A Fuzzy Lattice Approach to Automated Multimodal Image Fusion

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ABSTRACT
This paper presents a new technique for multimodal image fusion. Unlike most previous works on image fusion, this paper explores the use of fuzzy lattice theory in the fusion process. Our proposed image fusion algorithm involving infrared and visual images based on fuzzy lattice theory show better experimental result than the related research work. Finally the paper discusses several key topics for future research, including the applications of this technique to computer vision and other related fields.

Keywords
Multimodal fusion, infrared image, normal visual image, fuzzy, lattice, image registration, lattice.

1. INTRODUCTION
We present a technique for enhancing the underexposed visible image by fusing it with simultaneously captured image from no visible sensors, such as IR. Although IR sensors can capture image in low-light for night-vision applications, they lack the color and the relative luminance of visible spectrum sensors. On other hand RGB sensors capture color and correct relative luminance's, but are underexposed, noisy, and lack fine features due to the short exposure times necessary for image. Basically Multisensory Image Fusion (MIF) is the technique through which the images for the same target obtained by two or more sensors are processed and combined to create a single composite image which cannot be achieved with a single image. Our proposed image fusion technique has developed base on fuzzy and lattice [16, 17, and 19] concept. In technique we have used bilateral filter [18] for denoisification of sources image. After remove noise from images we continuing several process such image registration [9], fuzzification then we used lattice theory on those registered fuzzy images for fusion. In the following sections we will briefly describe the fuzzy and lattice concepts before presenting a fusion scheme for image. Experiments are carried out on a set of benchmark images. The fusion result is compared with some other fusion algorithms through some performance evaluation measures for fusion effect. This work organized as follow. The next section covers the theoretical basis of lattice. Then, a new image fusion approach for infrared and multispectral images based on the fuzzy-lattice concept. This is followed by a discussion of the image fusing experiments. Next, the experimental results are analyzed. Furthermore, the proposed method is compared with the previous methods which developed for image fusion, such as the discrete wavelet method (DWT) and the weighted average (WA) method and finally last section puts forward the conclusion drawn by this paper.

2. RELATED WORKS
Recently, a number of studies have shown that infrared (IR) imagery offers a promising alternative to visible imagery as the data relatively insensitive to illumination changes [3]. Multi-sensor data fusion seeks to combine information from multiple sensors and sources to achieve inferences that are not feasible from a single sensor or source [5]. The fusion of information from sensors with different physical characteristics enhances the understanding of our surroundings and provides the basis for planning, decision-making, and control of autonomous and intelligent machines [6]. In the past decades it has been applied to different fields such as pattern recognition, visual enhancement, object detection and area surveillance [10]. Multi-sensor data fusion is a process of combining images, obtained by sensors of different wavelengths to form a composite image. The composite image is formed to improve image content and to make it easier for the user to detect, recognize, and identify targets and increase situational awareness [11]. In 1997, Hall and Llinas gave a general introduction to multi-sensor data fusion [4]. Another in-depth review paper on multiple sensors data fusion techniques was published in 1998 [1, 2, and 12]. Further scientific papers on image fusion have been published with an emphasis on improving fusion quality and finding more application areas. Quite a few survey papers have been published recently, providing overviews of the history, developments, and the current state of the art of image fusion in the image-based application fields [13].

3. PROPOSED METHODS
3.1 Image fuzzification
The definitions of membership function depend on the exact requirement of particular application and on corresponding expert knowledge [14]. Image fuzzification is the first processing step in fuzzy-image processing theory. In our technique, we have used histogram-based gray level fuzzification technique. The shape of membership function of infra-red image and normal visual image defined as follow

$$\mu_{IR}(i, j, k) = 1 - \left(\left(G_{max,L} - H_{IR}(i, j, k)\right) / G_{max,L}\right) \ldots equ(1)$$

Where μ_{IR} is membership functional value of infra-red image and $Gmax_{,L}$ is maximum intensity level of given image. Similarly we also define membership function for normal visual image as

$$\mu_{VIS}(i, j, k) = 1 - \left(\left(G_{max,L} - H_{VIS}(i,j,k)\right) / G_{max,L}\right) ..equ\ (2)$$

Where μ_{VIS} is membership functional value of visual image and $G_{max,L}$ is maximum intensity value of given visual image.

