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Design of Centralized CRONE Controller Combined with MIMO-QFT Approach Applied to Non Square Multivariable Systems

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ABSTRACT
Motion control and robust path tracking were extended to non square MIMO systems having more outputs then inputs in this work. The Non square Relative Gain array (NRG) has been used to assess the performance of non-square control systems based on steady-state information. Using NRG and the SSE (Sum of Square Error), a square subsystem can be selected.
MIMO-QFT (Quantitative Feedback Theory) robust synthesis methodology permits to generate the appropriate equivalent MISO (Multi-input Single-Output) system structure from the MIMO (Multi-Input Multi-Output) structure. After that, the CRONE control approach based on third generation CRONE methodology was used to find the controller of the selected subsystem taking into account plant uncertainties. A fractional prefilter synthesis approach was already developed to find a non-integer prefilter expression in order to satisfy the performance specifications. A fully populated matrix controller structure has been proposed to govern perfectly the multivariable processes. In order to reduce the loop interactions, a coupling matrix has been designed. A numerical example has been treated in order to verify the proposed design.

General Terms
Path tracking design using CRONE control approach applied to non-square multivariable systems.

Keywords
Path tracking, Non square relative gain array (NRG), CRONE control design, Motion control, Coupling effect, Robotics.

1. INTRODUCTION
Multivariable systems with uncertainty can be considered one of the hardest problems in industry because most of complex industrial processes are always Multi-Input Multi- Output (MIMO) systems. MIMO systems are more difficult to control due to the existence of interactions among input and output variables.
Several researches have been used in robust generation and path tracking design. In industrial path tracking designs, a prefilter is used since it is easy to implement and adapt to reduce overshoots. One of most used prefilter is that of Davidson-Cole whose main property is eliminating overshoots on the plant output. It is possible by using the Davidson-Cole filter to limit the resonance of the feedback control loop, by a continuous variation on its two constitutive parameters time constant _ and real order n. A MIMO-QFT robust synthesis methodology has been used by Melchior [1] which is applied to a square MIMO system in path tracking design. The general problem in the QFT Two Degree Of Freedom (TDOF) system is how to generate the feedback controller and the prefilter [2]. Specifications of most QFT problems are to put the responses of the closed loop system into lower and upper bounds [2], [3], [4], [5], [6].
Nevertheless, the original MIMO-QFT design usually proposes the use of diagonal controller to control multivariable systems. A non-diagonal controller has been used to improve this structure. More design flexibility in the control of MIMO plants can be given by the non diagonal controller. Other alternative methods for non-diagonal multivariable QFT robust control system design have been introduced [7]. Garsia sanz et al. [8], [9], [10] extends the classic QFT diagonal controller design for square MIMO plants with uncertainty to a fully populated matrix controller structure. The QFT approach can be combined with different type of controller like CRONE control approach which is the aim of this work.
The CRONE control-system design is initially introduced by Oustaloup et al. [11], [12]. This methodology is based on fractional non-integer differentiation [13], [14]. The Crone control is a frequency design to provide the robust control of perturbed plants using the common unity feedback configuration. For the nominal state of the plant, this approach consists in determining the open-loop transfer function which guarantees the desired specifications like precision, overshoot and rapidity. The controller can be obtained from the ratio of the open loop transfer function to the nominal plant transfer function taking into account the plant right half-plane zeros and poles. There are three CRONE control generations [15], [16]. Only the used principle of the third generation is given in this paper.
The fractional non-integer differentiation allows to describe the open-loop transfer function. The optimal transfer function to meet the specifications is easier to obtain. Furthermore, the CRONE control design takes into account the plant genuine structured uncertainty domains. CRONE control design has already been applied to multivariable systems [15], [17].
Usually square MIMO systems are used in industry process. These type of systems have got an equal number of inputs and outputs. Yet, when one of actionnera is nonfunctional then the studied system become having more outputs than inputs. This paper resolves the problem of non square MIMO system which has more outputs than inputs in path tracking design. So, the aim of this work is to extend the CRONE control approach to non square MIMO systems using fractional prefilters in path tracking design. A combined CRONE control and MIMO-QFT structure using non diagonal controller have been developed to compare the utility of using fractional prefilter. The non square relative gain array (NRG) [18] is a useful tool to analyze a non square multivariable systems. NRG is used to determine the interaction measurements. This approach can help to square down the non square multivariable systems.
Section 2 briefly presented a method to control structure selection of non square multivariable systems. The MIMOQFT technique using a centralized controller is summarized in section 3. Section 4 outlines the CRONE CSD

methodology for multivariable plants. A fractional prefilter optimization is given in section 5. Finally, an example is employed to illustrate the effectiveness of the proposed methodology to control non square multivariable system in section 6.

