Performance Comparison between MPSK and MFSK in Rician Fading Channel based on MGF Method

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ABSTRACT
In this paper, analysis of symbol error probability (SEP) is carried over frequency non-selective slowly Rician fading channel with N branch receiver diversity using maximal ratio combining (MRC) technique. We assume that channel side information is known to the receiver. Symbol error rate expression of coherent M-ary phase shift keying (MPSK) and non coherent M-ary frequency shift keying (MFSK) are obtained through numerical computation based on moment generating function (MGF) approach in order to avoid complex numerical calculation. Error performance plots of these two modulation techniques has been drawn and compared for different values of Rician factor K, diversity order N and modulation order M.

Keywords
MPSK (M-ary phase shift keying), MFSK (M-ary frequency shift keying), MRC (maximal ratio combining), MGF (moment generating function), SEP (symbol error probability).

1. INTRODUCTION
In wireless communication due to high data rate requirement, there has been renewed interest in M-ary modulation because of their capability of sending multiple bits per transmitted symbol. Multipath fading is one of the boundary conditions of wireless communication and occurs due to multipath propagation, which can cause fluctuation in received signal's amplitude, phase and angle of arrival [1].Diversity is a technique which is based on the principle of providing multiple faded replica of same information bearing signal to the receiver [2].Several diversity techniques are used to combat the effect of fading and improve coverage capacity and reliability in wireless communication systems such as transmitter, receiver[3], space [4], frequency, polarization diversity [5]. Among all the conventional diversity combining scheme MRC is known to be optimum [6].

To describe the statistical behaviour of multipath fading, there are different models like Rayleigh, Rician and Nakagami-*m* and many more. Rician distribution used to model propagation paths consisting of one strong direct LOS component and many other weaker non-LOS components. Rician fading parameter K is measure of severity of fading, it can be define as the ratio of power in LOS component to power in other multipath component [7]. We include Rayleigh fading (when K=0) and AWGN (when K=∞, no fading) as special case. Probability density function (pdf) of received SNR explicitly includes the received signal statistics of all order, which helps to investigate error rate performance for given modulation scheme under specific channel model with or without diversity .Therefore in this paper pdf of received SNR with MRC diversity order under Rician fading channel is used to derive SEP with MGF approach . Although the SEP can be computed using direct numerical integration ,the MGF method perform better because direct integration has instability and inaccuracy due to presence of complex mathematical functions and infinite integration limits .Thus MGF approach save our computation time as well as its simple in numerical calculation time. Our result shows how these modulation techniques behave for different values of Rician fading parameter K, diversity order N and modulation order M.

2. PERFORMANCE MEASURE
2.1 Average Signal-to-Noise Ratio
Signal-to-noise ratio (SNR) is the most common performance measure characteristic of a digital communication system [8].

$$\bar{\gamma} = \int_0^\infty \gamma . p_\gamma(\gamma)\, d\gamma \quad \ldots\ldots\ldots\ldots\ldots (1)$$

Where γ denotes the instantaneous SNR at the receiver output and $p_\gamma(\gamma)$ denotes the probability density function (PDF) of γ.

2.2 Average Symbol Error Probability
This performance criterion average symbol error probability (SEP) is the one that is most revealing about the nature of the system behaviour. The primary reason for the difficulty in evaluating average SEP lies in the fact that the conditional SEP is, in general, a nonlinear function of the instantaneous SNR, the nature of the nonlinearity being a function of the modulation/detection scheme employed by the system. The average SEP can be written as [8]

$$P_S(E) = \int_0^\infty P_s\left(\frac{E}{\gamma}\right) . p_\gamma(\gamma)\, d\gamma \ldots\ldots (2)$$

Where, $P_s\left(\frac{E}{\gamma}\right)$ is conditional SEP.

3. METHOD
3.1 Moment generating function
The MGF for a non-negative random variable γ with distribution $p_\gamma(\gamma), \gamma \geq 0$, is defined as

$$M_\gamma(s) = \int_0^\infty p_\gamma(\gamma) e^{s\gamma}\, d\gamma \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Where $p_\gamma(\gamma)$ denotes the probability density function (PDF) of γ.

MGF for common Rician fading distribution with factor K and diversity order N is given by [10].

$$M_\gamma(s) = \left(\frac{N+K}{N+K-S\bar{\gamma}}\right)^N .exp\left(\frac{KS\bar{\gamma}}{N+K-S\bar{\gamma}}\right) \dots (4)$$

4. DIFFERENT M-ARY MODULATION TECHNIQUES

4.1 Error probability for MPSK

In MPSK, the phase of the carrier takes on one of possible values, $\theta_i = \frac{2(i-1)\pi}{M}$, where $i = 1,2 \dots\dots\dots M-1$, in each symbol interval T_S. The signal constellation of MPSK is two dimensional.