3.2 Fuzzy manipulation

We can modify the meaning of a fuzzy variable by modifying the numerical representation of the membership function. The most key hedges are intensity modification μi concentration μc and dilation μd.

$$\mu^{i}(x) = 2 * \mu^{2}(x) \text{ if } 0 \le x \le 0.5$$

$$\mu^{c}(x) = \mu^{2}(x)$$

$$\mu^{d}(x) = \mu^{0.5}(x)$$

We used in our technique as follow

$$\text{if } \left(\mu\ (i, j, k) \le 0.5\right) \textit{then}\ \mu\ (i, j, k) = 2 * \mu(i, j, k)^{2}$$

$$\textit{else}\ \mu(i, j, k) = \mu(i, j, k)$$

3.3 Lattice concept

In this section we introduce our proposed fuzzy-lattice based methodology for image fusion, the sequence of processing steps involved is shown in figure 5 and each step is briefed in the coming subsection.

We define a lattice as an algebraic system which is defining a partial ordering relation [16, 17]. A lattice L is a partially ordered set **< L, ≤ >** in which every pair of elements **(a, b)** ∈ **L** has a greatest lower bound (GLB) and a least upper bound (LUB). The greatest lower bound (GLB) of a sub set **{a, b} ≤ L** will be denoted by **(a*b)** and the least upper bound (LUB) by **(a ⊕ b).** It is customary to call the GLB **{a, b} = (a * b)** the meet or product of a and b, and the LUB **{a, b} = (a ⊕ b)** the join or sum of a and b. In certain cases the symbols ∩ and ∪ are also used to represent the meet and join respectively. It follows from definition of a lattice that both * and ⊕ are binary operation on L because of the uniqueness of the least upper bound (LUB) and greatest lower bound (GLB) of any subset of a partially ordered set as well as for any partial ordering relation ≤ on a set S, the converse relation ≥ is also a partial ordering relation on S.

For any the lattice structures say < L, ≥ > can be obtained from **< S, ≤ >** by simply turning it upside down. The partially ordered sets **< L, ≤ >** and **<S, ≥ >** are called duals of each other's .If **A ⊆ S** then LUB(A)with respect to the relation ≤ is the same as GLB (A) with respect to the relation ≥ and vice versa; In other word the GLB and LUB are interchanged if we interchange the relation ≥ and ≤. Now any statements about lattice involving the operation * and ⊕ and the relation ≥ and ≤ remains true if * is replaced by ⊕ , ⊕ by *, ≥ by ≤ and ≤by ≥ respectively. The properties of the two binary operation of meet and join denoted by * and ⊕ on al lattice **< L, ≤ >** for any **a, b, c** ∈ **L.** We have

$$(\mathrm{I})\ a * a = a;\ a \oplus a = a; (\text{idempotent})$$

$$(\mathrm{II})\ a*b=b*a; a\oplus b=b\oplus a; (\text{commutative})$$

$$(\mathrm{III})\ (a*b)*c=a*(b*c);\ (a\oplus b)\oplus c=a\oplus(b\oplus)(\text{associative})$$

$$(\mathrm{IV})\ a*(a\oplus b) = a; a \oplus (a*b) = a; (\text{absorption}).$$

An example of lattice depicted in figure [1].