2. CONTROL STRUCTURE FOR NON-SQUARE MULTIVARIABLE SYSTEMS

2.1 Non-square relative gain array (NRG)

Consider a $m \times n$ process transfer function $P$with $m \geq n$:

$$P(p) = \begin{pmatrix} p_{11} & \cdots & p_{1n} \\ \vdots & \ddots & \vdots \\ p_{m1} & \cdots & p_{mn} \end{pmatrix} \quad (1)$$

The non-square relative gain array is a way to measure interaction between inputs and outputs. The non-square relative gain array can be evaluated:

$$\Lambda^N(p) = P \otimes (P^*)^T \quad (2)$$

where the operator $\otimes$ is the Hadamard product and P^* represent the Moore-Penrose Pseudo-inverse transfer matrix of P.

$$\Lambda^N(p) = \begin{pmatrix} \lambda^N{}_{11} & \cdots & \lambda^N{}_{1n} \\ \vdots & \ddots & \vdots \\ \lambda^N{}_{m1} & \cdots & \lambda^N{}_{mn} \end{pmatrix} \quad (3)$$

The sum of all elements in each row RS and in each column RC is defined as:

$$RS = \left[\sum_{j=1}^{n} \lambda^N{}_{1j}, \sum_{j=1}^{n} \lambda^N{}_{2j}, \dots, \sum_{j=1}^{n} \lambda^N{}_{mj}\right]^T \quad (4)$$

$$= [rs(1), rs(2), \dots, rs(m)]^T \quad (5)$$

rs(i) is the sum of the ith row of the NRG.
and

$$CS = \left[\sum_{i=1}^{n} \lambda^N{}_{i1}, \sum_{i=1}^{n} \lambda^N{}_{ij}, \dots, \sum_{j=1}^{n} \lambda^N{}_{mj}\right]^T \quad (6)$$

$$= [rs(1), rs(2), \dots, rs(m)]^T \quad (7)$$

Some properties of non-square multivariable gain array (NRG) have been described by CHANG [18]:

- Sum of elements in each column of the NRG is equal to unity.

$$cs(j) = 1, \forall\, j$$

- Sum of elements in each row of the NRG is between zero and unity.

$$0 \leq rs(i) \leq 1, \forall\, i$$

- The NRG is invariant under input scaling and variant under output scaling.
- Any permutation of rows and columns in a transfer function matrix P results in the same permutation in the NRG.
- For an $m \times 1$ system, $P = [p_{11}; p_{21}, \dots, p_{m1}]^T$, so the NRG is described as follow :

$$\Lambda^N = [\lambda_{11}, \lambda_{12}, \dots, \lambda_{m1}]^T$$

with

$$\Lambda^N{}_{i1} = \frac{p_{i1}^2}{\sum_{k=1}^{m} p_{k1}^2}$$

- The elements of the NRG approach infinity as the non-square system matrix P becomes nearly singular.

2.2 Control structure selection

CHANG [18] was used an approach to design a control system for a non-square process. This approach help to square down the non-square system. Consider a non-square plant, P. It can be partitioned into a square subsystem Ps and a complementary (remaining) subsystem Pr. The control objective is to minimize the sum of square error (SSE) of uncontrolled outputs when the square subsystem is under perfect control. The row sum of the NRG (Non-square Relative Gain array) provides some information in this regard.

For an $m \times n$ process with $m > n$, if we choose n outputs for control the system can be partitioned into (see Figure 1):

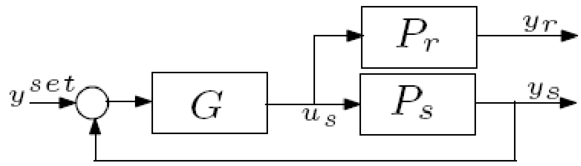


Fig 1 Control structure for TDOF MIMO system

then

$$\begin{bmatrix} y_s \\ - \\ y_r \end{bmatrix} = \begin{bmatrix} P_s \\ - \\ P_r \end{bmatrix} . u = \begin{bmatrix} p_{11} & \cdot & p_{1n} \\ \cdot & & \\ p_{n1} & \cdot & p_{nn} \\ - & - & - \\ p_{m1} & \cdot & p_{mn} \end{bmatrix} . u \quad (8)$$

where y_s is an $n \times 1$ output vector for the controlled outputs and y_r is an $(m-n)\times 1$ output vector for the remaining output. The objective is to minimize the Sum of Square Error (SSE) of the uncontrolled outputs for any variation in the controlled outputs.
The closed loop square subsystem gains are given by:

$$\check{u} = P_s^{-1} \check{y}_s^{set} \quad (9)$$

For all outputs, the steady state error is:

$$\check{e} = (I_{m\times n} - P P_s^{-1}) \check{y}_s^{set} \quad (10)$$

Choosing a particular square subsystem, P_s, the SSE is defined as:

$$SSE = \sum_{i=1}^{n} \|\check{e}(i)\|_2^2 = \sum_{i=1}^{n} \left\| (I_{m\times n} - P P_s^{-1}) \check{y}^{set}{}_{s,i} \right\|_2^2 \quad (11)$$

The row sum of NRG provides optimal solution to the problem for two special cases, namely, case of n = 1 and case of m = n + 1, and suboptimal solution of other cases [18].

2.2.1 Case of $m = n + 1$

A square subsystem is chosen, the sum of square error is given by:

$$SSE = \frac{rs(n+2-j)}{1 - rs(n+2-j)} \quad (12)$$

where j means the sum of square error when the j^th^ subset is chosen to form a square subsystem.

Since the value of the row sum is between zero and one so a small row sum gives a small SSE in the corresponding square subsystem. The small row sum of the NRG in the complementary system indicates a small SSE in the square subsystem. Then, the criterion for the selection of a square subsystem is to remove the controlled variable with the smallest row sum in the NRG. Other cases are well described in [18].

3. CENTRALIZED CONTROLLER

3.1 Coupling matrix

QFT approach has been developed to control MIMO systems. This approach, usually, uses diagonal controller. Whereas MIMO QFT procedures take into account the coupling effect between loops. They only use a diagonal controller G to govern the MIMO plant. This method has been improved using non-diagonal controllers [8], [9], [10]. Most design flexibility in the control of MIMO plants was given using a fully populated matrix. Also, the non-diagonal components can simply the diagonal controller design problem.