The conditional symbol error probability for MPSK is given by [9]

$$P_s\left(\frac{E}{\gamma}\right) = \frac{1}{\pi}\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}-\frac{\pi}{M}} exp\left[-\gamma sin^2\left(\frac{\pi}{M}\right).sec^2\theta\right] d\theta \dots (5)$$

After substitution of $P_s\left(\frac{E}{\gamma}\right)$ from (5) into (2) and by considering application of MGF from (3) we get

$$P_S(E) = \frac{1}{\pi}\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}-\frac{\pi}{M}} M_\gamma\left(-sin^2\left(\frac{\pi}{M}\right).sec^2\theta\right) d\theta \dots (6)$$

Using MGF relation for rician fading channel from (4)

$$M_\gamma\left(-sin^2\left(\frac{\pi}{M}\right).sec^2\theta\right)$$

$$= \left(\frac{N+K}{N+K+sin^2\left(\frac{\pi}{M}\right).sec^2\theta\bar{\gamma}}\right)^N .exp\left(-\frac{K.sin^2\left(\frac{\pi}{M}\right).sec^2\theta.\bar{\gamma}}{N+K+-sin^2\left(\frac{\pi}{M}\right).sec^2\theta\bar{\gamma}}\right) \dots (7)$$

Finally by putting (7) into (6), the symbol error probability of coherent MPSK over Rician fading channel with Rician parameter K is

$$P_S(E)$$

$$= \frac{1}{\pi}.\left(\frac{N+K}{\bar{\gamma}}\right)^N \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}-\frac{\pi}{M}} \frac{exp\left[\frac{-K.sin^2\left(\frac{\pi}{M}\right).sec^2\theta}{\frac{N+K}{\bar{\gamma}}+sin^2\left(\frac{\pi}{M}\right).sec^2\theta}\right]}{\left(\frac{N+K}{\bar{\gamma}}+sin^2\left(\frac{\pi}{M}\right).sec^2\theta\right)^N} d\theta \dots (8)$$

The symbol error probability for coherent MPSK over Rayleigh fading channel can be obtained by substitution of K=0 in (8), gives

$$P_S(E) = \frac{1}{\pi}\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}-\frac{\pi}{M}} \frac{1}{\left(1+\frac{\bar{\gamma}}{N}sin^2\left(\frac{\pi}{M}\right).sec^2\theta\right)^N} d\theta \dots (9)$$

When K approaches infinity, (8) reduce to (5)

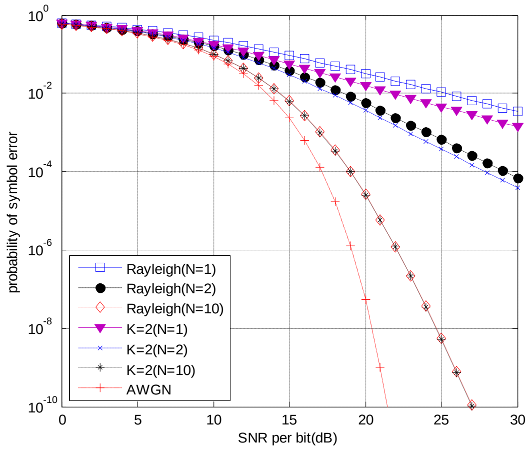

Fig.1: SEP for MPSK over Rician, Rayleigh and AWGN fading channel for different values of K and N and M=8

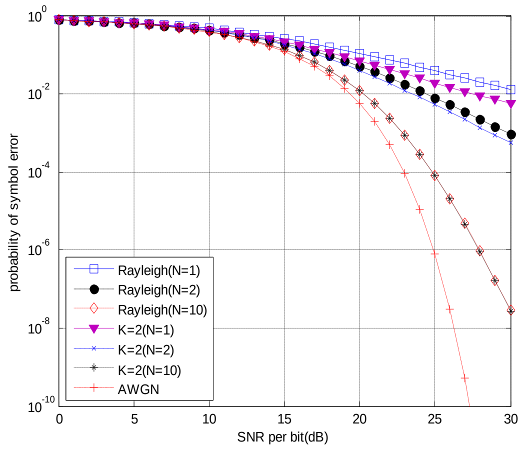

Fig.2: SEP for MPSK over Rician, Rayleigh and AWGN fading channel for different values of K and N and M=16