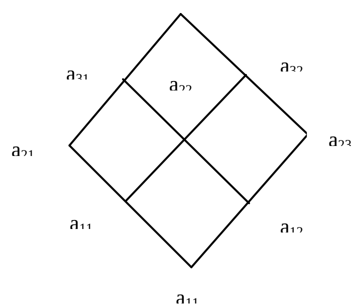

Figure 1

Where we represent a general structure of asimple lattice.In this structure, each element are convey a proper relation such as increseing or decresing order. Where o is greatest lower bound (GLB) and 1 is least lower upper bound (LUB).and also each elements cover by other elements by proper relation. The present section is devoted to computational representation for commonly increasing operator. We will apply lattice definition and concepts to Infrared and Normal Visual image whose membership values fit in a lattice. Now the operation minimum **(**∩**)** and maximum **(** ∪ **)** are point wise are induced on the lattice L by corresponding operation on L :(A ∪ B) (x) =A(x) ∨ B(x) and A∩B(x) =A(x) ∧ B(x) respectively. In our proposed technique we represent fuzzy data by simple lattice structures. We have defined proper membership functions for infrared and normal visual images which are already mentioned. After fuzzification of both input images we get separate fuzzy data such as infrared and normal visual fuzzy data. Now we represent both fuzzy data by a single lattice structure and also represent infrared and normal visual fuzzy data by two lattice structures which are ordered reverse relation.

3.3.1 Single fuzzy-lattice image fusion

For single lattice structure, we consider 3X3 size image window which generate from infrared and normal visual fuzzy data with increase ordered relation and they are depicted in figure [2].We define HIR and HVIS are infrared and Normal visual image as inputs. So fuzzy membership

functions for two input image are

$$\mu_{IR}(i, j, k) = 1-\left(\left(G_{max,L} - H_{IR}(i, j, k)\right) / G_{max,L}\right) \ldots \text{equ}(1)$$

Where $\mathbf{\mu_{IR}}$ is fuzzy data of infrared image and $\mathbf{G_{max,L}}$ is maximum intensity level of infrared image.

$$\mu_{VIS}(i, j, k) = 1-\left(\left(G_{max,L} - H_{VIS}(i,j,k)\right) / G_{max,L}\right) .. \text{equ}\ (2)$$

Where μ_{VIS} is fuzzy data of visual image and $G_{max,L}$ is maximum intensity level of visual image. Now we construct a lattice **L (μ, ≤, *, ⊕)** from $\mathbf{\mu_{IR}}$ and $\mathbf{\mu_{VIS}}$ data sets. Where we have taken large membership value between $\mathbf{\mu_{IR}}$ and $\mathbf{\mu_{VIS}}$ as well as construct a new matrix $\mathbf{\mu_{R}}$. From new matrix $\mathbf{\mu_{R}}$ we consider a ***3X3*** sub-image window. Which graphically represent in Figure [2]. After lattice construction we calculate specific mathematical computation on two fuzzy data sets $\mathbf{\mu_{IR}}$ and $\mathbf{\mu_{VIS}}$ such that

$\text{MAX}\{c_i,d_i\} = c_i \oplus d_i$ and $\text{MIN}\{c_i,d_i\} = c_i * d_i$

Where $c_i = \mu_{IR}(i,j,k)$

$d_i = \mu_{VIS}(i,j,k)$ are i^{th} data points.

Also consider $\mathbf{a_i}$ and $\mathbf{b_i}$ are $\mathbf{i^{th}}$ data points such that