A measurement index has been defined in order to quantify the loop interactions in MIMO control systems.

The MIMO-QFT structure is given by figure 2. This figure is composed of a $n \times n$ multivariable uncertain plant P_s, a fully populated matrix controller G and a prefilter F.

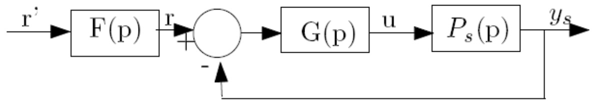


Fig 2 Two-degrees-of-freedom control system: MIMO structure

- $P_s(p)$ is $n \times n$ transfer function matrix. P_s represents a linear time invariant uncertain and minimum-phase plant. The plant P_s and its inverse P_s^* must be stable and having no unstable modes. $P_s \in ⟧P_s$ where $⟧P_s$ is a set of possible uncertain plants.

$$P(p) = \begin{pmatrix} p_{11}(p) & \cdots & p_{1n}(p) \\ \vdots & \ddots & \vdots \\ p_{n1}(p) & \cdots & p_{nn}(p) \end{pmatrix} \quad (13)$$

- $G(p)$ is the non-diagonal controller

$$G(p) = \begin{pmatrix} g_{11}(p) & \cdots & g_{1n}(p) \\ \vdots & \ddots & \vdots \\ g_{n1}(p) & \cdots & g_{nn}(p) \end{pmatrix} \quad (14)$$

- $F(p)$ is the non-diagonal controller

$$F(p) = \begin{pmatrix} f_{11}(p) & \cdots & f_{1n}(p) \\ \vdots & \ddots & \vdots \\ f_{n1}(p) & \cdots & f_{nn}(p) \end{pmatrix} \quad (15)$$

The plant inverse is denoted P_s^* so:

$$P_s^{-1}(p) = P_s^*(p) = \left[p_{ij}^*(p)\right] = \Lambda(p) + B(p) \quad (16)$$

where:

$$\Lambda(p) = \begin{pmatrix} p_{11}^*(p) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & p_{nn}^*(p) \end{pmatrix}$$

and

$$B(p) = \begin{pmatrix} 0 & \cdots & p_{1n}^*(p) \\ \vdots & \ddots & \vdots \\ p_{n1}^*(p) & \cdots & 0 \end{pmatrix}$$

Elements q_{ij} of matrix Q are described by:

$$q_{ij}(p) = \frac{1}{p_{ij}^*(p)} \quad (17)$$

The controller, $G(p)$, is divided to this form:

$$G(p) = G_d(p) + G_b(p) \quad (18)$$

where

$$G_d(p) = \begin{pmatrix} g_{11}(p) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & g_{nn}(p) \end{pmatrix}$$

and

$$G_b(p) = \begin{pmatrix} 0 & \cdots & g_{1n}(p) \\ \vdots & \ddots & \vdots \\ g_{n1}(p) & \cdots & 0 \end{pmatrix}$$

Λ and G_d are respectively the diagonal part of $P(p)$ and $G(p)$, wheras B is the balance of P_s^* and G_d is the balance of G.

The controlled system for the reference tracking problem can be written as:

The controlled system for the reference tracking problem, can be written as:

$$\begin{aligned} y_s &= (I + P_s G)^{-1} P_s G r \\ &= T_{y_s/r} F r' \end{aligned} \quad (19)$$

(19) can be reformulated using (16) and (18):

$$T_{y_{s/r}} r = (I + \Lambda^{-1} G_d)^{-1} \Lambda^{-1} G_d r + (I + \Lambda^{-1} G_d)^{-1} \Lambda^{-1} \left(G_b r - (B + G_b) T_{y_s/r} r\right) \quad (20)$$

Observing the expression of the closed-loop transfer function matrix (20), two different terms can be found which are the diagonal $T_{y_{s/r-d}}$ and the non diagonal $T_{y_{s/r-b}}$ terms.

$$T_{y_{s/r-d}} = (I + \Lambda^{-1} G_d)^{-1} \Lambda^{-1} G_d \quad (21)$$

$$\begin{aligned} T_{y_{s/r-b}} &= (I + \Lambda^{-1} G_d)^{-1} \Lambda^{-1} \left(G_b - (B + G_b) T_{y_s/r}\right) \\ &= (I + \Lambda^{-1} G_d)^{-1} \Lambda^{-1} C \end{aligned} \quad (22)$$

The only part that related to the non-diagonal terms of both plant inverse B and controller G_b is the matrix C. For reference tracking problems, C is called the coupling matrix.

$$C = G_b - (B + G_b) T_{y_s/r} \quad (23)$$

Each elements of this matrix respect:

$$c_{ij} = g_{ij}\left(1 - \delta_{ij}\right) - \sum_{k=1}^{n} (p_{ik}^* + g_{ik}) t_{kj} \left(1 - \delta_{ij}\right) \quad (24)$$

(21) is equivalent to n reference tracking single-input single-output where plants are equal to the elements of Λ^{-1} and (22) is equivalent to the same n previous systems with internal disturbances $c_{ij} r_j$. So, the i^{th} MISO structure is as follow:

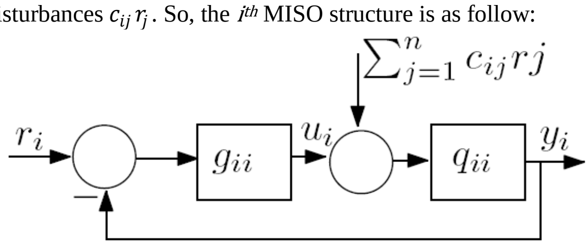


Fig 3 i^{th} equivalent MISO system

To analyse the coupling element, Garsia-Sanz [8], [9] supposes that $\forall\ k \neq j, |t_{jj}(p_{ij}^* + g_{ij})| \gg |t_{kj}(p_{ik}^* + g_{ik})|$. This assumption leads to two simplifications which facilitate the coupling expression:

1. S1 $c_{ij} = g_{ij} - t_{jj}\left(p_{ij}^* + g_{ij}\right); i \neq j$
2. S2 $t_{jj} = \frac{g_{jj}\, q_{jj}}{1 + g_{jj}\, q_{jj}}$

The coupling effects c_{ij} using S1 and S2 becomes:

$$c_{ij} = g_{ij} - \frac{g_{jj}\left(p_{ij}^* + g_{ij}\right)}{\left(p_{jj}^* + g_{jj}\right)}; i \neq j \quad (25)$$

3.2 Controller design

The optimum non diagonal controller is found making the coupling effect (25) equal to zero.

$$g_{ij}^{opt} = F_{pd}\left(g_{jj}\frac{p_{ij}^{*N}}{p_{jj}^{*N}}\right), i \neq j \tag{26}$$

The proposed controller is deduced from a sequential procedure. The first step is pairing outputs and inputs using RGA methods. The sequential controller approach composed of n stages which is the number of loops. The second step is the design of the diagonal controller g_{kk}. The controller g_{kk} is calculated using the CRONE control approach [12], [14] for the inverse of the equivalent plant $[p_{kk}^{*e}(p)]_k^{-1}$so that obtaining robust stability and robust performance specifications. The equivalent plant is introduced by Horowitz [7] and ameliorated by Franchek et al [19].

$$[p_{ij}^{*e}(p)]_k = [p_{ij}^{*e}(p)]_{k-1} - \frac{\left([p_{i(k-1)}^{*e}(p)]_{k-1}+[g_{i(k-1)}(p)]_{k-1}\right)}{\left([p_{(k-1)(k-1)}^{*e}(p)]_{k-1}+[g_{(k-1)(k-1)}(p)]_{k-1}\right)} \times \left([p_{(k-1)j}^{*e}(p)]_{k-1}+[g_{(k-1)j}(p)]_{k-1}\right) \tag{27}$$

After that, the non diagonal part of the controller is designed to minimize the coupling effects (see(26)).
Finally, a fractional prefilter F is determined which does not present any difficulty to make the closed loop transfer function into the desired specifications.

4. CRONE CONTROL

The CRONE control approach is a frequential approach based on fractional derivative. The object of this method is how to design a controller which allows a degree of stability robustness. There are three generations describing the control law. In this paper we will focus on the third generation CRONE control because the plant frequency uncertainty domains are of various types. The third-generation CRONE can manage the robustness/performance tradeoff. Also, it is able to synthesis controllers for plants with positive real part zeros or poles, with lightly damped mode, and/or time delay [20]. For multivariable plant (MIMO) two methods have been developed [21], multivariable and multi-SISO approach.
The objective is to succeed output feedback decoupling. Therefore, the decoupling and diagonal open-loop transfer matrix will allow a diagonal nominal closed-loop transfer matrix:

$$\beta_0(p) = \begin{pmatrix} \beta_{0_1}(p) & \dots & \dots & 0 \\ 0 & \ddots & \dots & \dots \\ \dots & \dots & \beta_{0_i}(p) & 0 \\ 0 & \dots & 0 & \beta_{0_n}(p) \end{pmatrix} \tag{28}$$

The nominal sensitivity $S_0(p)$, the nominal complementary sensitivity $T_0(p)$, input sensitivity $S_{U0}(p)$ and input disturbance sensitivity $S_{I0}(p)$, transfer function matrices are:

$$S_0(p) = [I + \beta_0(p)]^{-1} = diag[S_{0j}(p)]_{1\leq j\leq n} \tag{29}$$

$$T_0(p) = [I + \beta_0(p)]^{-1}\beta_0(p) = diag[T_{0j}(p)]_{1\leq j\leq n} \tag{30}$$

$$S_{U0}(p) = G(p)[I + \beta_0(p)]^{-1} = G(p)\, S_0(p) \tag{31}$$

$$S_{I0}(p) = [I + \beta_0(p)]^{-1}Q(p) = T_0(p)G^{-1}(p) \tag{32}$$

With:

$$T_{0j}(p) = \frac{\beta_0(p)}{1+\beta_0(p)} \tag{33}$$

$$S_{0j}(p) = \frac{1}{1+\beta_0(p)} \tag{34}$$

The open-loop transfer functions $\beta_{0i}(p)$ are used to satisfy some objectives:

- accuracy specifications at low frequencies,
- required nominal stability margins of the closed-loops
- specifications on the n control efforts at high frequencies.