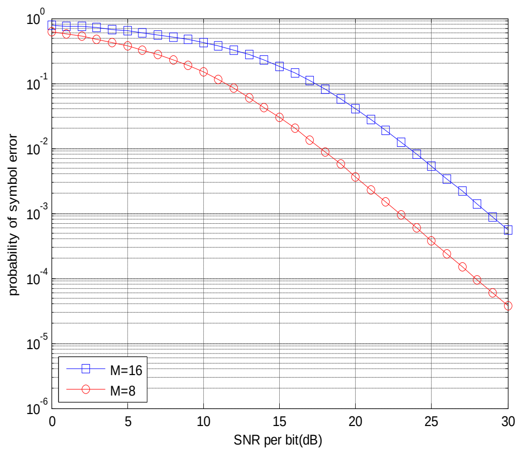

Fig.3: SEP for MPSK over Rician fading channel for different values of M and k=2, N=2

TABLE 1: SEP of MPSK for different values of K, M and N (SNR=20)

Values of K and N	symbol error probability	
	M=8	M=16
K=0 &N=1	0.0324	0.1099
K=0 &N=2	0.0056	0.0509
K=0 &N=10	2.6882e-05	0.0121
K=2 &N=1	0.0156	0.0695
K=2 &N=2	0.0036	0.0406
K=2 &N=10	2.5297e-06	0.0119
K=inf	5.2532e-08	0.0057

In Fig.1 and Fig.2 symbol error probability of MPSK over Rician, Rayleigh and AWGN fading channel is plotted for different values of K and N. And in Fig.3 comparison is made using M=8 & M=16 and for K=2 &N=2. The numerical values of the SEP are summarized in the table 1. From curves, we can say that the performance improves over the same SNR with increase in value of diversity order N while keeping Rician fading parameter K and modulation order M fixed. These curves provide information to estimate required transmitted energy to achieve certain SEP.

4.2 Error probability for MFSK

In an M-ary FSK scheme, the transmitted signals are defined by

$$s_i(t) = \sqrt{\frac{2E}{T}} cos2\pi(f_o + (i-1)\Delta f)t$$

Where f_0T is taken as an integer.

The conditional symbol error probability of MFSK is given by [9]

$$P_s\left(\frac{E}{\gamma}\right) = \frac{1}{M}\sum_{d=2}^{M} -1^d . \binom{M}{d} . exp\left(-\frac{(d-1)\bar{\gamma}}{d}\right) \dots (10)$$

After substitution of $P_s\left(\frac{E}{\gamma}\right)$ from (10) into (2) and by considering application of MGF from (3) we get

$$P_S(E) = \frac{1}{M}\sum_{d=2}^{M} -1^d . \binom{M}{d} . M_\gamma\left(-\frac{d-1}{d}\right) \dots (11)$$

Using MGF relation for rician fading channel from (4)

$$M_\gamma\left(-\frac{d-1}{d}\right) = \frac{1+K}{1+K+\left(\frac{d-1}{d}\right)\bar{\gamma}} . exp\left(-\frac{K\left(\frac{d-1}{d}\right)\bar{\gamma}}{1+K+\left(\frac{d-1}{d}\right)\bar{\gamma}}\right) \dots (12)$$

Finally by putting (12) into (11), the symbol error probability of coherent MDPSK over Rician fading channel with Rician parameter K is [10]

$$P_S(E) = \frac{1}{M}\sum_{d=2}^{M} -1^d . \binom{M}{d} . \left(\frac{N}{N+K+\left(\frac{d-1}{d}\right)\bar{\gamma}}\right)^N . exp\left(-\frac{K\left(\frac{d-1}{d}\right)}{\frac{N+K}{\bar{\gamma}}+\left(\frac{d-1}{d}\right)}\right) \dots (13)$$

The symbol error probability for coherent MDPSK over Rayleigh fading channel can be obtained by substitution of K=0 in (13), gives

$$P_S(E) = \frac{1}{M}\sum_{d=2}^{M} -1^d . \binom{M}{d} . \left(\frac{1}{1+\left(\frac{d-1}{d}\right)\bar{\gamma}}\right) \dots (14)$$

When K approaches infinity, (13) reduce to (10)