$$a_i = L(i,j,k) \text{ and } b_i = L(i,j+1,k)$$

$$e_i \Rightarrow \text{minimum}(c_i,d_i)$$

$$f_i \Rightarrow \text{maximum}(c_i,d_i)$$

Now from above calculation we define a factor ∇ gi Such that

$$\nabla g_i = (e_i * f_i)/(e_i + f_i) \text{ where } 1 \le i \le n$$

$$F(i,j,k) = \sum_{i=1}^{n} (a_i - b_i) * \nabla g_i \ldots equ(3)$$

$$\text{Where } g_{n+1} = 0 \text{ and } 1 \le i \le n$$

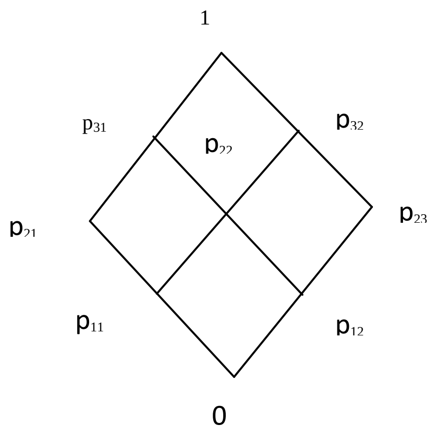

Figure 1

3.3.2 Double lattice image fusion

Similarly for dual lattice we carry out following mathematical model, we define HIR and YVIS are infrared

and Normal visual image as inputs. So fuzzy membership functions for two input image are

$$\mu_{IR}(i,j,k) = 1 - \left(\left(G_{max,L} - H_{IR}(i,j,k)\right)/G_{max,L}\right) \ldots equ(1)$$

Where $\boldsymbol{\mu_{IR}}$ is fuzzy data of infrared image and $\mathbf{G_{max,L}}$ is maximum intensity level of infrared image. And for visual image fuzzy member function is defined as

$$\mu_{VIS}(i,j,k) = 1 - \left(\left(G_{max,L} - H_{VIS}(i,j,k)\right)/G_{max,L}\right) .equ(2)$$

Where $\boldsymbol{\mu_{VIS}}$ is fuzzy data of visual image and $\mathbf{G_{max,\ L}}$ is maximum intensity level of visual image. Now we construct a lattice $\mathbf{L_1}$ **(**$\boldsymbol{\mu_{IR}}$**,** $\le$**, *,** $\oplus$**)** from $\boldsymbol{\mu_{IR}}$ and another lattice $\mathbf{L_2}$ **(**$\boldsymbol{\mu_{VIS}}$**,** $\ge$**, *,** $\oplus$**)** from $\boldsymbol{\mu_{VIS}}$ data sets. Which are representing in figure [3] and [4] after lattice construction we calculate following mathematical computation such that

$$L_1(\mu_{IR}, \le, *, \oplus) \text{ and } L_2(\mu_{VIS}, \ge, *, \oplus)$$

Now we consider ai and bi is ith lattice point of two lattices L1 and L2 respectively.

Now

$$a_i = L_1(i,j,k) \text{ and } b_i = L_2(i,j,k)$$

$$c_i = \text{maximum } (a_i \oplus b_i)$$

$$d_i = \text{minimum } (a_i \oplus b_i)$$

We aggregate lattice data point of two lattices. Let $\mathbf{e_i}$ a factor that measure relative data value lattice points. We have calculated the discrimination between two consecutive lattice points. This indicated difference between membership values of lattice data points. This measure defined by a symbol $\Delta\mathbf{f}$ of i^{th} lattice points of two defined lattice .Which given as

$$\alpha_i = (c_i + d_i) - 1/2 * (c_i * d_i)$$

$$\Delta f = (c_i + d_i)/(c_i * d_i)$$

$$F(i,j,k) = (\alpha_i * \Delta f) \ldots\ldots equ\ (4)$$

Where F (i, j, k) is fused data from two lattice L1 $(\mu_{IR}, \le, *, \oplus)$ and $L_2(\mu_{VIS}, \ge, *, \oplus)$ respectively.