The third generation of CRONE CSD uses complex non integer order integration over a selected frequency range [ω_A, ω_B]. The generalized template is a straight line of any direction in the Nichols chart created by the complex fractional order $n_f = a + ib$ (figure 4):

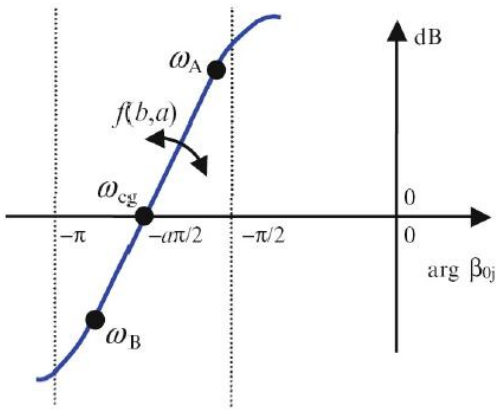


Fig 4 The generalized template

Its phase location at frequency ω_{cg} is given by the real part of n_f and the imaginary part defines its direction [22]. When the generalized template is based on band-limited complex non-integer integration, then the transfer function is [23], [24]:

$$\beta_{0i}(p) = C^{sign\,(b)}\left(\frac{1+p/\omega_h}{1+p/\omega_l}\right)^a \times \left(Re_{/i}\left\{\left(C_g\frac{1+p/\omega_h}{1+p/\omega_l}\right)^{ib}\right\}\right)^{-qsign\,(b)} \tag{35}$$

$$C = ch\left[b\left(arctan\left(\frac{\omega_{cg}}{\omega_l} - \frac{\omega_{cg}}{\omega_h}\right)\right)\right] \tag{36}$$

$$C_g = \left(\frac{1+\left(\frac{\omega_{cg}}{\omega_l}\right)^2}{1+\left(\frac{\omega_{cg}}{\omega_h}\right)^2}\right)^{1/2} \tag{37}$$

The corner frequencies are placed such that:

$$\omega_l < \omega_A < \omega_{cg} < \omega_B < \omega_h \tag{38}$$

In the open-loop transfer function, the generalized template is taken into account when the plant is stable and minimum phase:

$$\beta_{0ii}(p) = \beta_{li}(p)\beta_{0i}(p)\beta_{hi}(p) \tag{39}$$

where

$$\beta_{li}(p) = C_{li}\left(\frac{\omega_{li}}{p} - 1\right)^{n_{li}} \tag{40}$$

$$\beta_{hi}(p) = \frac{C_{li}}{\left(\frac{p}{\omega_{hi}}+1\right)^{n_{hi}}} \quad (41)$$

The accuracy of each closed-loop is fixed by the order n_{li} but the order n_{hi} allows the elements of the controller to be proper. Consider that Q_0 is the nominal plant transfer matrix such that $Q_0(p) = \left[q_{0_{ij}}(p)\right]_{i,j\in N}$:

$$\beta_0 = Q_0 G = diag[\beta_{0i}]_{i=j} = diag\left(\frac{n_i}{d_i}\right)_{i\in N} \quad (42)$$

where:
$\beta_{0i} = \frac{n_i}{d_i}$ the element of the i^{th} column and row.

The objective of CRONE control for MIMO plants is to determine a decoupling controller for the nominal plant. Q_0 being not diagonal, the problem is to find a decoupling and stabilizing controller G. The controller exists if the following hypotheses are true [23]:

$$H_1: [Q(p)]^{-1} \text{ exists}, \quad (43)$$

$$H_2: Z_+[Q(p)] \cap P_+[Q(p)] = 0 \quad (44)$$

where $Z_+[Q(p)]$ and $P_+[Q(p)]$ are respectively the positive real part zero and pole sets.
The controller G is described by:

$$G(p) = Q_0^{-1}(p)\beta_0(p) \quad (45)$$

For each nominal open-loop $\beta_0(p)$, many generalized templates can border the same required magnitude-contour of the Nichols chart or the same resonant peak M_{p0i}. The optimal one minimizes the robustness cost function:

$$J = \sum_{i=1}^{n}\left(M_{p_{max\ i}} - M_{p_{min\ i}}\right) \quad (46)$$

where:

$$M_{p_{max\ i}} = \max_Q \sup_\omega \left(T_{ii}(j\omega)\right) = \max_Q \sup_\omega \left(\frac{\beta_{ii}(j\omega)}{1+\beta_{ii}(j\omega)}\right) \quad (47)$$

$$M_{p_{min\ i}} = \min_Q \sup_\omega \left(T_{ii}(j\omega)\right) = \min_Q \sup_\omega \left(\frac{\beta_{ii}(j\omega)}{1+\beta_{ii}(j\omega)}\right) \quad (48)$$

This optimization can be done while respecting the following set for $\omega\epsilon R$ and $i, j\epsilon N$:

$$\inf_Q \left|T_{ij}(j\omega)\right| \geq T_{ijl}(\omega) \quad (49)$$

$$\sup_Q \left|T_{ij}(j\omega)\right| \geq T_{iju}(\omega) \quad (50)$$

$$\sup_Q \left|S_{ij}(j\omega)\right| \geq S_{iju}(\omega) \quad (51)$$

$$\sup_Q \left|GS_{ij}(j\omega)\right| \geq G\, S_{iju}(\omega) \quad (52)$$

$$\sup_Q \left|S_{ij}(j\omega)\right| \geq SQ_{iju}(\omega) \quad (53)$$

where Q is the nominal or perturbed plant.
A non-linear optimization method permits the extraction of the independent parameters of each open loop transfer function. Respecting other specifications taken into account by constraints on sensitivity function magnitude, this optimization is based on minimization of the stability degree variations.

5. FRACTIONAL PREFILTER OPTIMIZATION

A Davidson-Cole (DC) filter is described by the following transmittance:

$$F(p) = \frac{1}{(1+\tau p)^n} = \frac{1}{\left(1+\frac{p}{\omega}\right)^n} \quad (54)$$

which uses real poles and prevents frequency resonance. The choice of identical poles can lead to the largest possible energy on bandwidth (figure.5 (a)).
Davidson-Cole prefilter [25] (see (54)), at high frequencies, reduces energy of the signal. As can be seen in figure.5 (b), it continuously controls the bandwidth (time constant τ) and the selectivity (real order n).
As analog or digital filter, it can be used as prefilter to reduce overshoots in position control.