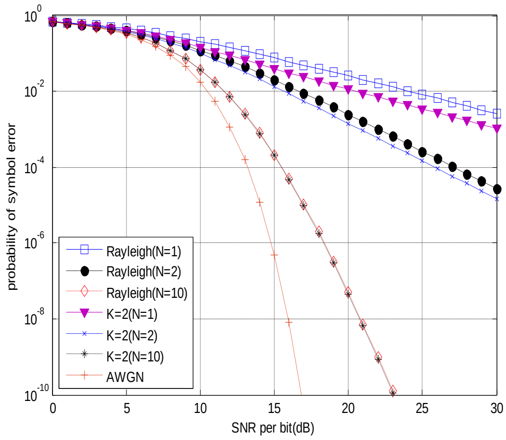

Fig.4: SEP for MFSK over Rician, Rayleigh and AWGN fading channel for different values of K and N and M=8

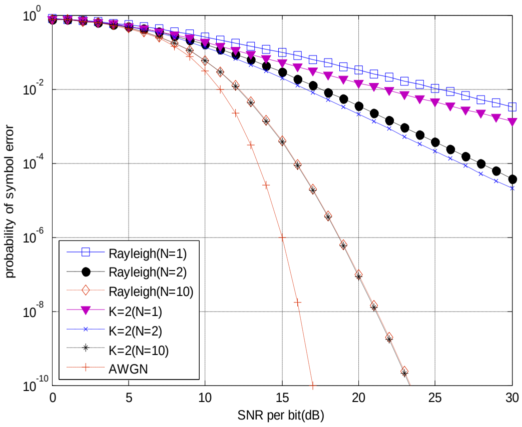

Fig.5: SEP for MFSK over Rician, Rayleigh and AWGN fading channel for different values of K and N and M=16

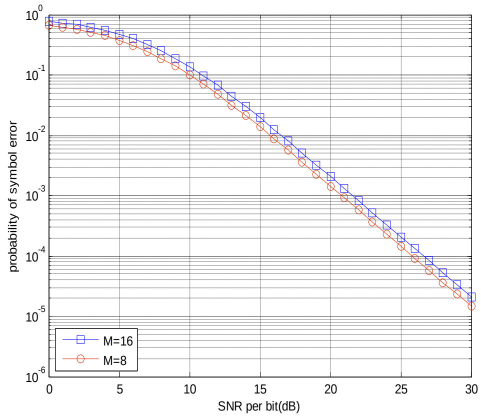

Fig.6: SEP for MFSK over Rician fading channel for different values of M and K=2, N=2

TABLE 2: SEP of MFSK for different values of K, M and N (SNR=20)

Values of K and N	symbol error probability	
	M=8	**M=16**
K=0 &N=1	0.0253	0.0322
K=0 &N=2	0.0024	0.0035
K=0 &N=10	5.0902e-08	9.8735e-08
K=2 &N=1	0.0113	0.0146
K=2 &N=2	0.0014	0.0021
K=2 &N=10	4.5318e-08	8.7985e-08
K=inf	6.7506e-22	1.4466e-21

In Fig.4 and Fig.5 symbol error probability of MFSK over Rician, Rayleigh and AWGN fading channel is plotted for different values of K and N. In Fig.6 comparison is made using M=8 & M=16 and for K=2 &N=2. The numerical values of the SEP are summarized in the table 2. From curves, we can say that the performance improves over the same SNR with increase in value of N while keeping K and M fixed. These curves provide information to estimate required transmitted energy to achieve certain SEP.

5. RESULTS AND DISCUSSION

In fig 7-11 we show performance comparison for MPSK and MFSK is made under Rician fading with MRC diversity combining for different values of Rician fading parameter K, diversity order N and modulation order M.