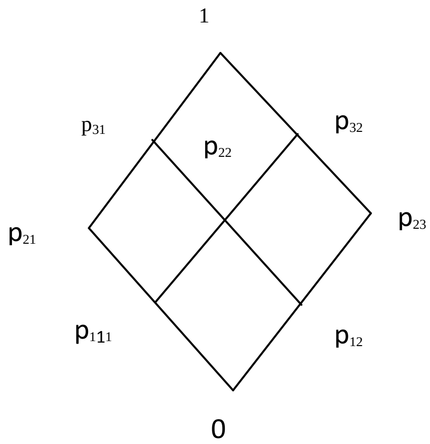

Figure 3

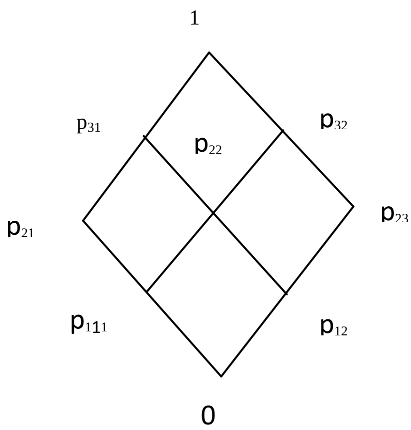

Figure 4

Whenever we attempt to fused multimodal image data, first we are concerned about the nature of data. Generally in multimodal image data such as IR-image may have a large number of high scale data and as well as low scale data. Which indicate higher details of image, similarly in visual image may have large number low scale data and apparently small number of higher scale data. Where low scale data in normal image indicate lower detailed and smoothness of image. Now we have given a membership value to both IR image and visual image respectively. Potentially inn IR image have obtained large number of higher membership values in fuzzy plane, and similarly in normal image we also get higher membership value which are may be complement with IR membership. Now we have extracted higher order membership grade values from both IR and visual image and those membership values we represent and alignments by a single lattice structure. Generally any higher membership values in IR may be lower in normal image spatially vs. now we have measure the basic changes of information between two membership values in both images in fuzzy plane. This change of information in fuzzy data is used to aggregation of fuzzy information Similarly we can represent IR-membership and visual membership by two distinct lattice structures base on increasing and decreasing order respectively which graphically represent in figure 3 and 4.

Step 5: carry out fuzzy manipulation using equation (ii) on F matrix.
Step 6: defuzzify final matrix F by eq (6)
Step7: output fused image.