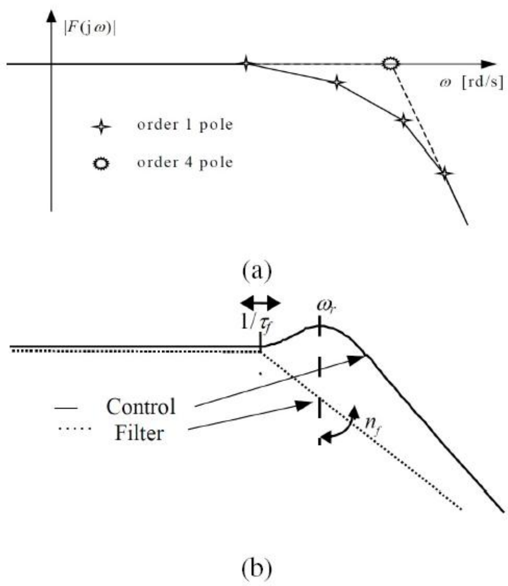


Fig 5 (a): Pole assignment for a maximum energy in a given pass band; (b) : Frequency response of the Davidson-Cole filter

The reference sensitivity transfer function S_{ref} between control u and input r is given by (figure 6):

$$S_{ref}(p) = \frac{F(p)G(p)}{1+G(p)Q(p)} \quad (55)$$

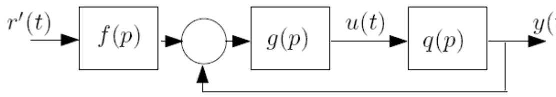


Fig 6 Unity feedback control loop with prefilter

In order to keep the control signals under its maximum value, the frequency constraint is:

$$\forall\ \omega > 0, \tau > 0, \left|S_{ref}(j\omega)\right| \leq \gamma \quad (56)$$

where $\gamma = \frac{u_{max}}{e_{max}}$, with u_{max} the maximum static constraint value on the control signal and e_{max} is a constant signal to apply on the prefilter input.
The desired range of the closed-loop transfer function is described by two bounds in frequency domain which are detailed bellow:

$$\forall\ \omega > 0, \tau > 0, |T_{RL}(j\omega)| \leq |t_{rii}(j\omega)| \leq |T_{RU}(j\omega)| \quad (57)$$

This equation becomes:

$$\forall\ \omega > 0, \tau > 0, |T_{RL}(j\omega)| \leq |t_{rii}(j\omega)|_{min} \quad (58)$$

$$|t_{rii}(j\omega)|_{max} \leq |T_{RU}(j\omega)| \quad (59)$$

with the closed loop transfer function:

$$t_{rii}(j\omega) = \frac{f_{ii}(j\omega)g_i(j\omega)q_{ii}(j\omega)}{1+g_i(j\omega)q_{ii}(j\omega)} \quad (60)$$

By considering the integral gap criterion, we can obtain the optimized parameters of the Davidson-Cole filter can be obtained. The integral gap analytic expression for step response is:

$$I_e \leq n\tau \quad (61)$$

For $m \times m$ MIMO systems, the integral gap criterion is calculated as MISO sub-system [8], so in the case of
$F = diag\ [f_{ii}]$ (49) becomes:

$$I_e \leq n_1\tau_1 + n_2\tau_2 + \cdots + n_m\tau_m \quad (62)$$

We can find the optimal parameters of $(n,\ \tau)$ using the optimization toolbox of MATLAB.

6. APPLICATION

Considering a 2×3 MIMO uncertain system. It's transfer function matrix $P(p)$ is :

$$P(p) = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \\ p_{31} & p_{32} \end{pmatrix} \quad (63)$$

$$p_{ij}(p) = \frac{k_{ij}}{1+A_{ij}p} \quad (64)$$

12 cases of plant are given in table 1.

6.1 Pairing rules

From first to fourth plant conditions (table 1), $P(0)$ is :

$$P(0) = \begin{pmatrix} 1 & 0.5 \\ 1 & 2 \\ 0.7 & 5 \end{pmatrix} \quad (65)$$

So, the NRG matrix Λ^N is described by:

$$\Lambda^N = \begin{pmatrix} 0.7127 & -0.0645 \\ 0.4683 & -0.0554 \\ -0.1810 & 1.1199 \end{pmatrix} \quad (66)$$

The sum of elements of each row of Λ^N is:

$$RS = [0.6482\ 0.4129\ 0.9389]^T \quad (67)$$

Observing *RS* array (Eq.67), it is clear that the second output which corresponds to the minimum value of *RS* is the variable that to be eliminated. Then, the rest of Λ^N becomes $\Lambda^{N'}$:

$$\Lambda^{N'} = \begin{pmatrix} [0.7127] & -0.0645 \\ -0.1810 & [1.1199] \end{pmatrix} \quad (68)$$

According to the NRG matrix $\Lambda^{N'}$, the paired variables are $\{u_1/y_1;\ u_2/y_3\}$. The same technique is used for all other plants. From five to eight plant cases, the controller structure becomes $\{u_1/y_1;\ u_2/y_3\}$ and for the rest of plant cases the variables are paired according to this form $\{u_1/y_1;\ u_2/y_2\}$.
Multi-loop controllers are now to be found to control the square subsystem.