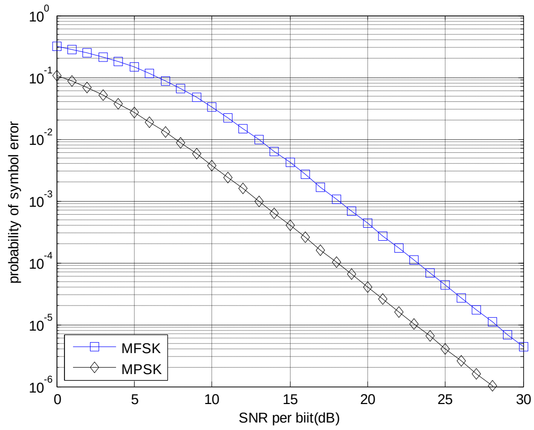

Fig.7: SEP comparison of BFSK and BPSK for M=2, K=2 and N=2

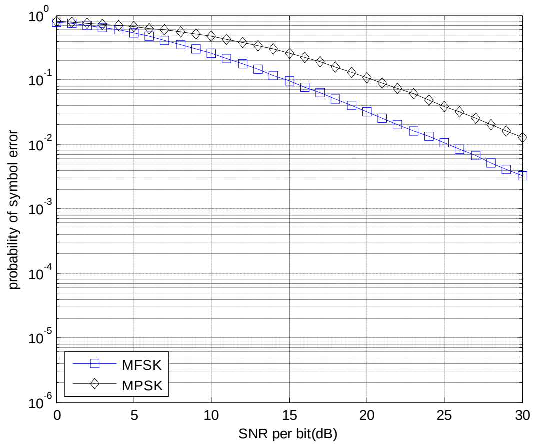

Fig.8: SEP comparison of MFSK and MPSK for M=16, K=0 and N=1

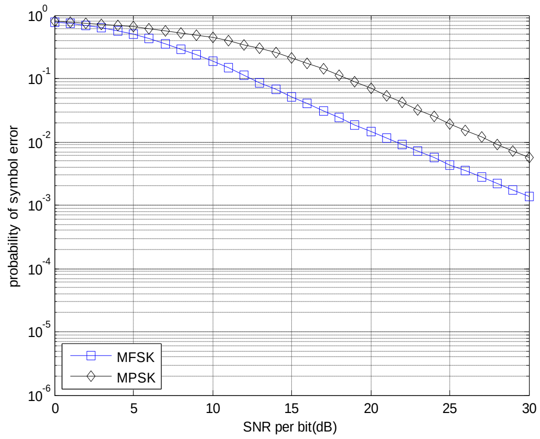

Fig.9: SEP comparison of MFSK and MPSK for M=16, K=2 and N=1

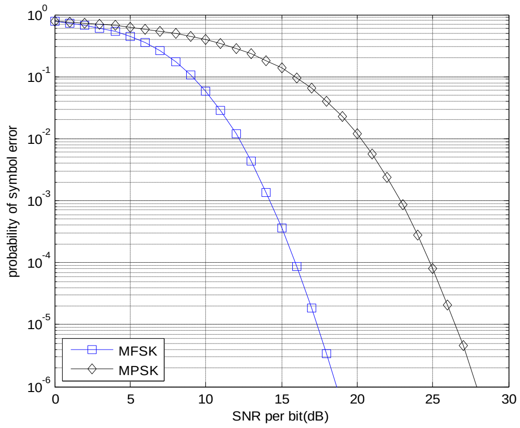

Fig.10: SEP comparison of MFSK and MPSK for M=16, K=2 and N=10

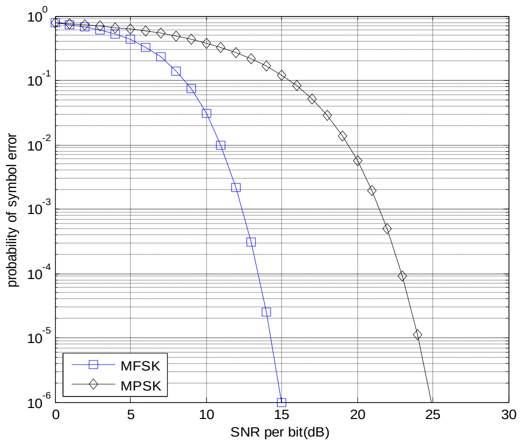

Fig.11: SEP comparison of MFSK and MPSK for M=16, K=Inf

TABLE 3: Comparison of SEP for MFSK and MPSK for different values of K, M and N (SNR=20)

Values of K & N	MFSK		MPSK	
	M=8	M=16	M=8	M=16
K=0 & N=1	0.0256	0.0322	0.0324	0.1099
K=0 & N=10	5.0902 e-08	9.8735 e-08	2.668 e-05	0.0121
K=2 & N=1	0.0113	0.0146	0.0156	0.0695
K=2 & N=10	4.5318 e-08	8.7985 e-08	2.529 e-06	0.0119
K=inf	6.7506 e-22	1.4466 e-21	5.253 e-08	0.0057

In case of BPSK and BFSK ,error performance of BPSK is better than BFSK for M=2, but for higher values of M say 8 & 16 performance of MFSK is better than MPSK because in MFSK modulation order is inversely proportional to SEP so as the value of M increases SEP decreases more sharply than the MPSK case .

Plots show that symbol error probability rises with increase in modulation order M by keeping other parameter fixed and SEP decreases as values of K and N increases by keeping other parameters fixed.

6. CONCLUSION

In this paper Error performance, based on MGF approach (which is better in terms of applicability), of two modulation techniques under Rician fading channel with MRC diversity combining are analyzed and SEP is calculated. Based on numerical calculation SEP of non-coherent MFSK and MPSK is graphically plotted and compared. Many of these results can be extended for other cases of diversity reception to combat fading effects.

7. REFERENCES

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