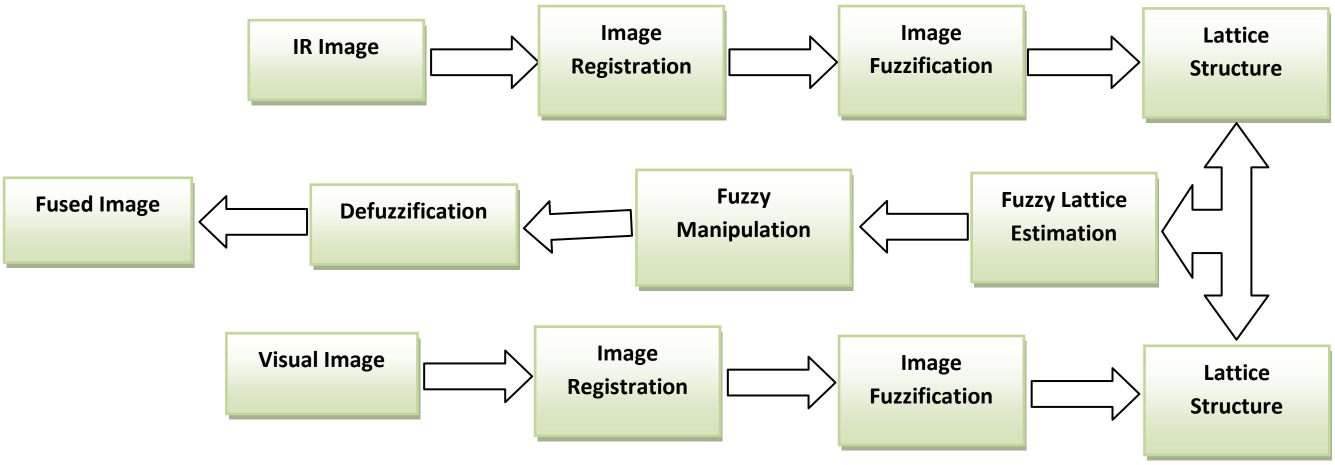

Figure 5

Algorithm I: Single lattice Fusion (SLF)
Step 1: read input images (visual and infrared).
Step2: measure size of images (number of row and columns in the image matrix).
Step3: carry out fuzzification on both input images. Using equation(1 and 2).
Step4: compare the element wise corresponding values of the two fuzzified matrix and select the larger value in each case and generate the final matrix say F.
Step5: calculate the value of ∇g and F using equation(3).
Step 6: carry out fuzzy manipulation using eq (ii) on F matrix.
Step 7: defuzzify final matrix F by equation (5).
Step8: output fused image.

Algorithm II: Double lattice Fusion (DLF)
Step 1: read input images (visual and infrared)
Step2: measure size of images (number of row and columns in the image matrix).
Step3: carry out fuzzification on both images. Using equation eq (1 and 2)
Step 4: calculate the value of α_i, ∇f and F using equation (4).

3.4 Defuzzification

We have used gray-scale level defuzzification model that defined as follow .For single lattice

$$H(i,j,k)=G_{max,L}-G_{max,L}*(t/s*(1-G(i,j,k))^{S})...equ(5)$$

And for dual lattice

$$H(i,j,k)=G_{max,L}-G_{max,L}*(G(i,j,k)/s*(1-(s/t)*G(i,j,k))^{t}*s)^{S})))...equ(6)$$

Where $G_{max,L}$ maximum gray level of image and **t** and **s** is scaling factor of membership function, whose value belongs **2 ≤ t ≤ 4** and **2 ≤ s ≤ 6.**

In defuzzification method here we have defined heuristic based defuzzification function which mapping membership grade value [0, 1] to crisp set [0,255] plane. The transformation of fuzzy data to crisp plane is more crucial step and may have more efficient defuzzification functions which depend on expert knowledge.

4. PERFORMANCE EVALUATION

The simplest approach to assessing the quality of a fused image F is to compare it with a known reference image R.

Standard Deviation (SD): For a fused image of size **N ×M**, its standard deviation can be estimated by

$$SD=\sqrt{\frac{1}{NM}\sum_{i=1}^{N}\sum_{j=1}^{M}C_f(i,j)-\bar{m})^2}$$

Where $C_f(i,j)$ is the **(i, j)** th pixel intensity value and $\bar{m}$ is the sample mean of all pixel values of the image. SD is

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estimates the signal strength efficiently in the absence of noise.

Entropy (EN):

Entropy is used to evaluate the information content of an image [10]. From definition of entropy, the information content of an image is calculated by following equation.

$$EN = -\sum_{i=0}^{G} p(i)\log_2\{p(i)\}$$

Where **G** is the number of gray levels in the image's histogram (which can be 255 for a typical 8-bit image) and ***p(i)*** is probability of each gray level in the histogram of the image.

Average gradient (AG):

The average gradient of an image ***g (I)*** is the measure of its sharpness in terms of gradient values. The average gradient is defined [10] by:

$$\bar{g}_k = \bar{g}(I_{F_k}) = \frac{1}{XY}\sum_{x=1}^{X}\sum_{y=1}^{Y}\sqrt{(\frac{\partial}{\partial x}I_{F_k})^2 + (\frac{\partial}{\partial y}I_{F_k})^2}$$

Where this partial derivatives are the differentiation operators. The average gradient $\bar{g}$ is devious of sensitivity of image.