6.2 Performance specifications

6.2.1 Tracking specifications

The tracking tolerances must be achieved by $|T_{y/r-d} = t_{r_{ii}}|$(see (57)). The tracking specifications of the closed loop transfer function are enforced to be into the following upper and lower bounds:

$$T_{RU_{ii}}(p) = \frac{25(1+0.08p)(1+p/25)}{(p^2+7.5p+25)(1+0.002p)} \quad (69)$$

$$T_{RL_{ii}}(p) = \frac{96}{(p+1.5)(p+8)^2} \quad (70)$$

6.2.2 Controller specifications

The fourth plant is considered as the nominal case. Some specifications must be satisfied for all plant cases:

- For both outputs zero steady-state error
- Settling time as short as possible
- Robustness according to disturbances and parametric variations
- A first overshoot less than 10%.

Some elements of the open-loop transfer matrix can be initialized using these specifications.

6.2.3 Optimization

Respecting all specifications, the initial values for the parameters of the first fractional open-loop transfer function are:

- $\omega_r = 5.53434\ rad/s$
- $\omega_l = 12.0215\ rad/s$
- $\omega_h = 242.859\ rad/s$
- $\|\beta_{0_1}(j\omega)\|_{\omega=\omega_r} = 3.07508\ dB$
- $n_l = 1$
- $n_h = 2$

And for the second

- $\omega_r = 11.0606\ rad/s$
- $\omega_l = 2.29876\ rad/s$
- $\omega_h = 56.7086\ rad/s$
- $\|\beta_{0_2}(j\omega)\|_{\omega=\omega_r} = 2.26705\ dB$
- $n_l = 1$
- $n_h = 2$

Table 1 Different conditions of uncertain MIMO systems

N	k_{11}	k_{22}	k_{12}	k_{21}	k_{31}	k_{32}	A_{11}	A_{22}	A_{12}	A_{21}	A_{31}	A_{32}
1	1	2	0.5	1	0.7	5	2	2	4	4.5	2	2
2	1	2	0.5	1	0.7	5	0.5	1	1	3	3	2
3	1	2	0.5	1	0.7	5	0.2	0.4	0.5	2	1.5	2
4	1	2	0.5	1	0.7	5	0.7	0.8	0.3	1	2	2
5	4	5	1	2	3	6	1	2	2	4	4.5	2
6	4	5	1	2	3	6	0.5	1	1	3	3	2
7	4	5	1	2	3	6	0.2	0.4	0.5	2	1.5	3
8	4	5	1	2	3	6	0.7	0.8	0.3	1	2	3
9	10	8	2	4	1	7	1	2	2	4	4.5	3
10	10	8	2	4	1	7	0.5	1	1	3	3	3
11	10	8	2	4	1	7	0.2	0.4	0.5	2	1.5	3
12	10	8	2	4	1	7	0.7	0.8	0.3	1	2	3

Taking into account all specifications, the optimal values for the various parameters of open loop transfer function matrix are:

- For the first loop: $C_{h1}.C_{l1} = 0.739879$, $a_1 = 2.02427$, $b_1 = 0.515991$, $q_1 = 2$ and $C_1 = 1.1006$
- For the second loop: $C_{h2}.C_{l2} = 7.80658$, $a_2 = 1.23807$, $b_2 = -0.511694$, $q_2 = 1$ and $C_2 = 4.82348$

6.3 Controller design

Taking into account the desired specifications for the first loop and the model uncertainty of $(p_{11}^{*})^{-1}$, the CRONE control approach is used to find the first loop controller g_{11}:

$$g_{11} = \frac{1742.7881\,(p+85.2)(p+0.519)}{p(p+380)(p+10.4)} \tag{71}$$

Then, minimizing the coupling effect c_{21} the controller g_{21} can be designed (see (26)):

$$g_{21} = \frac{-108.4402\,(p+0.519)(p+0.5)}{p(p+10.4)(p+0.2222)} \tag{72}$$

Next, respecting the second loop controller specifications and using the CRONE control approach the controller g_{22} for the equivalent plant $(P_{22}^{*e})^{-1}$ is calculated:

$$[p_{22}^{*e}]_2 = [p_{22}^{*}]_1 - \frac{([p_{21}^{*}]_1+[g_{21}]_1)([p_{12}^{*}]_1+[g_{12}]_1)}{[p_{11}^{*}]_1+[g_{11}]_1} \tag{73}$$

now, the controller expression is :

$$g_{22} = \frac{365.2146\,(p+100)(p+1.42)(p+0.542)(p+0.13)}{p(p+90.7)(p+21.8)(p+1.97)(p+0.177)} \tag{74}$$

Finally, the controller g_{12} is calculated to minimize the coupling effect:

$$g_{12} = \frac{-91.3037\,(p+0.542)(p+0.13)}{p(p+0.5)(p+0.177)} \tag{75}$$

6.4 Prefilter synthesis

The first step consists of selecting the maximum and the minimum plants. The plants number one and eleven are respectively the minimum and the maximum plants. Secondly, the ratio $\frac{u_{max}}{e_{max}} = 1$ is fixed. The optimized parameters are obtained by minimizing the integral gap criterion (61) with $m = 2$ while respecting the frequency bound inequality (55) and the performance specification (56):

$$n_1 = 1.0353;\ \tau_1 = 0.31636 \tag{76}$$

$$n_2 = 1.1273;\ \tau_1 = 0.405$$

Using the module "Frequency Domain System Identification" of the CRONE software [26], the integer order approximation of these prefilters is determined :

