it could need different amounts of liquid in the output at different times, according to the requisite density of the liquid.

Hence using the above proposition the constructed FPN is as follows

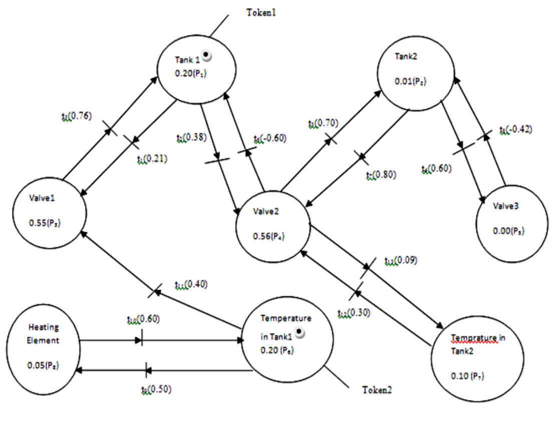


Fig. 4 The values of FPN for fixed event

The following firing of transitions sequences
t_1 t_5 t_2 t_3 t_4 t_8 t_7 t_6 – flow of the liquid
t_9 t_{10} t_{11} t_5 t_2 t_{13} t_{12} t_6 t_4 t_8 t_7 t_6 t_1 – flow of temperature
prove that the flow is from lower level to upper level. It can be seen from the above figure, two or more concepts of the FPN in the lower level pass through the interface and influence on concept in the FPN on the upper level, an analogous interface exists for the inverse transmission of information.

8 .CONCLUSION

Fuzzy Petri net are defined to model Fuzzy cognitive maps. In the example considered it is show that the behavior of complex system utilizes existing experience in the operation of the system. It is extremely difficult to describe the entire system by a precise mathematical model. However, Fuzzy Petri nets is more attractive and useful to represent it in a graphical way showing the relation between the state concepts. This representation can be automated using Petri net tools. Hence an effective way of intelligent and autonomous control system may be considered.

9. ACKNOWLEGEMENT

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