5. RESULTS AND ANALYSIS

We have used three fusion strategies which are discrete wavelet fusion, weighted average fusion and our proposed fusion technique. In this paper, we used those multimodal fusion approaches and perform a comparative evaluation. This work has been simulated using MATLAB 7.9.0version in a machine of the configuration 3.40 GHz Intel Xeon (TM) CPU and 2GB of Physical Memory. We analyze the performance of our algorithm using the thermal and visual image database. We give some typical examples to illustrate respectively the fusion on a benchmark example. This example is used to test the effectiveness of fusion algorithm. It is considered as a benchmark [21] example for fusion algorithms. Here we give results of applying one of our developed fusion methods to the benchmark images [21]. It is shown that our proposed methods perform very well for those examples. We carry out quality estimation to our proposed technique. From the testing results, it can be observed that fusion of IR and visual images can enhance features in both kinds of images and more impressively it can reveal potential information that more than IR images or visual image. Thus, it can be concluded that multi-sensors fusion does give us to improve our capability of doing for detection. We used entropy, spatial frequency index, mean and standard division for performance analysis to select on benchmark images. The result sets of our estimation on the images which are represented on following table 1.We used following method on infrared, visual and fused images. From data set we conclude that fused image always contain more information, clarity and active than infrared and visual images. The Entropy, spatial index, means and standard value of fused images are better than their infrared and visual images. We have compared our proposed technique with best well known technique such as discrete wavelet technique (DWT), weighted average fusion technique (WAFT). The corresponding result set represent by table 2.

We have carried out these methods in a standard bench mark images. We have calculate entropy, spatial index, mean and standard division of fused output images. From the table we conclude that our proposed technique good as others well known fusion technique.

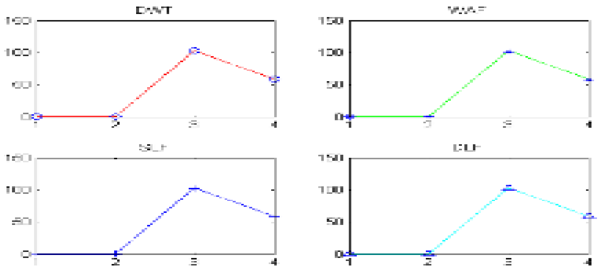

Figure 5

Table 1.Data Analysis

Image Name	Entropy	Spatial frequency	Mean	Standard Division
Visual image[1a]	0.3523	0.2638	73.6138	57.9258
Infrared image[1b]	0.1121	0.2144	67.1763	30.9547
Fused image[1c]	0.3620	0.2887	78.6842	58.9862
Visual image[2a]	0.2925	0.0793	81.8236	51.8249
Infrared image[2b]	0.1589	0.0331	57.9168	32.7285
Fused image[2c]	0.3278	0.0831	85.2727	59.986
Visual image[3a]	0.3232	0.1563	84.9177	55.8532
Infrared image[3b]	0.1487	0.2543z	73.2073	33.1689
Fused image[3c]	0.3662	0.2887	103.0397	57.9236

Table 2.Comparisoin data analysis of different algorithms

Fusion technique	Entropy	Spatial frequency	Mean	Standard Division
DWT	0.3627	0.28624	103.1308	57.9214
WAF	0.3214	0.2752	101.4853	56.8586
SLF	0.36206	0.28563	102.3386	57.9138
DLF	0.3662	0.28827	103.1327	57.9236

RESULT DATASET

Visual image [1a] IR image [1b] fused IMAGE [1c]

Visual image [2a] IR image [2b] fused image [2c]

Visual image [3a] IR image [3b] fused image [3c]

Comparisons study

DWTF WAF SLF DLF

6. CONCLUSIONS

In this work we have done a primary study on multiple sensor fusion, designed and implemented. In fuzzy-lattice based fusion method, combination of the input images, that produced fuse the images. The fuzzy-lattice based fusion algorithm provides additional flexibility for control over information used in the fusion process. The output is a fused image containing enhanced visual and thermal information. The performances of the proposed method are tested infrared image and the normal multi-spectral image. Both subjectively qualitative analysis and objectively quantitative evaluation verify the validity of the new method. The proposed method can improve the spatial details while preserving the spectral information of the multi-spectral image. Compared with the traditional discrete wavelet transform method of the same wavelet transform level, and weighted average fusion method where the new method has the advantage of preserving more spatial details and spectral information.

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