$$F = \begin{pmatrix} F_{1DC} & 0 \\ 0 & F_{2DC} \end{pmatrix} \tag{77}$$

where

$$F_{1DC} = \frac{2.5657(p+107)(p+25.6)}{(p+87.74)(p+26.61)(p+3.01)}$$

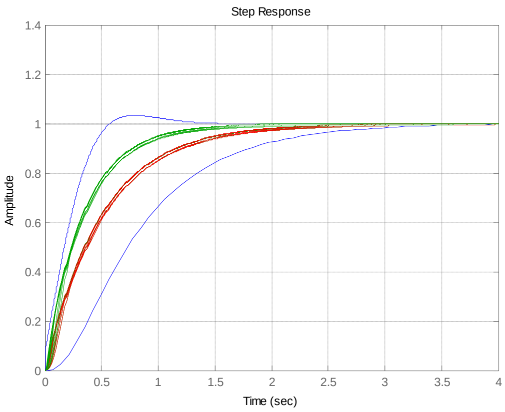
$$F_{2DC} = \frac{0.99433(p+539.9)(p+36.41)}{(p+361.1)(p+25.89)(p+2.09)}$$

Under all operating conditions, the resultant time domain closed-loop tracking responses are illustrated using fractional and classical prefilters F_{cl} in figure 7. The classical prefilter is described by the following expression:

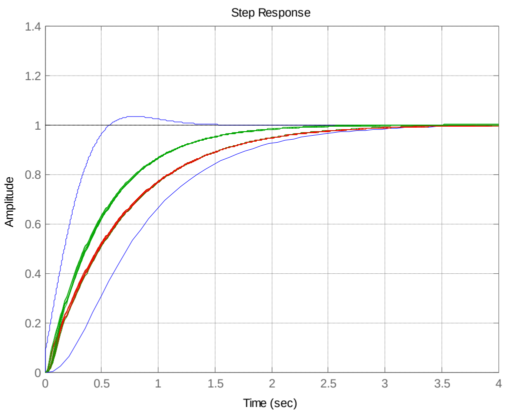
$$F_{cl}=\begin{pmatrix}F_{cl1} & 0\\ 0 & F_{cl2}\end{pmatrix} \qquad (78)$$

$$F_{cl1}=\frac{2}{p+2}$$

$$F_{cl2}=\frac{1.5}{p+1.5}$$



(a)



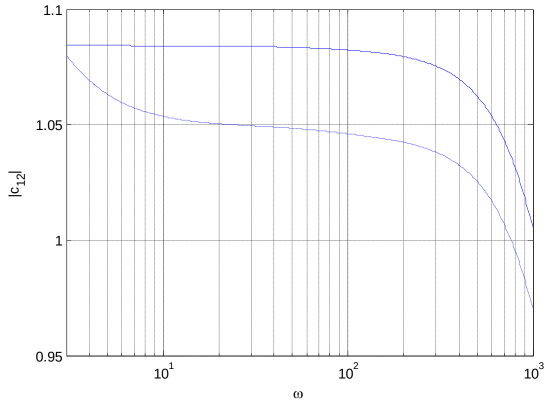
(b)

Fig 7 (a),(b) : Closed loop tracking response with classical (red) and fractional prefilters (green), tracking references (blue)

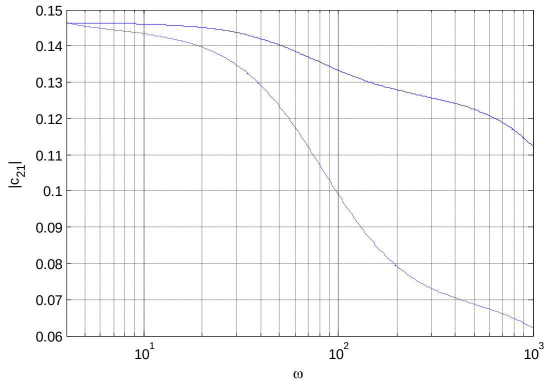
The closed loop tracking specifications are respected. All plant cases are under upper and lower bounds. The comparison between the two types of prefilters shows the benefit of using fractional prefilters.

The frequency plot of the obtained coupling reduction for the nominal plant is shown by figure 8.

A significant reduction in the coupling effect using the non diagonal controller is observed. The non diagonal controller is more flexible than the diagonal controller while governing a MIMO system.



(a)



(b)

Fig 8 (a) $c_{12_{g_{12}=0}}$ (solid line), $c_{12_{g_{12}=g_{12}^{opt}}}$ (dashed line) and (b) $c_{21_{g_{21}=0}}$ (solid line), $c_{21_{g_{21}=g_{21}^{opt}}}$ (dashed line)

7. CONCLUSION

Motion control by fractional prefilter was extended to non-square MIMO systems which is based on MIMO-QFT robust control design combined with CRONE control methodology. In terms of control structure selection, the NRG method has been used. After selection of the square subsystem the CRONE control methodology is used to find the robust controller. The QFT structure uses a non diagonal controller to solve the MIMO reference tracking problem. A coupling matrix has been defined to quantify the amount of loop interaction. The off-diagonal elements of the matrix controller can reduce the cross-coupling. The controller and fractional prefilter have been synthesized in order to put the transfer function of closed loop inside the specified bounds. Both parameters of fractional Davidson-Cole prefilter are optimized on the multiple SISO systems tracking into account the tracking specifications. Validation of this method is applied on a 2x3 MIMO system example.

Thus a prospect is envisaged which concerns the validation of this approach using a real process.

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